Discrete-Time Fourier Transform (DTFT)

1 Preliminaries

- **Definition:** The Discrete-Time Fourier Transform (DTFT) of a signal $x[n]$ is defined to be

$$X(e^{j\omega}) \;=\; \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}. \tag{1}$$

  In other words, the DTFT of $x[n]$ is the z-transform $X(z)$ evaluated at $z = e^{j\omega}$. Noting that $|e^{j\omega}| = 1$, the DTFT of $x[n]$ is $X(z)$ evaluated on the unit circle (the circle with center at the origin and radius 1).

- **Motivation from redundancy in z-transform:** We have seen earlier that the inverse z-transform can be computed using a contour integral using any contour Γ around the origin lying in the ROC. This means that there is a lot of redundancy in the z-transform, i.e., given the values of $X(z)$ on the contour Γ, we can find $x[n]$ and hence the values of $X(z)$ everywhere in the complex plane. So, we only need to consider the values of $X(z)$ evaluated on the contour Γ to adequately represent the signal $x[n]$. Γ can be any contour enclosing the origin and lying in the ROC. A particularly convenient contour is the unit circle since it forms the boundary between stability and instability (recall that a system is BIBO stable if and only if the poles of the system lie within the unit circle; equivalently, a system with impulse response $h[n]$ is BIBO stable if and only if the poles of $H(z)$ lie within the unit circle). Taking Γ to be the unit circle, the evaluation of $X(z)$ on the unit circle gives the DTFT of $x[n]$.

- **Motivation from response of LTI systems to sinusoidal input signals:** Consider any given FIR system. Recall that any FIR system is of the form

$$y[n] \;=\; \sum_{k=-M_1}^{M_2} h[k]x[n-k] \tag{2}$$

  with $h[n]$ being the impulse response. Consider the case where the input signal $x[n]$ is sinusoidal. The general complex-valued sinusoidal signal is of the form $x[n] = Ae^{j\phi}e^{j\omega n}$. A is called the magnitude, ϕ is called the phase, and ω is called the frequency of the sinusoidal signal $x[n]$. From (2), the output signal $y[n]$ is obtained to be

$$\begin{aligned} y[n] &= \sum_{k=-M_1}^{M_2} h[k]Ae^{j\phi}e^{j\omega(n-k)} \\ &= Ae^{j\phi}e^{j\omega n}\sum_{k=-M_1}^{M_2} h[k]e^{-j\omega k} \\ &= Ae^{j\phi}e^{j\omega n}H(e^{j\omega}) \end{aligned} \tag{3}$$

where the last step in (3) is obtained by noting that $\sum_{k=-M_1}^{M_2} h[k]e^{-j\omega k} = H(e^{j\omega})$. Hence,

$$y[n] = A|H(e^{j\omega})|e^{j\phi + j\angle H(e^{j\omega})}e^{j\omega n}, \tag{4}$$

i.e., the output signal $y[n]$ is the same as the input signal $x[n]$ except for a magnitude gain of $|H(e^{j\omega})|$ and a phase shift of $\angle H(e^{j\omega})$. Note that the frequency of the output signal is the same as the frequency of the input signal. Thus, $H(e^{j\omega})$ represents, in a sense, the response of the system to a sinusoidal input signal of frequency ω. Hence, $H(e^{j\omega})$ is called the *frequency response* of the system, $|H(e^{j\omega})|$ is referred to as the *magnitude response* of the system, and $\angle H(e^{j\omega})$ is referred to as the *phase response* of the system.

We have shown above that the output signal of any FIR LTI system given a sinusoidal input signal is a *scaled* and *shifted* sinusoidal signal of the same frequency as the input signal. We will see later that this is true, in the sense of steady-state response, for any BIBO stable (both FIR and IIR) LTI system. Thus, $H(e^{j\omega})$ appears naturally when we seek to compute the output signal corresponding to a sinusoidal input signal.

2 Basic properties of the DTFT

- 2π **Periodicity:** $X(e^{j(\omega+2\pi)}) = X(e^{j\omega})$ valid for any signal $x[n]$.

