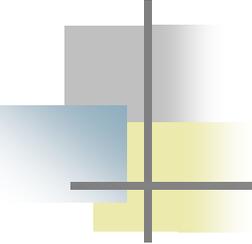


# Lecture 5

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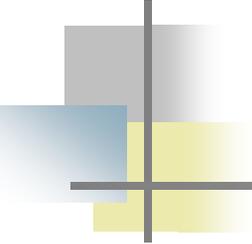
## Present Worth



# Judging proposed investments

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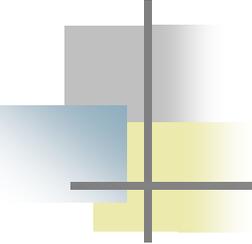
- There are many ways of judging proposed investments:
  - All based on a minimum rate of return  $i^*$
- How to determine  $i^*$ ?
  - At least as high as the interest rate
  - Also based on other available opportunities
  - Discussed in more detail in Chapter 18



# Judging proposed investments

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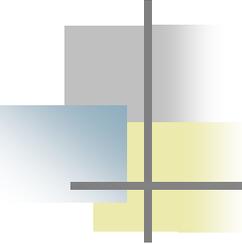
- Four different methods:
  - Present worth
  - Annual equivalent cash flow
  - Internal rate of return
  - Benefit/cost ratios
- All are mathematically equivalent:
  - Slightly different pluses and minuses



# Judging proposed investments

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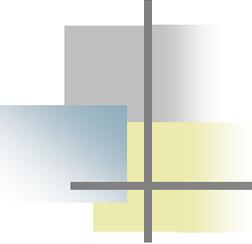
- One alternative might have:
  - Higher initial cost, but
  - Lower annual cost or longer life
- Must convert to comparable terms
- Alternatives may also have different income tax implications:
  - Compare based on *after*-tax performance!



# Calculation of present worth

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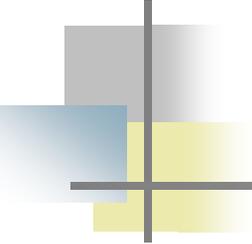
- Based on *discounting!*
  - Future costs and benefits discounted to present
  - Discount rate = minimum rate of return  $i^*$
  - Tells us how much we care about the future
- Present worth is the most intuitive method:
  - All costs and benefits are converted to year 0
  - Easy to interpret
- But can be difficult to implement for projects with different lives



# Example

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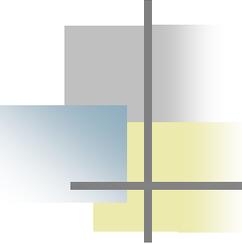
- Current labor cost is \$9200/year
- Option to build new equipment:
  - First cost \$15,000
  - Labor \$3300/year
  - Power \$400/year
  - Maintenance \$1100/year
  - Property tax and insurance \$300/year
  - Income tax \$1040/year
  - Total annual cost \$6140/year



# Example

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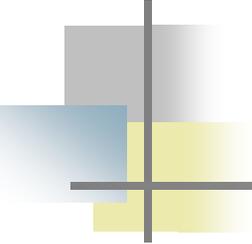
- Note:
  - Only need to account for changes in property tax, insurance, etc.
- Assumptions:
  - Lifetime of equipment is 10 years
  - Minimum rate of return  $i^* = 9\%$



# Example--results

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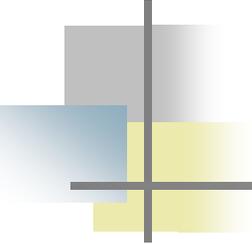
- Present worth (cost) of current option:
  - $\$9200 (P/A, 9\%, 10) = \$59,050$
- Present worth (cost) of new equipment:
  - $\$6140 (P/A, 9\%, 10) = \$39,407$
  - First cost =  $\$15,000$
  - Total =  $\$54,407$
- Is the new equipment better?



# Projects with different lives

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- Cannot just bring back to present worth
- For example:
  - 20 years of service at a cost of \$20,000 may (or may not) be worth more than
  - 10 years of service at a cost of \$15,000
- When using present worth method:
  - Must compare options with *equivalent lives*



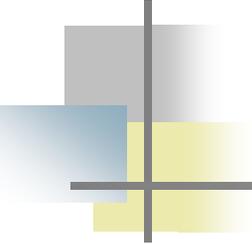
# Example

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- Compare options **A** and **B** at  $i^* = 11\%$ :
  - A: First cost = \$50,000
    - Annual cost = \$9,000/year for 20 years
    - Salvage value = \$10,000 in year 20
  - B: First cost = \$120,000
    - Annual cost = \$7,000/year for 40 years
    - Salvage value = \$20,000 in year 40
      - Salvage value should be *subtracted* from cost!

