Lecture 23

Uncertainty Analysis

Learning Objectives

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- Calculate how sensitive the results are to variations in the data
- (Typically using two or three estimates)

Complications of Sensitivity Analysis

- All analyses may not favor the same decision!
- If ***most*** favor the same decision:
  - ***And*** it is not ***too*** bad in the other analysis
  - Then that one is reliably the best
- What if one decision is:
  - Much better in some analyses
  - ***But*** much worse in others?

More Complications

- How far should parameters be varied?
  - Plus or minus 10%?
  - Plus or minus 100?
- What if several parameters are varied at the same time?
- If a decision is better in some analyses:
  - But worse in others

  Which values are most likely?

Uncertainty

- We may not know all the costs and benefits of a new technology
- Our uncertainty can be expressed in ***probabilities!***
  - E.g., earn $100 with probability .9
  - But lose $100 with probability .1

Example

- Option 1:
  - Earn $100 with probability .9
  - Lose $100 with probability .1
- Option 2:
  - Earn $100 with probability .95
  - Lose $300 with probability .05
- Which one would you prefer?
  - Why?

Decision Rules

- Maximize the probability of gain (minimize the probability of loss):
  - ***Prefer option 2***
- Minimize the ***amount*** of the largest possible loss:
  - ***Prefer option 1***
- Weight each outcome by its probability of occurring

Expected Value

- **Option 1:**
  - Earn $100 with probability .9
  - Lose $100 with probability .1
  - Expected value = .9 (100) + .1 (-100) = **80**
- **Option 2:**
  - Earn $100 with probability **.95**
  - Lose **$300** with probability .05
  - Expected value = .95(100) + .05(-300) = **80**

Decision Rules

- **Expected value**
  - **Weighting each outcome by its probability**

  is not easily "fooled"
- **It looks at all aspects of the problem:**
  - **And combines them in a logical way**
- **Useful for both routine uncertainties and extreme events *(e.g., bankruptcy)***