Proof: By definition, $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$. Since $e^{j\theta} = \cos(\theta) + j\sin(\theta)$, we have

$$\begin{aligned} e^{-j(\omega+2\pi)n} &= \cos((\omega+2\pi)n) - j\sin((\omega+2\pi)n) \\ &= \cos(\omega n) - j\sin(\omega n) \\ &= e^{-j\omega n}. \end{aligned} \tag{5}$$

Hence,

$$\begin{aligned} X(e^{j(\omega+2\pi)}) &= \sum_{n=-\infty}^{\infty} x[n]e^{-j(\omega+2\pi)n} \\ &= \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n} \\ &= X(e^{j\omega}). \end{aligned} \tag{6}$$

- **Conjugate Symmetry, i.e.,** $X(e^{-j\omega}) = X^*(e^{j\omega})$ valid for any real signal $x[n]$.

  Proof: By definition, $X(e^{-j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{j\omega n}$. Also, if $x[n]$ is real,

$$\begin{aligned} X^*(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x^*[n](e^{-j\omega n})^* \\ &= \sum_{n=-\infty}^{\infty} x[n]e^{j\omega n}. \end{aligned} \tag{7}$$

  Hence, $X(e^{-j\omega}) = X^*(e^{j\omega})$ for any real signal $x[n]$.

- **Convolution in time domain is equivalent to multiplication in frequency domain:** If the DTFT of $x_1[n]$ is $X_1(e^{j\omega})$ and the DTFT of $x_2[n]$ is $X_2(e^{j\omega})$, then the DTFT of $x_1[n] * x_2[n]$ is $X_1(e^{j\omega})X_2(e^{j\omega})$.

  Proof: We know that if the z-transform of $x_1[n]$ is $X_1(z)$ and the z-transform of $x_2[n]$ is $X_2(z)$, then the z-transform of $x_1[n] * x_2[n]$ is $X_1(z)X_2(z)$. Evaluating at $z = e^{j\omega}$, it follows that the DTFT of $x_1[n] * x_2[n]$ is $X_1(e^{j\omega})X_2(e^{j\omega})$.

3 Response of LTI systems to sinusoidal input signals

We saw earlier that if the input signal to an FIR LTI system is a sinusoidal signal, then the output signal is a scaled and shifted copy of the input signal. This can be generalized, in the sense of steady-state response, for any BIBO stable LTI system. The general form of an IIR LTI system is known to be

$$y[n] = \sum_{l=1}^{N} a_l y[n-l] + \sum_{k=0}^{M} b_k x[n-k]. \tag{8}$$

Let the given initial conditions be some $y[-1], \ldots, y[-N]$ and let the input signal be a sinusoid $x[n] = Ae^{j\phi}e^{j\omega n}$ for $n \geq 0$ and 0 for $n < 0$, i.e., $x[n] = Ae^{j\phi}e^{j\omega n}u[n]$. Then one method to compute the output signal is by using the one-sided z-transform. This method gives, as seen in the lecture notes on difference equations,

$$\begin{aligned} Y(z) &= \underbrace{\frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{l=1}^{N} a_l z^{-l}} X(z)}_{\text{Forced response}} \\ &\quad + \underbrace{\frac{\sum_{l=1}^{N} a_l z^{-l} \sum_{n_1=-l}^{-1} y[n_1] z^{-n_1}}{1 - \sum_{l=1}^{N} a_l z^{-l}}}_{\text{Homogeneous response}}. \end{aligned} \tag{9}$$

If the IIR system is BIBO stable, then the roots of the N^{th} order equation $1 - \sum_{l=1}^{N} a_l z^{-l} = 0$ are all within the unit circle so that the inverse z-transform of the homogeneous response will consist of components that go to 0 as $n \to \infty$ (this can be easily seen by using the method of guessing exponentials). Hence, in steady-state (i.e., as $n \to \infty$), the output signal is simply the forced response. This is intuitively obvious since we expect the *transients* caused due to initial conditions to die out if the system is stable. To find the inverse z-transform of the homogeneous response, we can use a partial fraction expansion which will contain terms corresponding to the poles of the system and terms corresponding to poles introduced by the input signal. Note that if the input signal is $x[n] = Ae^{j\phi}e^{j\omega n}u[n]$, then $X(z) = Ae^{j\phi}\frac{1}{1-e^{j\omega}z^{-1}}$ so that $X(z)$ has a pole at $e^{j\omega}$. If $H(z)$ does not have a pole at $e^{j\omega}$, then $Y(z)$ has a pole of multiplicity 1 at $e^{j\omega}$. In this case, it can be shown that the partial fraction expansion of $Y(z)$ includes the term $\frac{1}{1-e^{j\omega}z^{-1}}$ with coefficient $Ae^{j\phi}H(e^{j\omega})$, i.e.,

