Lecture 5

Present Worth
Judging proposed investments

- There are many ways of judging proposed investments:
  - All based on a minimum rate of return $i^*$

- How to determine $i^*$?
  - At least as high as the interest rate
  - Also based on other available opportunities
  - Discussed in more detail in Chapter 18
Judging proposed investments

- Four different methods:
  - Present worth
  - Annual equivalent cash flow
  - Internal rate of return
  - Benefit/cost ratios

- All are mathematically equivalent:
  - Slightly different pluses and minuses
Judging proposed investments

- One alternative might have:
  - Higher initial cost, but
  - Lower annual cost or longer life
- Must convert to comparable terms
- Alternatives may also have different income tax implications:
  - Compare based on *after*-tax performance!
Calculation of present worth

- Based on *discounting!*
  - Future costs and benefits discounted to present
  - Discount rate = minimum rate of return $i^*$
  - Tells us how much we care about the future

- Present worth is the most intuitive method:
  - All costs and benefits are converted to year 0
  - Easy to interpret

- But can be difficult to implement for projects with different lives
Example

- Current labor cost is $9200/year
- Option to build new equipment:
  - **First cost**: $15,000
  - Labor: $3300/year
  - Power: $400/year
  - Maintenance: $1100/year
  - Property tax and insurance: $300/year
  - **Income tax**: $1040/year
  - **Total annual cost**: $6140/year
Example

Note:
- Only need to account for changes in property tax, insurance, etc.

Assumptions:
- Lifetime of equipment is 10 years
- Minimum rate of return $i^* = 9\%$
Example--results

- Present worth (cost) of current option:
  - $9200 \times (P/A, 9\%, 10) = \$59,050$

- Present worth (cost) of new equipment:
  - $6140 \times (P/A, 9\%, 10) = \$39,407$
  - First cost = $\$15,000$
  - Total = $\$54,407$

- Is the new equipment better?
Projects with different lives

- Cannot just bring back to present worth
- For example:
  - 20 years of service at a cost of $20,000 may (or may not) be worth more than
  - 10 years of service at a cost of $15,000
- When using present worth method:
  - Must compare options with equivalent lives
Example

- Compare options A and B at $i^* = 11\%$:
  
  **A**: First cost = $50,000
  
  - Annual cost = $9,000/year for 20 years
  
  - Salvage value = $10,000 in year 20
  
  **B**: First cost = $120,000
  
  - Annual cost = $7,000/year for 40 years
  
  - Salvage value = $20,000 in year 40

  - Salvage value should be *subtracted* from cost!
Example

- Present worth (cost) of option B:
  - First cost = $120,000
  - $7000 \((P/A, 11\%, 40)\) = $62,657
  - -$20,000 \((P/F, 11\%, 40)\) = -$308
  - Total = $182,349

- This option provides **40 years** of service
Example

- Must convert option A to 40 years!
  - First cost $50,000
  - $50,000 (P/F, 11%, 20) = $6201
  - $9000 (P/A, 11%, 40) = $80,559
  - -$10,000 (P/F, 11%, 20) = -$1240
  - -$10,000 (P/F, 11%, 40) = -$154
  - Total = $135,326
  - First cost, salvage value appear twice!
Example

- Which option is better?
  - Option B has:
    - Longer lifetime
    - Lower annual cost
    - Higher salvage value at end of life
  - But two copies of option A can provide 40 years of service with lower present worth!
Projects with different lives

- To evaluate based on present worth:
  - Must convert lifetimes of all projects to their least common multiple!
  - In this example, that was easy:
    - Least common multiple of 20 and 40 is 40
  - In some problems, it can get complicated:
    - Least common multiple of 7 and 12 is 84!
    - Would need 12 copies of one, 7 of the other
Projects with *perpetual* lives

- Some projects may last so long that they can be modeled as *perpetual*!
- Even projects with perpetual lives can have a *finite* present worth:
  - Why?
- General formula for perpetual lives:
  - \( P = \frac{A}{i^*}, \) or \( A = P i^* \)
Example

- **First cost** = $50,000
- **Annual cost** = $9,000/year forever
- **Interest rate** $i^* = 11\%

**Present worth:**

- $50,000 + $9,000/.11 = $131,818
Perpetual lives

- Some perpetual costs are not annual
  - For example, every 20 years we may:
    - Need to purchase new equipment ($50,000)
    - Get salvage value of old equipment ($10,000)

- To convert perpetual recurrent costs to present worth:
  - First convert to annual
  - Then divide by \( i^* \) to get present worth
Example

- Every 20 years we:
  - Need to purchase new equipment
    - $50,000
  - Get salvage value of old equipment
    - $10,000

- Annualized cost is:
  - $40,000 \( (A/F, 11\%, 20) \) = $623
  - Present worth = \( $623/i^* \) = $5664
Example

- Present worth of continuing project A in perpetuity:
  - First cost in year 0 = $50,000
  - Annual cost $9,000/i = $81,818
  - $40,000 (A/F, 11%, 20)/i = $5664
    - (Replacement cost minus salvage value)
  - Total present worth = $137,482
    - Only slightly greater than 2 copies ($135,326)
Perpetual lives

- Why use perpetual lives?
- Avoids the need to analyze numerous copies of a project:
  - If least common multiple of lives is large
- Can simply convert all projects to their perpetual equivalent
  - (Assuming an \textit{infinite} number of copies)
Projects with different lives

- The comparison methods so far:
  - Least common multiple of lifetimes
  - Perpetual lifetimes

make sense if the best option would be used for an extended period of time

- This may not always be the case:
  - E.g., computers (due to rapid change)
Review

- What is the single most important pitfall to avoid when using present worth to compare projects?