Lecture 7

Internal Rate of Return

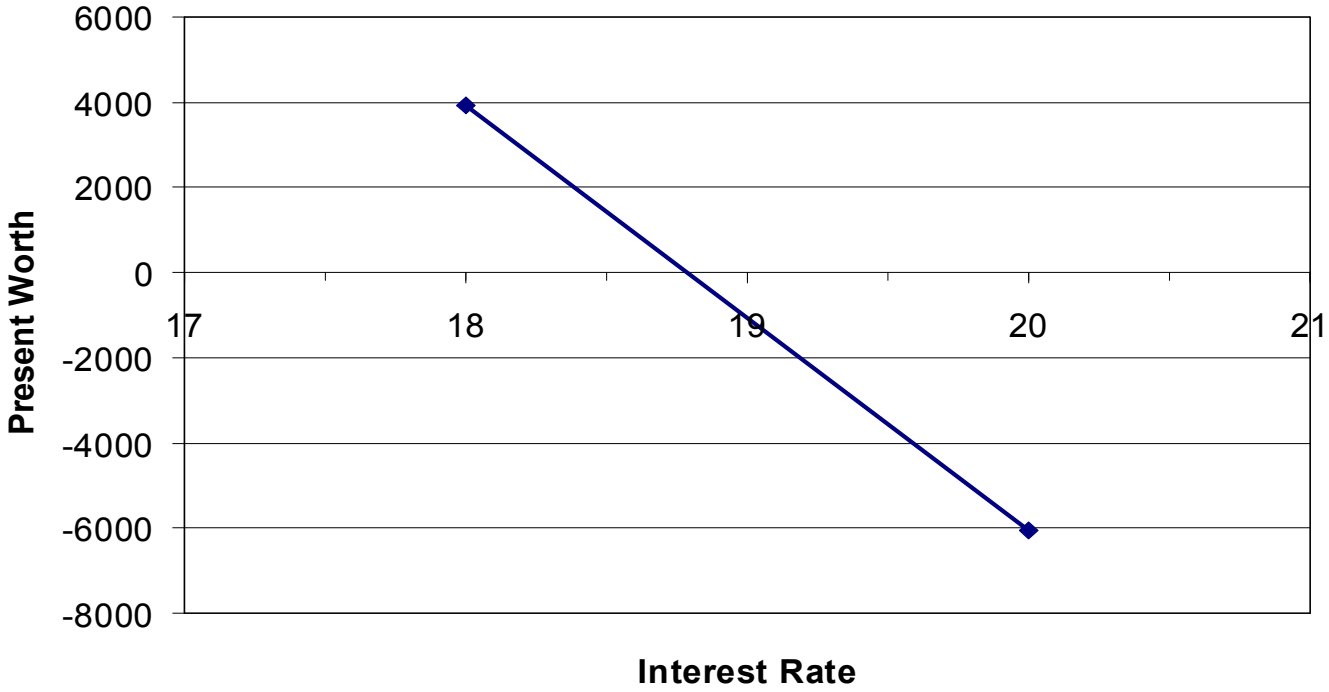
Judging proposed investments

- Don’t need to pre-specify minimum rate of return i* in judging projects
- Internal rate of return (IRR) is:
  - The interest rate that makes the present worth of a project exactly 0
- Project is desirable if i* < IRR,
  - Not otherwise (since present worth would be negative)

Example

- Consider a stream of costs and revenues over time
- Present worth equals:
  - $3924 at 18%
  - -$6051 at 20%

Example



- *Assume* function is linear in between
  - (True function actually lies slightly below)

Example

- Present worth equals:
  - $3924 at 18%, -$6051 at 20%
- Slope = Delta Y/Delta X
  - = (-$6051 - $3924)/(20 - 18) = -4987.5
- Want distance x such that height y = 0:
  - 3924 - 4987.5x = 0 $\Rightarrow$ x=.8 $\Rightarrow$ IRR=18+.8
  - (True value = 18.76)

Internal rate of return

- Note that we don't need to consider *present* worth:
  - As long as it is all in one time period

Example

- A factory starts operation at the end of year 0:
  - Capital costs arise in years -2, -1, and 0
- Can convert to present worth in year -2,
  - Or to value in year 0

Example

- Assume expenses of:
  - $300K for land in year -2
  - $800K for construction in year -1
  - $700K for construction in year 0
  - $200K for inventory in year 0

Example

- Convert to *future* value in year 0
  - For year -2: $300 (F/P, 10%, 2) = $363
  - For year -1: $800 (F/P, 10%, 1) = $880
  - For year 0: $900
  - Total: $2143
- Can then convert future revenues to year 0 to compute "present worth"

Another example

- Find IRR of a $10K, 7%, 20-year bond:
  - Bought for $8K
  - (Can't do without knowing purchase price!)
- Bond pays:
  - 7% of $10K = $700/year for 20 years
  - $10K in year 20
- Would IRR be higher or lower than 7%?

Another example

- Present worth of bond at i* = 9%:
  - $700 (P/A, 9%, 20) = $6390
  - $10,000 (P/F, 9%, 20) = $1784
  - Purchase price = -$8000
  - Total = $174

Another example

- Present worth of bond at i* = 10%:
  - $700 (P/A, 10%, 20) = $5959
  - $10,000 (P/F, 10%, 20) = $1486
  - Purchase price = -$8000
  - Total = -$555

Another example

- Present worth equals:
  - $174 at 9%
  - -$555 at 10%
- Would you expect the IRR to be:
  - Closer to 9%?
  - Closer to 10%?

Another example

- Present worth equals:
  - $174 at 9%, -$555 at 10%
- Slope = DeltaY/Delta X
  - = (-$555 - $174)/(10 - 9) = -729
- Want distance x such that height y = 0:
  - $174 - 729x = 0 \Rightarrow x = .24$
  - IRR = 9 + .24 = 9.24

More realistic example

- Now assume that the bond pays:
  - $350 every six months for 20 years
  - $10K in year 20
- Would IRR be higher or lower than the previous value?
  - *And why?*

More realistic example

- Present worth at i* = 4% for 6 months:
  - $350 (P/A, 4%, 40) = $6927
  - $10,000 (P/F, 4%, 40) = $2083
  - Purchase price = -$8000
  - Total = $1010

More realistic example

- Present worth at i* = 5% for 6 months:
  - $350 (P/A, 5%, 40) = $6006
  - $10,000 (P/F, 5%, 40) = $1420
  - Purchase price = -$8000
  - Total = -$574

More realistic example

- Present worth equals:
  - $1010 at 4%, -$574 at 5% per 6 months
- Slope = Delta Y/Delta X
  - = (-$574 - $1010)/(5 - 4) = -1584
- Want distance x such that height y = 0:
  - 1010 - 1584x = 0 $\Rightarrow$ x = .64
  - IRR = 4 + .64 = 4.64 per 6 months!

More realistic example

- How to convert to annual interest rate?
  - *Nominal* annual interest rate

    = 2 (4.64) = 9.28%
  - *Effective* annual interest rate

    = $(1.0464)^2$ - 1 = 1.095 - 1 = .095

    or 9.5%

Judging proposed investments

- Don’t need to specify exact value of i* in judging projects:
  - Only a range
- In last example, bond is desirable if:
  - Minimum attractive rate of return < 9.5%
  - Not otherwise
- Bounding i* is easier than estimating it!

Review

- We learned how to
  - Find internal rate of return of a project:
    - Convert all costs, benefits to one time period
    - Try different interest rates until you find:
      - One where value of project is positive
      - One where value is negative
    - Interpolate to estimate IRR
  - Compare against minimum acceptable rate of return i* to assess desirability