$$Y(z) \;=\; \Big[\text{Partial fraction components arising from poles of } H(z)\Big] + \frac{Ae^{j\phi}H(e^{j\omega})}{1-e^{j\omega}z^{-1}}.$$

As discussed above, since the system is stable, the poles of $H(z)$ are within the unit circle so that the partial fraction components arising from poles of $H(z)$ yield terms that go to 0 as $n \to \infty$. Hence, the steady-state response of the system is simply the inverse z-transform of $\frac{Ae^{j\phi}H(e^{j\omega})}{1-e^{j\omega}z^{-1}}$, i.e., in steady state,

$$y[n] \;=\; A|H(e^{j\omega})|e^{j\phi+j\angle H(e^{j\omega})}e^{j\omega n}. \tag{10}$$

This result matches what we saw earlier in the case of FIR LTI systems.

Above, we assumed that $e^{j\omega}$ is not a pole of $H(z)$. This is definitely true since $|e^{j\omega}| = 1$. Remember that we are considering stable IIR LTI systems so that the poles of $H(z)$ are all within the unit circle.

The above analysis yields the general result:

> For any BIBO stable LTI system, if the input signal is sinusoidal, then, at steady-state, the output signal is a scaled (with magnitude gain $|H(e^{j\omega})|$) and shifted (with phase shift $\angle H(e^{j\omega})$) copy of the input signal.

Now that we know how to compute the output of an LTI system given a sinusoidal input signal, we can easily generalize the technique to a sum of sinusoidal signals by appealing to superposition, i.e., if the input signal is formed as the sum of a set of sinusoidal signals

$$x[n] \;=\; \sum_{i=1}^{L} A_i e^{j\phi_i} e^{j\omega_i n}, \tag{11}$$

then the output signal is the sum of the corresponding output signals to each of the constituent sinusoidal signals:

$$y[n] \;=\; \sum_{i=1}^{L} A_i |H(e^{j\omega_i})| e^{j\phi_i + j\angle H(e^{j\omega_i})} e^{j\omega_i n}. \tag{12}$$

Zeros and poles: If $e^{j\omega_0}$ is a zero of the system, i.e., if $H(e^{j\omega_0}) = 0$, then the steady-state response of the system given a sinusoidal input signal of frequency ω_0 is zero. This is the motivation for the usage of the name *zero* for values of z for which $H(z)$ is zero.

Example 3.1 Consider the system $y[n] = x[n] - x[n-1]$. Given the input signal $e^{j\omega n}$, find the output signal: For the given system, we have $H(z) = 1 - z^{-1}$ and $H(e^{j\omega}) = 1 - e^{-j\omega} = 1 - \cos(\omega) + j\sin(\omega)$. Hence,

$$\begin{aligned} |H(e^{j\omega})| &= \sqrt{(1-\cos(\omega))^2 + \sin^2(\omega)} \\ \angle H(e^{j\omega}) &= \tan^{-1}\left(\frac{\sin(\omega)}{1-\cos(\omega)}\right) \end{aligned} \tag{13}$$

and the output signal is $|H(e^{j\omega})|e^{j\angle H(e^{j\omega})}e^{j\omega n}$. If $\omega = 0$, then $|H(e^{j\omega})|$ is zero so that the output signal is zero. This can also be seen from the facts that the zero of the system is $z = 1$ and that $e^{j0} = 1$. In time domain, what this means is that if the input signal is a sinusoid with frequency zero (i.e., is a constant), then the output signal is zero.

