Lecture 6

Annual Equivalent Cash Flow

Judging proposed investments

- Another way of judging investments:
  - Again based on minimum rate of return i*
- With annual equivalent cash flow:
  - All costs and benefits (present or future) converted to equivalent annual amounts
- Easy for projects with different lives:
  - Yields the cost of one year of service

Calculation of annual amount

- Reminders
  - Convert future to annual:

    $$A = F\, i/[(1+i)^n - 1]$$

  - Convert present to annual:

    $$A = P\, i \left[\frac{(1+i)^n}{(1+i)^n - 1} \right]$$

Same example as last time

- Current labor cost is $9200/year
- Option to build new equipment:
  - First cost $15,000
  - Labor $3300/year
  - Power $400/year
  - Maintenance $1100/year
  - Property tax and insurance $300/year
  - Income tax $1040/year
  - Total annual cost $6140/year

Reminders

- Note:
  - Only need to account for *changes* in property tax, insurance, etc.
- Assumptions:
  - Lifetime of equipment is 10 years
  - Minimum rate of return i* = 9%

Example--results

- Annual equivalent of current option:
  $9200
- Annual equivalent of new equipment:
  - Annual cost $6140
  - <u>$15,000 (A/P, 9%, 10) = $2337</u>
  - Total = $8477
- Is the new equipment better?

Example--results

- What happens if we increase i*?
  - Annual equivalent of current option: $9200
  - Annual equivalent of new equipment:
    - Annual cost $6140
    - $15,000 (A/P, 16%, 10) = $3104
    - Total = $9244
  - Now, the new equipment is *worse!*

Example--results

- *Ratio* of results is same (for a given i*):
  - Regardless of the method chosen
- Annual equivalent:
  - $9200/$8477 = 1.085
    - Current option is 8.5% more expensive
- Present worth:
  - $59,050/$54,407 = 1.085
    - Mathematically equivalent!

Projects with different lives

- Cannot just bring back to present worth
  - But *can* just convert to annual equivalent!
  - The cost (or benefit) of one year of service
    - Regardless of the project lifetime!
- No need to convert to *equal lives*
  - (As with present worth)

Projects with different lives

- When converting to annual equivalent,
  - Use appropriate lifetime for each option
- What about benefits (or costs) accruing after one option has already expired?
  - Can imagine continuing that option with another equivalent choice
  - We are choosing a *policy*,
    - Not making a one-time choice

Example

- Compare options A and B at $i^* = 11\%$:

  A: First cost = $50,000
  - Annual cost = $9,000/year for 20 years
  - <u>Salvage value = $10,000 in year 20</u>

  B: First cost = $120,000
  - Annual cost = $7,000/year for 40 years
  - Salvage value = $20,000 in year 40
    - Salvage value should be *subtracted* from cost!

Example

- Annual equivalent (cost) of option A:
  - Annual cost = $9000/year
  - $50,000 (A/P, 11%, 20) = $6279/year
  - -$10,000 (A/F, 11%, 20) = - $156/year
  - Total = $15,123/year
- No need to convert to 40 years!
  - A year of service of option A is comparable to a year of service of option B

Example

- Annual equivalent (cost) of option B:
  - Annual cost = $6000/year
  - $120,000 (A/P, 11%, 40) = $13,406/year
  - <u>-$20,000 (A/F, 11%, 40) = - $34/year</u>
  - Total = $19,372/year
- So life extension is not worthwhile:
  - Salvage value is *greater,* but worth *less!*

Projects with different lives

- When using annual equivalent:
  - No need to convert lifetimes of all projects to their least common multiple!
  - Avoids complications with messy lifetimes
    - (E.g., comparing projects of 7 and 12 years)

Projects with *perpetual* lives

- What about *perpetual lives?*
  - How can we convert a cost in the present to an annual equivalent cost for an infinite number of years?
- We know that:
  - $P = A\,[1 - 1/(1+i)^n]/i$
    - $= A/i - A/[(1+i)^n i]$
- Taking limits gives $P = A/i$, or $A = Pi$

Example

- At 8% interest for 100 years:
  - A = .08004 P
- At 8% interest for infinite years:
  - A = .08 P
- Difference is only 1/20th of 1% of .08!

Example

- At 3% interest:
  - The future is discounted less,
  - So the difference between 100 years and perpetual lifetime is greater
    - But still only 1/6th of 1% of .03!
    - (A = .03 P versus .03005 P for perpetual life)

Perpetual lives

- The previous examples show that the distant future is discounted *sharply!*
- Discounting may not be appropriate for analyzing very long-term problems:
  - Ecological damage
  - Conservation
  - Global warming
  - Species extinction

Example

- Every 20 years we:
  - Need to purchase new equipment
    - $50,000
  - Get salvage value of old equipment
    - $10,000
- Annualized cost is:
  - $40,000 (A/F, 11%, 20) = $623

Example

- Present worth of continuing project A in perpetuity (at i* = 11%):
  - Annual cost = $9000
  - First cost $50,000 = $5500
  - $40,000 (A/F, 11%, 20) = $623
    - (Annual equivalent cost continues forever)
  - Total annual equivalent = $15,123
    - Exactly same as annual equivalent for 20 years!

Projects with different lives

- Like the previous methods:
  - Least common multiple of lifetimes
  - Perpetual lifetimes

  annual equivalent cost makes sense only if the best option would be used for an extended period of time
- This may not always be the case

Review

- We learned how to
  - Find annual equivalent cost of a project:
    - Annual cost
    - First cost (convert from present to annual)
    - Salvage value (convert from future to annual)
  - Compare options with different lives:
    - Choose option with lowest annualized cost
      - (Or highest annual equivalent benefit)