# Example

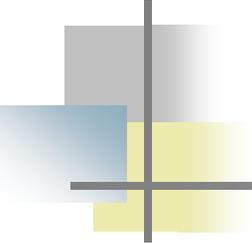
- Present worth (cost) of option **B**:
  - First cost = \$120,000
  - \$7000 (P/A, 11%, 40) = \$62,657
  - -\$20,000 (P/F, 11%, 40) = - \$308
  - Total = \$182,349
- This option provides *40 years* of service



# Example

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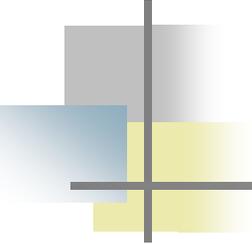
- Must convert option **A** to 40 years!
  - First cost = \$50,000
  - \$50,000 (P/F, 11%, 20) = \$6201
  - \$9000 (P/A, 11%, **40**) = \$80,559
  - -\$10,000 (P/F, 11%, 20) = - \$1240
  - -\$10,000 (P/F, 11%, 40) = - \$154
  - Total = **\$135,326**
    - First cost, salvage value appear *twice!*



# Example

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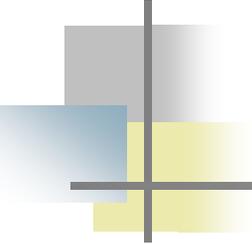
- Which option is better?
  - Option **B** has:
    - Longer lifetime
    - Lower annual cost
    - Higher salvage value at end of life
  - But two copies of option **A** can provide 40 years of service with *lower present worth!*



# Projects with different lives

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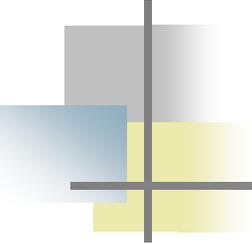
- To evaluate based on present worth:
  - Must convert lifetimes of all projects to their *least common multiple!*
  - In this example, that was easy:
    - Least common multiple of 20 and 40 is 40
  - In some problems, it can get complicated:
    - Least common multiple of 7 and 12 is 84!
    - Would need 12 copies of one, 7 of the other



# Projects with *perpetual* lives

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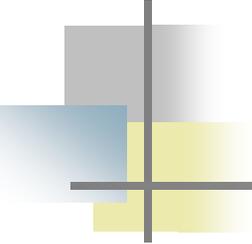
- Some projects may last so long that they can be modeled as *perpetual!*
- Even projects with perpetual lives can have a **finite** present worth:
  - Why?
- General formula for perpetual lives:
  - $P = A/i^*$ , or  $A = P i^*$



# Example

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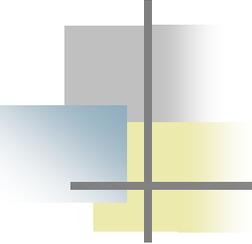
- First cost = \$50,000
  - Annual cost = \$9,000/year forever
  - Interest rate  $i^*$  = 11%
- Present worth:
  - $\$50,000 + \$9,000/.11 = \$131,818$



# Perpetual lives

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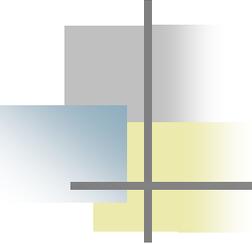
- Some perpetual costs are not annual
  - For example, every 20 years we may:
    - Need to purchase new equipment (\$50,000)
    - Get salvage value of old equipment (\$10,000)
- To convert perpetual recurrent costs to present worth:
  - First convert to annual
  - Then divide by  $i^*$  to get present worth



# Example

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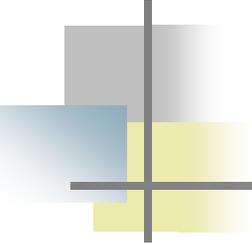
- Every 20 years we:
  - Need to purchase new equipment
    - \$50,000
  - Get salvage value of old equipment
    - \$10,000
- Annualized cost is:
  - $\$40,000 (A/F, 11\%, 20) = \$623$
  - Present worth =  $\$623/i^* = \$5664$



# Example

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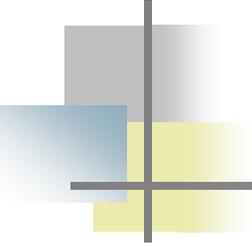
- Present worth of continuing project **A** in perpetuity:
  - First cost in year 0 = \$50,000
  - Annual cost \$9,000/i = \$81,818
  - \$40,000 (A/F, 11%, 20)/i = \$5664
    - (Replacement cost minus salvage value)
  - Total present worth = **\$137,482**
    - Only slightly greater than 2 copies (**\$135,326**)



# Perpetual lives

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- Why use perpetual lives?
- Avoids the need to analyze numerous copies of a project:
  - If least common multiple of lives is large
- Can simply convert all projects to their perpetual equivalent
  - (Assuming an *infinite* number of copies)



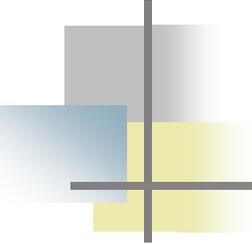
# Projects with different lives

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- The comparison methods so far:
  - Least common multiple of lifetimes
  - Perpetual lifetimes

make sense if the best option would be used for an extended period of time

- This may not always be the case:
  - E.g., computers (due to rapid change)



# Review

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- What is the single most important pitfall to avoid when using present worth to compare projects?