Example 3.2 Consider the system $y[n] = x[n] + x[n-1]$. Find the output signal if the input signal is $x[n] = e^{j\pi n} + e^{j\frac{\pi}{2}n}$: For this system, we have $H(z) = 1 + z^{-1}$ and $H(e^{j\omega}) = 1 + e^{-j\omega} = 1 + \cos(\omega) - j\sin(\omega)$. The system has a zero at $z = -1$, i.e., at $\omega = \pi$. Hence, we expect that the output due to the input component $e^{j\pi n}$ is zero. This can also be verified in time domain by noting that $e^{j\pi n} = (-1)^n$ so that $x[n] + x[n-1] = 0$ for all n. Therefore, the output signal given the input signal $x[n] = e^{j\pi n} + e^{j\frac{\pi}{2}n}$ is $|H(e^{j\frac{\pi}{2}})|e^{j\angle H(e^{j\frac{\pi}{2}})}e^{j\frac{\pi}{2}n}$. In this case, $H(e^{j\frac{\pi}{2}}) = 1 - j$ so that $|H(e^{j\frac{\pi}{2}})| = \sqrt{2}$ and $\angle H(e^{j\frac{\pi}{2}}) = -\frac{\pi}{4}$. Hence, the output signal is $y[n] = \sqrt{2}e^{-j\frac{\pi}{4}}e^{j\frac{\pi}{2}n}$.

Example 3.3 Consider the system $y[n] = -0.5y[n-1] + x[n]$. The transfer function of this system is $H(z) = \frac{1}{1+0.5z^{-1}}$ and the frequency response of the system is $H(e^{j\omega}) = \frac{1}{1+0.5e^{-j\omega}}$. Note that this system is BIBO stable. Given a sinusoidal input signal $Ae^{j\phi}e^{j\omega n}u[n]$, the output signal at steady-state is given by $A|H(e^{j\omega})|e^{j\phi + j\angle H(e^{j\omega})}e^{j\omega n}$.

Example 3.4 Consider the system $y[n] = -0.1y[n-1] + x[n]$. Find the output signal at steady-state if the input signal is $x[n] = \cos(\pi n)u[n]$: Using the fact that

$$\cos(\pi n) = \frac{e^{j\pi n} + e^{-j\pi n}}{2}, \tag{14}$$

the output signal can be written as the sum of the responses to the input signals $0.5e^{j\pi n}$ and $0.5e^{-j\pi n}$.

Example 3.5 The system $y[n] = x[n - n_0]$ with n_0 being a constant represents a delay by n_0 samples. If the input signal is a sinusoid, then we expect that the output signal is also a sinusoid with the same magnitude as the input signal but with a phase shift relative to the input signal. This can be verified by noting that $H(e^{j\omega}) = e^{-j\omega n_0}$ so that $|H(e^{j\omega})| = 1$ and $\angle H(e^{j\omega}) = -\omega n_0$. In this example, the phase shift $\angle H(e^{j\omega})$ is a linear function of ω. Such a system is called a *linear phase* system.

4 Inverse DTFT

The inverse DTFT problem is to find $x[n]$ given its DTFT $X(e^{j\omega})$. As with the inverse z-transform, there are many ways to find the inverse DTFT. Two methods are explained below. Of these, the first method which uses the inverse z-transform to evaluate the inverse DTFT is easier to apply.

4.1 Inverse DTFT: Using $z = e^{j\omega}$

This method simply involves substituting z whereever we see $e^{j\omega}$ and then taking the inverse z-transform with a ROC including the unit circle.

Example 4.1 Given $H(e^{j\omega}) = \frac{1-e^{-j\omega}}{1+0.5e^{-j\omega}}$, find $h[n]$: Substituting z for $e^{j\omega}$, we find that

$$H(z) = \frac{1 - z^{-1}}{1 + 0.5z^{-1}} = -2 + \frac{3}{1 + 0.5z^{-1}}. \tag{15}$$

Hence, $h[n] = -2\delta[n] + 3(-0.5)^n u[n]$.

