Lecture 7

Internal Rate of Return
Judging proposed investments

- Don’t need to pre-specify minimum rate of return $i^*$ in judging projects
- **Internal rate of return (IRR)** is:
  - The interest rate that makes the present worth of a project exactly 0
- Project is desirable if $i^* < \text{IRR}$,
  - Not otherwise (since present worth would be negative)
Example

Consider a stream of costs and revenues over time

Present worth equals:

- $3924 at 18%
- -$6051 at 20%
Example

Assume function is linear in between

(True function actually lies slightly below)
Example

- Present worth equals:
  - $3924 at 18%, -$6051 at 20%
- Slope = Delta Y/Delta X
  - \[ \frac{-6051 - 3924}{20 - 18} = -4987.5 \]
- Want distance x such that height y = 0:
  - \[ 3924 - 4987.5x = 0 \Rightarrow x = 0.8 \Rightarrow \text{IRR} = 18 + 0.8 \]
  - (True value = 18.76)
Internal rate of return

Note that we don’t need to consider present worth:
- As long as it is all in one time period
Example

- A factory starts operation at the end of year 0:
  - Capital costs arise in years -2, -1, and 0
  - Can convert to present worth in year -2,
  - Or to value in year 0
Example

- Assume expenses of:
  - $300K for land in year -2
  - $800K for construction in year -1
  - $700K for construction in year 0
  - $200K for inventory in year 0
Example

- Convert to future value in year 0
  - For year -2: $300 (F/P, 10%, 2) = $363
  - For year -1: $800 (F/P, 10%, 1) = $880
  - For year 0: $900
  - Total: $2143

- Can then convert future revenues to year 0 to compute “present worth”
Another example

- Find IRR of a $10K, 7%, 20-year bond:
  - Bought for $8K
  - (Can’t do without knowing purchase price!)

- Bond pays:
  - 7% of $10K = $700/year for 20 years
  - $10K in year 20

- Would IRR be higher or lower than 7%?
Another example

- Present worth of bond at $i^* = 9\%$:
  - $700 \text{ (P/A, 9\%, 20)} = 6390$
  - $10,000 \text{ (P/F, 9\%, 20)} = 1784$
  - Purchase price = -8000
  - Total = 174
Another example

- Present worth of bond at $i^* = 10\%$:
  - $700 \ (P/A, \ 10\%, \ 20) = 5959$
  - $10,000 \ (P/F, \ 10\%, \ 20) = 1486$
  - Purchase price = -$8000$
  - Total = -$555$
Another example

- Present worth equals:
  - $174 at 9%
  - -$555 at 10%

- Would you expect the IRR to be:
  - Closer to 9%?
  - Closer to 10%?
Another example

- **Present worth equals:**
  - $174 at 9%, -$555 at 10%

- **Slope = \Delta Y/\Delta X**
  - \( (-$555 - $174)/(10 - 9) = -729 \)

- **Want distance \( x \) such that height \( y = 0 \):**
  - \( 174 - 729x = 0 \Rightarrow x = .24 \)
  - **IRR = 9 + .24 = 9.24**
More realistic example

- Now assume that the bond pays:
  - $350 every six months for 20 years
  - $10K in year 20
- Would IRR be higher or lower than the previous value?
  - And why?
More realistic example

- Present worth at $i^* = 4\%$ for 6 months:
  - $350 \ (P/A, \ 4\%, \ 40) = 6927$
  - $10,000 \ (P/F, \ 4\%, \ 40) = 2083$
  - Purchase price = -8000
  - Total = 1010
More realistic example

- Present worth at $i^* = 5\%$ for 6 months:
  - $350 \times (P/A, 5\%, 40) = 6006$
  - $10,000 \times (P/F, 5\%, 40) = 1420$
  - Purchase price = -8000
  - Total = -574
More realistic example

- Present worth equals:
  - $1010 at 4%, -$574 at 5% per 6 months

- Slope = Delta Y/Delta X
  - $574 - $1010)/(5 - 4) = -1584

- Want distance x such that height y = 0:
  - 1010 - 1584x = 0 \Rightarrow x = .64
  - IRR = 4 + .64 = 4.64 per 6 months!
More realistic example

- How to convert to annual interest rate?
  - *Nominal* annual interest rate
    
    \[ = 2 \times (4.64) = 9.28\% \]
  - *Effective* annual interest rate
    
    \[ = (1.0464)^2 - 1 = 1.095 - 1 = .095 \]
    
    or 9.5\%
Judging proposed investments

- Don’t need to specify exact value of $i^*$ in judging projects:
  - Only a range
- In last example, bond is desirable if:
  - Minimum attractive rate of return < 9.5%
  - Not otherwise
- Bounding $i^*$ is easier than estimating it!
Review

- We learned how to
  - Find internal rate of return of a project:
    - Convert all costs, benefits to one time period
    - Try different interest rates until you find:
      - One where value of project is positive
      - One where value is negative
    - Interpolate to estimate IRR
  - Compare against minimum acceptable rate of return $i^*$ to assess desirability