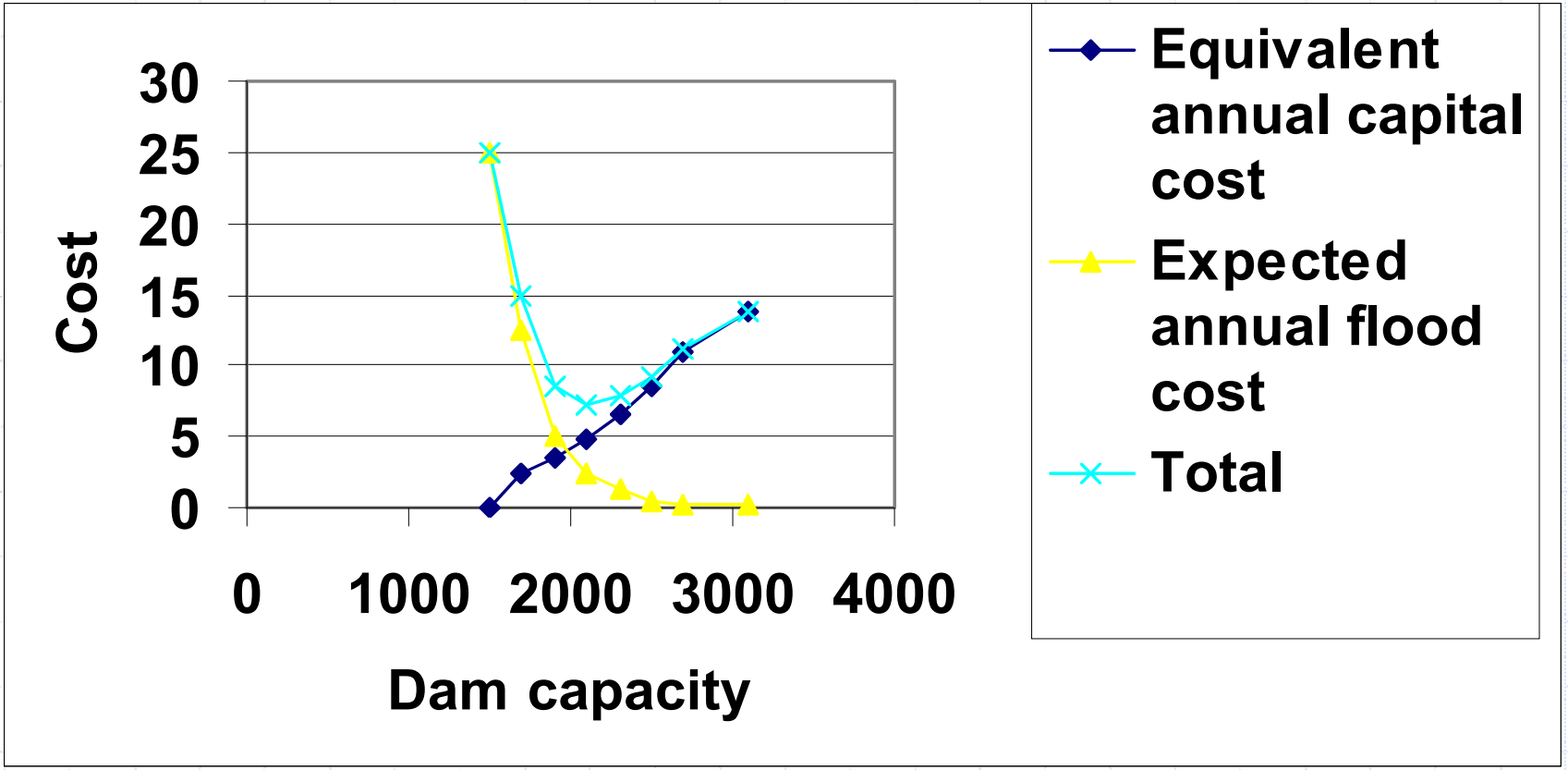
Extreme Example

- Consider the decision of whether to increase the capacity of a dam:
  - To prevent flooding
- Probability of flood depends on dam capacity (currently 1500 feet^3/second)
- Cost of a flood = $250K:
  - ***Expected*** cost of a flood = P(flood) $250K

Extreme Example (continued)

Capacity	Annual cost	P(flood)	Flood cost	Total
1500	0	0.1	25.0	25.0
1700	2.5	0.05	12.5	15.0
1900	3.6	0.02	5.0	8.6
2100	4.8	0.01	2.5	7.3
2300	6.5	0.005	1.25	7.8
2500	8.6	0.002	0.5	9.1
2700	10.9	0.001	0.25	11.2
3100	13.7	0.0005	0.125	13.8

Extreme Example (continued)