Example 4.2 Given $H(e^{j\omega}) = \cos(j\omega)$, find $h[n]$: Using the fact that

$$\cos(j\omega) = \frac{e^{j\omega} + e^{-j\omega}}{2}, \tag{16}$$

we find that $H(z) = \frac{z+z^{-1}}{2}$ so that

$$h[n] = \frac{\delta[n+1] + \delta[n-1]}{2}. \tag{17}$$

4.2 Inverse DTFT: Using integration

By definition, $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$. Multiplying both sides by $e^{j\omega k}$ with some constant k and integrating over a 2π interval yields:

$$\begin{aligned}\int_0^{2\pi} X(e^{j\omega})e^{j\omega k}d\omega &= \int_0^{2\pi} \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}e^{j\omega k}d\omega \\ &= \sum_{n=-\infty}^{\infty} x[n] \int_0^{2\pi} e^{j\omega(k-n)}d\omega \end{aligned} \tag{18}$$

The following result is easy to prove:

$$\int_0^{2\pi} e^{j\omega m}d\omega = \begin{cases} 2\pi \text{ if } m = 0 \\ 0 \text{ if } m \neq 0. \end{cases} \tag{19}$$

Hence, in the right hand side of (18), only one term in the summation is non-zero. The non-zero term is the one for which $(k-n) = 0$, i.e., $n = k$. Hence,

$$\int_0^{2\pi} X(e^{j\omega})e^{j\omega k}d\omega = 2\pi x[k] \tag{20}$$

so that

$$x[k] = \frac{1}{2\pi} \int_0^{2\pi} X(e^{j\omega})e^{j\omega k}d\omega. \tag{21}$$

5 Plotting the DTFT: The Bode Plot

A convenient pictorial representation of the DTFT $H(e^{j\omega})$ is obtained by plotting $|H(e^{j\omega})|$ and $\angle H(e^{j\omega})$. Such a pair of plots is called the Bode plot and it consists of the magnitude plot (plot of $|H(e^{j\omega})|$ vs. ω) and the phase plot (plot of $\angle H(e^{j\omega})$ vs. ω).

6 More Properties of the DTFT

- If the DTFT of $x[n]$ is $X(e^{j\omega})$, then the DTFT of $x^*[n]$ is $X^*(e^{-j\omega})$.

  Proof: Let the DTFT of $x^*[n]$ be denoted by $X_a(e^{j\omega})$. Then, by definition,

$$\begin{aligned} X_a(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x^*[n]e^{-j\omega n} \\ &= \sum_{n=-\infty}^{\infty} x^*[n](e^{j\omega n})^* \\ &= \sum_{n=-\infty}^{\infty} (x[n]e^{j\omega n})^* \\ &= (\sum_{n=-\infty}^{\infty} x[n]e^{-j(-\omega)n})^* \\ &= X^*(e^{-j\omega}). \end{aligned} \tag{22}$$

  Hence, the DTFT of $x^*[n]$ is $X^*(e^{-j\omega})$.

- If the DTFT of $x[n]$ is $X(e^{j\omega})$, then the DTFT of $x^*[-n]$ is $X^*(e^{j\omega})$.

  Proof: Let the DTFT of $x^*[-n]$ be denoted by $X_a(e^{j\omega})$. Then, by definition,

$$\begin{aligned} X_a(e^{j\omega}) &= \sum_{n=-\infty}^{\infty} x^*[-n]e^{-j\omega n} \\ &= \sum_{n=-\infty}^{\infty} (x[-n]e^{j\omega n})^* \\ &= (\sum_{n=-\infty}^{\infty} x[-n]e^{j\omega n})^* \\ &= (\sum_{n=-\infty}^{\infty} x[-n]e^{-j\omega(-n)})^* \\ &= (\sum_{n_1=-\infty}^{\infty} x[n_1]e^{-j\omega n_1})^* \end{aligned} \tag{23}$$

  where $n_1 = -n$. Hence, $X_a(e^{j\omega}) = X^*(e^{j\omega})$, i.e., the DTFT of $x^*[-n]$ is $X^*(e^{j\omega})$.

- **Multiplication in time domain is equivalent to convolution in frequency domain.** If the DTFT of $x_1[n]$ is $X_1(e^{j\omega})$ and the DTFT of $x_2[n]$ is $X_2(e^{j\omega})$, then the DTFT of $x_1[n]x_2[n]$ is $\frac{1}{2\pi}\int_0^{2\pi} X_1(e^{j\tilde{\omega}})X_2(e^{j(\omega-\tilde{\omega})})d\tilde{\omega}$.

  Remark: The signal $x_1[n]x_2[n]$ is a pointwise multiplication of the signals $x_1[n]$ and $x_2[n]$.

