Lecture 5

Present Worth

Judging proposed investments

- There are many ways of judging proposed investments:
  - All based on a minimum rate of return i*
- How to determine i*?
  - At least as high as the interest rate
  - Also based on other available opportunities
  - Discussed in more detail in Chapter 18

Judging proposed investments

- Four different methods:
  - Present worth
  - Annual equivalent cash flow
  - Internal rate of return
  - Benefit/cost ratios
- All are mathematically equivalent:
  - Slightly different pluses and minuses

Judging proposed investments

- One alternative might have:
  - Higher initial cost, but
  - Lower annual cost or longer life
- Must convert to comparable terms
- Alternatives may also have different income tax implications:
  - Compare based on *after*-tax performance!

Calculation of present worth

- Based on *discounting!*
  - Future costs and benefits discounted to present
  - Discount rate = minimum rate of return i*
  - Tells us how much we care about the future
- Present worth is the most intuitive method:
  - All costs and benefits are converted to year 0
  - Easy to interpret
- But can be difficult to implement for projects with different lives

Example

- Current labor cost is $9200/year
- Option to build new equipment:
  - First cost — $15,000
  - Labor — $3300/year
  - Power — $400/year
  - Maintenance — $1100/year
  - Property tax and insurance — $300/year
  - Income tax — $1040/year
  - Total annual cost — $6140/year

Example

- Note:
  - Only need to account for *changes* in property tax, insurance, etc.
- Assumptions:
  - Lifetime of equipment is 10 years
  - Minimum rate of return $i^* = 9\%$

Example--results

- Present worth (cost) of current option:
  - \$9200 (P/A, 9%, 10) = \$59,050
- Present worth (cost) of new equipment:
  - \$6140 (P/A, 9%, 10) = \$39,407
  - <u>First cost</u> = <u>\$15,000</u>
  - Total = \$54,407
- Is the new equipment better?

Projects with different lives

- Cannot just bring back to present worth
- For example:
  - 20 years of service at a cost of $20,000

  may (or may not) be worth more than
  - 10 years of service at a cost of $15,000
- When using present worth method:
  - Must compare options with *equivalent lives*

Example

- Compare options A and B at $i^* = 11\%$:

  A: First cost = \$50,000

  - Annual cost = \$9,000/year for 20 years
  - Salvage value = \$10,000 in year 20

  B: First cost = \$120,000

  - Annual cost = \$7,000/year for 40 years
  - Salvage value = \$20,000 in year 40
    - Salvage value should be *subtracted* from cost!

Example

- Present worth (cost) of option B:
  - First cost = $120,000
  - $7000 (P/A, 11%, 40) = $62,657
  - -$20,000 (P/F, 11%, 40) = - $308
  - Total = $182,349
- This option provides *40 years* of service

Example

- Must convert option A to 40 years!
  - First cost = \$50,000
  - \$50,000 (P/F, 11%, 20) = \$6201
  - \$9000 (P/A, 11%, 40) = \$80,559
  - -\$10,000 (P/F, 11%, 20) = - \$1240
  - -\$10,000 (P/F, 11%, 40) = - \$154
  - Total = \$135,326
    - First cost, salvage value appear *twice!*

Example

- Which option is better?
  - Option B has:
    - Longer lifetime
    - Lower annual cost
    - Higher salvage value at end of life
  - But two copies of option A can provide 40 years of service with *lower present worth!*

Projects with different lives

- To evaluate based on present worth:
  - Must convert lifetimes of all projects to their *least common multiple!*
  - In this example, that was easy:
    - Least common multiple of 20 and 40 is 40
  - In some problems, it can get complicated:
    - Least common multiple of 7 and 12 is 84!
    - Would need 12 copies of one, 7 of the other

Projects with *perpetual* lives

- Some projects may last so long that they can be modeled as *perpetual!*
- Even projects with perpetual lives can have a finite present worth:
  - Why?
- General formula for perpetual lives:
  - $P = A/i^*$, or $A = P\,i^*$

Example

- First cost = $50,000
  - Annual cost = $9,000/year forever
  - Interest rate i* = 11%
- Present worth:
  - $50,000 + $9,000/.11 = $131,818

Perpetual lives

- Some perpetual costs are not annual
  - For example, every 20 years we may:
    - Need to purchase new equipment ($50,000)
    - Get salvage value of old equipment ($10,000)
- To convert perpetual recurrent costs to present worth:
  - First convert to annual
  - Then divide by i* to get present worth

Example

- Every 20 years we:
  - Need to purchase new equipment
    - $50,000
  - Get salvage value of old equipment
    - $10,000
- Annualized cost is:
  - $40,000 (A/F, 11%, 20) = $623
  - Present worth = $623/i* = $5664

Example

- Present worth of continuing project A in perpetuity:
  - First cost in year 0 = $50,000
  - Annual cost $9,000/i = $81,818
  - <u>$40,000 (A/F, 11%, 20)/i = $5664</u>
    - (Replacement cost minus salvage value)
  - Total present worth = $137,482
    - Only slightly greater than 2 copies ($135,326)

Perpetual lives

- Why use perpetual lives?
- Avoids the need to analyze numerous copies of a project:
  - If least common multiple of lives is large
- Can simply convert all projects to their perpetual equivalent
  - (Assuming an *infinite* number of copies)

Projects with different lives

- The comparison methods so far:
  - Least common multiple of lifetimes
  - Perpetual lifetimes

  make sense if the best option would be used for an extended period of time
- This may not always be the case:
  - E.g., computers (due to rapid change)

Review

- What is the single most important pitfall to avoid when using present worth to compare projects?