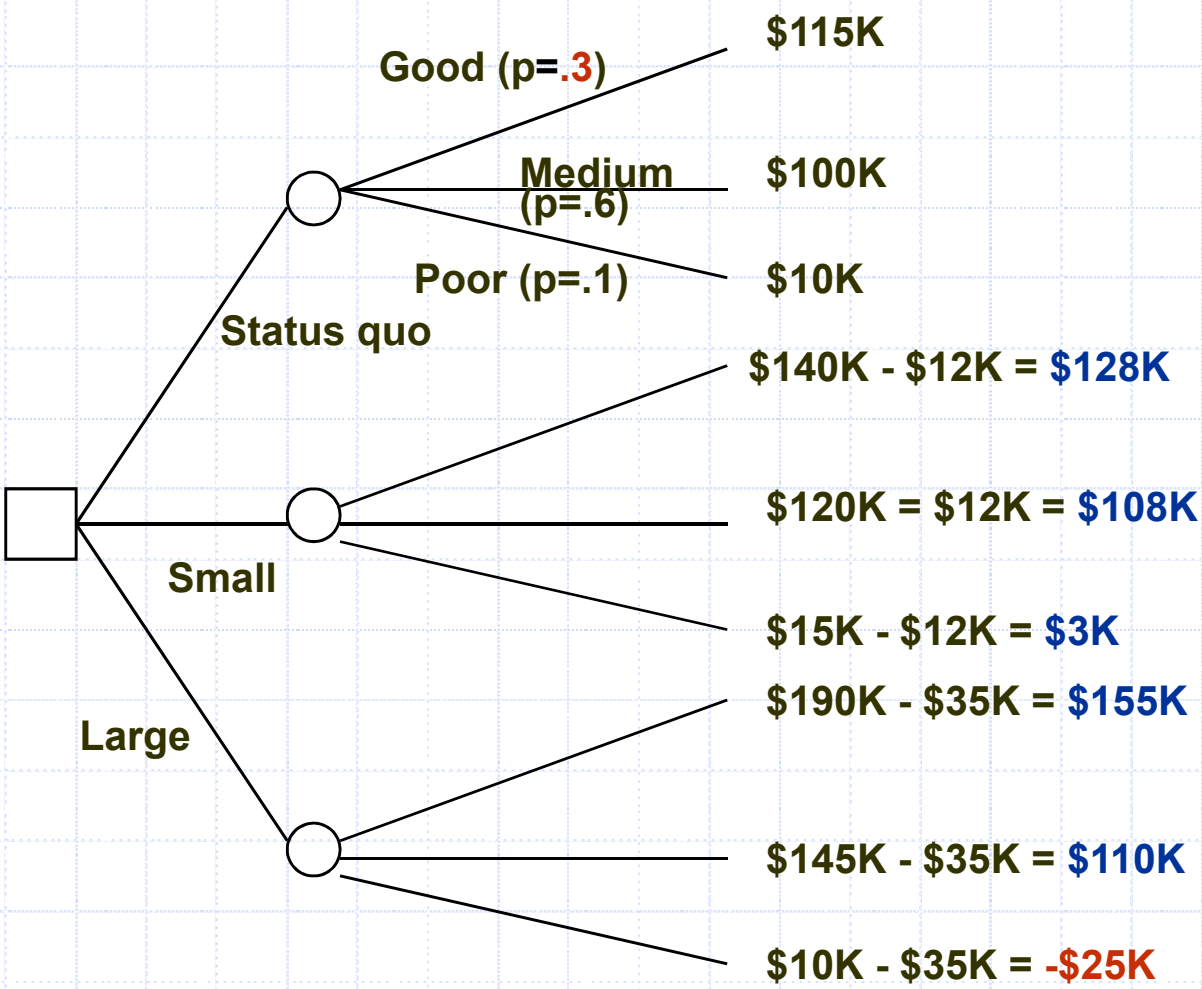
Extreme Events

- **Need information on:**
  - Event ***frequencies***
    - (Risk analysis is a field in itself)
  - Event ***consequences***
  - Costs of different alternatives