  Remark: Convolution of two functions $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ in the frequency domain is defined to be $\frac{1}{2\pi}\int_0^{2\pi} X_1(e^{j\tilde{\omega}})X_2(e^{j(\omega-\tilde{\omega})})d\tilde{\omega}$. Note the similarity of this definition of convolution in frequency domain with the definition of convolution in time domain (essentially, summation is replaced by integration, and there is an additional $\frac{1}{2\pi}$ factor).

  Proof: By definition,

$$
\begin{aligned}
X_1(e^{j\tilde{\omega}}) &= \sum_{n_1=-\infty}^{\infty} x_1[n_1]e^{-j\tilde{\omega}n_1} \\
X_2(e^{j(\omega-\tilde{\omega})}) &= \sum_{n_2=-\infty}^{\infty} x_2[n_2]e^{-j(\omega-\tilde{\omega})n_2}.
\end{aligned} \tag{24}
$$

  Hence,

$$
\begin{aligned}
X_1(e^{j\tilde{\omega}})X_2(e^{j(\omega-\tilde{\omega})}) &= \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} x_1[n_1]x_2[n_2]e^{-j\tilde{\omega}n_1}e^{-j(\omega-\tilde{\omega})n_2} \\
&= \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} x_1[n_1]x_2[n_2]e^{-j\omega n_2}e^{-j\tilde{\omega}(n_1-n_2)}.
\end{aligned} \tag{25}
$$

  Therefore,

$$
\frac{1}{2\pi}\int_0^{2\pi} X_1(e^{j\tilde{\omega}})X_2(e^{j(\omega-\tilde{\omega})})d\tilde{\omega} = \sum_{n_1=-\infty}^{\infty}\sum_{n_2=-\infty}^{\infty} x_1[n_1]x_2[n_2]e^{-j\omega n_2}\frac{1}{2\pi}\int_0^{2\pi} e^{-j\tilde{\omega}(n_1-n_2)}d\tilde{\omega}.
$$

  Using (19), only the term for which $n_1 = n_2$ is nonzero. Hence,

$$
\frac{1}{2\pi}\int_0^{2\pi} X_1(e^{j\tilde{\omega}})X_2(e^{j(\omega-\tilde{\omega})})d\tilde{\omega} = \sum_{n_1=-\infty}^{\infty} x_1[n_1]x_2[n_1]e^{-j\omega n_1}. \tag{26}
$$

  By definition, the DTFT of $x_1[n]x_2[n]$ is $\sum_{n_1=-\infty}^{\infty} x_1[n_1]x_2[n_1]e^{-j\omega n_1}$. Hence, the DTFT of $x_1[n]x_2[n]$ is equal to $\frac{1}{2\pi}\int_0^{2\pi} X_1(e^{j\tilde{\omega}})X_2(e^{j(\omega-\tilde{\omega})})d\tilde{\omega}$.

- **Parseval's Theorem:** If the DTFT of $x[n]$ is $X(e^{j\omega})$, then $\sum_{n=-\infty}^{\infty} x[n]x^*[n] = \frac{1}{2\pi}\int_0^{2\pi} X(e^{j\omega})X^*(e^{j\omega})d\omega$.

  Proof: Consider (26) in the special case $\omega = 0$. We get

  $$\frac{1}{2\pi}\int_0^{2\pi} X_1(e^{j\tilde{\omega}})X_2(e^{-j\tilde{\omega}})d\tilde{\omega} \quad = \quad \sum_{n_1=-\infty}^{\infty} x_1[n_1]x_2[n_1] \qquad (27)$$

  which yields the statement of the Parseval's theorem by taking the signal $x_2[n]$ to be $x_1^*[n]$.

  Remark: Recall that, for instance, the power dissipation in a resistor is dependent on the square of the current or the voltage. In general, the square of a signal is associated with power and the summation (or integration) of the square of a signal is associated with energy of the signal. Parseval's theorem essentially states that the energy in the time-domain signal is equal to the energy in the frequency-domain DTFT.

General Note: Since the DTFT is periodic with period 2π, we can use any interval of length 2π to characterize the DTFT. Throughout this lecture, we have used $[0, 2\pi]$ as the interval (in all integrations, etc.). Equivalently, we could have used $[-\pi, \pi]$ or any other interval of length 2π.