Lecture 8

Internal Rate of Return
Judging proposed investments

- IRR gets more complicated when comparing multiple alternatives
  - (Rather than evaluating a single project)

Why?
- Desirability depends on both
  - IRR
  - size of initial investment
Example

Consider two alternatives:

- Invest $1 at an IRR of 100%
- Invest $1,000,000 at an IRR of 20%

Which investment would you prefer?
Example

Consider two alternatives:

- Invest $1 at an IRR of 100%
- Invest $1,000,000 at an IRR of 20%

The more expensive project has:

- Smaller IRR

but

- Larger present worth!
Judging proposed investments

- If you are going to pick only one alternative from several,
  - Need to compare them **against each other**!
    - (based on **differences** in cost)
  - not only against the base rate of return $i^*$
- Need to evaluate each **incremental** investment to see if it is worthwhile
Example

- Compare options A and B:
  - A: First cost = $1420
    - Annual benefit = $256/year for 40 years
    - Rate of return = 18%
  - B: First cost = $1684
    - Annual benefit = $300/year for 40 years
    - Rate of return = 17.8%

- You can only do one of these!
Example

- Option B has:
  - Slightly lower rate of return,
  - but
    - Higher initial investment
- Present worth of benefit may be greater than option A!
Example

- Need to evaluate the incremental investment to see if it is worthwhile:
  - Delta first cost = $1684 - 1420 = $264
  - Delta annual benefit = $300 - 256 = $44 (for 40 years)
  - Rate of return = 16.6%

- Is option B worthwhile?
  - (Depends on i*)
Example

- Option A has IRR 18%, first cost $1420
  - (B - A) has IRR 16.6%, first cost $264
- If i* = 15%, then:
  - Option A is worthwhile
  - The delta for option B is also worthwhile
- If i* = 17%, then:
  - Option A is worthwhile, but not B
Example

- Option A has IRR 18%
  - (B - A) has IRR 16.6%
- If i* = 20%, then:
  - Neither option A nor option B is good
Judging proposed investments

- To compare multiple alternatives with:
  - Different initial investments
  - Same lifetimes
- Look at differences between options
- Compare them against each other!
  - Not only against the base rate of return $i^*$
Judging proposed investments

- To compare multiple alternatives with:
  - Same initial investments
  - **Different** lifetimes

  can just compare IRR values directly

- Assume that each option is repeated:
  - The one with the better IRR in early years will still have the better IRR later on!
Judging proposed investments

To compare multiple alternatives with:

- **Different** initial investments
- **Different** lifetimes

Must first convert to equal lifetimes:

- Then look at **differences** between options
- Compare them **against each other**!
  - Not only against the base rate of return $i^*$
Judging proposed investments

- Possible mistakes:
  - Highest IRR is not necessarily best
    - Another project with a larger investment might yield a larger total benefit!
  - Project with largest initial investment is also not necessarily best
  - But we know that options with IRR < i* will never be chosen!
Judging proposed investments

- Compute incremental rate of return:
  - Based on a smaller investment that is already known to acceptable!

Example:
- \( i^* = 15\% \)
- IRR of option \( A = 12\% \)
- Option \( B \) may not be worthwhile
  - Even if IRR of \( (B - A) \) is > \( i^* \)!
Can also plot graphically

- Overall IRR is where it crosses X axis
  - *Incremental* IRR is where 2 curves cross
Judging proposed investments

- If \( i^* < \text{IRR of } (B - A) \), then:
  - Option B is better

- If \( \text{IRR of } (B - A) < i^* < \text{IRR of } A \):
  - Option A is better

- If \( i^* > \text{IRR of } A \), then:
  - Neither option has positive present worth
  - *Don’t do either one!*
Combinations of investments

- Buy a mine for $1.5 million, n=8:
  - Annual benefit = $391,000
  - IRR = 20%

- Put aside money each year at 4% for 8 years (equal annual deposits), to pay $1.5M in year 8:
  - Annual amount A equals $163,000/year
  - Remaining mine revenue = $228,000/year
Combinations of investments

- Combined investment:
  - *Pay* $1.5 million in year 0
  - *Receive* $228,000/year for 8 years
  - *Receive* $1.5 million in year 8
  - IRR = 15.2% for this combination
    - But investing at 4% is not worth it even if \( i^* < 15\% \)!
    - Must judge both investments *separately*!
Note

- In some cases:
  - Sequences of investments over time
  - Rather than one investment period at start there may be \textit{no} IRR
  - (or more than one!)
- Increasing discount rate makes benefits worth less, but costs \textit{also} worth less
Example with no IRR

- Pay $15,000 in year 0
- Receive $5,000 in year 1:
  - $6,000 in year 2
  - $7,000 in year 3
- Pay $10,000 in year 4
- Small i* makes present worth of years 1-3 positive, *but makes cost count more*
Example with two IRR values

- Pay $700 in year 0
- Receive $200/year in years 1-10
  - $100/year in years 11-20
- Pay $3,000 in year 21
- @IRR function requires a guess:
  - With a guess of 2%, we get IRR = 2.8%
  - With a guess of 25%, we get IRR = 26.2%
Judging proposed investments

- @IRR function may create problems when there are multiple IRR values:
  - This happens for cash flow series with two or more reversals of sign
  - E.g., - + + + -
  - Or - + + - + + - + + +
- See Grant and Ireson, Appendix B
Review

- What is the single most important pitfall to avoid when using present worth to compare projects?
- What is the single most important pitfall to avoid when using internal rate of return to compare projects?