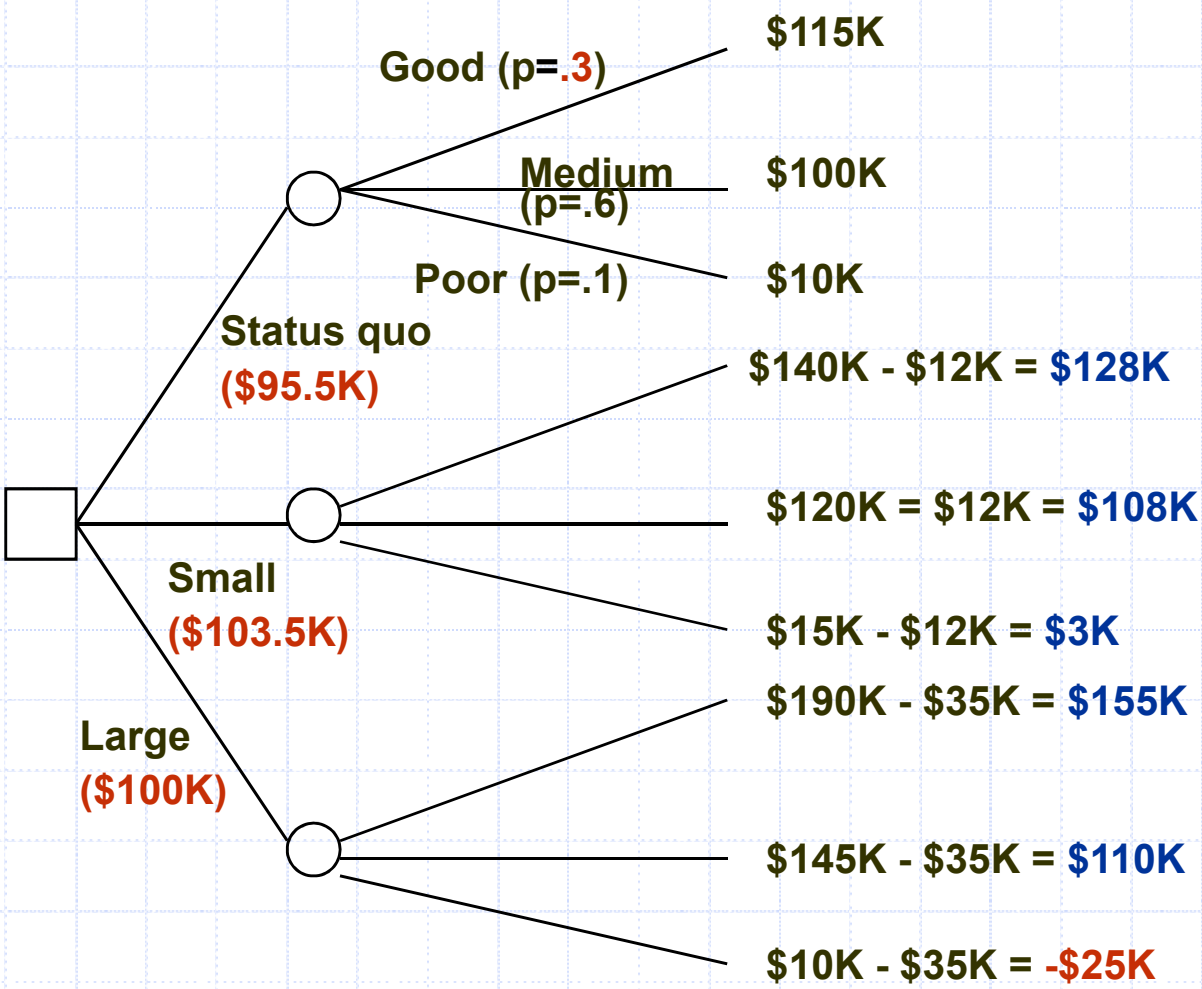
Routine Example

- Consider the decision of whether:
  - To maintain the status quo, or
  - Expand (small addition)—costs $12K
  - Expand (large addition)—costs $35K
- Business conditions can be:
  - Good (.3), medium (.6), or poor (.1)
  - Influences future cash flows

Routine Example (continued)



Routine Example (continued)



Uncertainty Analysis

- **Small problems:**
  - **Few uncertain variables**
  - **Few values per variable**

  can be solved using decision trees
- **Larger problems can be solved using Monte Carlo simulation (320):**
  - **Many variables, infinite number of values**

Observations

- "Utility" of large gains or losses may be non-linear:
  - Is winning $10 million really 10 times as good as winning $1 million?
  - Maybe losing $25K is enough to put your business into bankruptcy!
- Utility theory is covered in 516:
  - Introduction to Decision Analysis

Expected *Utility*

- Option 1:
  - Earn $1,000,000 with probability .9
  - Lose $1,000,000 with probability .1
- Option 2:
  - Earn $1,000,000 with probability .95
  - Lose $3,000,000 with probability .05
- Losing $3,000,000 may be much worse than losing $1,000,000!

Review

- We learned about:
  - The importance of taking uncertainties into account
    - Sensitivity analysis
    - Expected value
    - Expected utility
  - Quantitative methods for doing so
    - Decision trees
    - Monte Carlo simulation