Lecture 2

Classifications of Cost, and Financial Mathematics
What Do Organizations Produce?

- **Physical output (products)**
- **Monetary income (profits)**
What Inputs Do They Use?

- **People’s services (labor)**

- **Materials and supplies**
  - Raw materials used to make their final products
  - Indirect materials (lubrication oil, etc.)
  - Electric power and other energy inputs

- **Capital (money), which is used to pay for:**
  - Land and buildings
  - Producer goods (e.g., tools, equipment)
  - Taxes (why are taxes an “input”?)
What Inputs Do We Need to Consider?

**Direct costs:**
- Costs associated with providing a particular product or service
- Typically materials and labor (wages and salaries)

**Overhead (or indirect costs):**
- Costs that cannot be traced directly to a particular product/service, because they help support multiple products or services
- Examples: Depreciation, taxes, insurance, maintenance
- Supervisors, engineers, and other administrative/clerical personnel
- Can also include materials and labor for inspection, testing, etc.

**Operating expenses:**
- The costs of doing business (typically **not** including depreciation)
- Includes both direct and indirect costs, but **not** capital
- Examples: Materials and supplies, wages and salaries, fuel, water, electric power, taxes, insurance
First Cost

The cost or total amount of investment required for getting an activity started:

- Occurs only once for any given activity
- Typically assumed to be paid in year 0
- Typically used for capital (land, buildings, tools, equipment), not operating expenses
Fixed Costs

Costs that **remain constant**:  
- Don’t vary with *level of production*

**Examples:**  
- Depreciation, maintenance, taxes, insurance, lease rentals, interest, sales programs, administrative expenses, research, heat, light, janitorial services

**Fixed costs are only relatively fixed**
Variable Costs

Costs that **vary with activity level**:
- E.g., with number of units produced
- Typically only **direct costs**
- May (or may not) remain constant **per unit of product**

**Examples:**
- Materials costs, direct labor, direct electric power
Incremental or Marginal Cost

- Refer to essentially the same concept:
  - Additional cost of making one more unit

- Let’s say:
  - Fixed cost $50, variable cost $1 per unit

- If we make 10 units:
  - Total cost is $60
  - Average cost is total cost/number of units:
    - $60/10 = $6 per unit
  - Marginal cost is the extra cost of increasing production by 1 unit: $1 per unit
### Average Versus Marginal Cost

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**Cost vs. Number of Units**

- **Total**
- **Average**
- **Marginal**

![Graph showing the relationship between cost and number of units](image-url)
Incremental or Marginal Cost

- This is the correct value to look at in deciding whether to increase production
  - It’s the extra cost we would have to pay!

- In our example:
  - Marginal cost << average cost
  - High fixed cost creates *economies of scale*

- Marginal cost can be > average cost:
  - ?
  - ?
Sunk Costs

- **Sunk cost is any cost that occurred in the past:**
  - Cannot be changed by a future decision or action

- **Examples:**
  - ____________________?
  - ____________________?

- **Sunk costs are irrelevant for making decisions:**
  - Sunk costs should be ignored in your choice!
  - (Except if they affect tax liability and depreciation)
Sunk Costs

Why are sunk costs irrelevant?
- Decisions should be made on the basis of differences between choices
- Identical factors can be canceled out

Sunk costs are already spent:
- Remain constant regardless of what you do
- Should be ignored in making decisions

This principle is difficult to apply:
- Why: ____________________________?
Sunk Costs

How should you consider sunk costs?

- *To learn what went wrong!*
- So you can avoid that in future decisions

Example:

- Ignore sunk costs in deciding whether to finish a half-completed project
- Study them to learn:
  - Why your project went wrong
  - How to avoid similar problems in the future
Financial Mathematics
Note!

We will **assume no inflation!**

- In the discussion that follows
- (And for the next several weeks)
**Notation**

- $i =$ interest rate (per time period)
- $n =$ # of time periods
- $P =$ money at *present*
- $F =$ money in *future*
  - After $n$ time periods
  - Equivalent to $P$ now, at interest rate $i$
- $A =$ payment at end of each time period
  - E.g., *annual*
Assumptions

- **Assume all cash flow occurs at the end of each time period**
  - For example, all year 1 payments are due on December 31 of year 1
- **The present is the end of period 0**
Overview

• Converting from P to F, and from F to P
• Converting from A to P, and from P to A
• Converting from F to A, and from A to F
Present to Future, and Future to Present
Converting from Present to Future

To find $F$ given $P$:

- Compound forward in time
Derive by Recursion

Invest an amount $P$ at rate $i$:
- Amount at time 1 = $P(1+i)$
- Amount at time 2 = $P(1+i)^2$
- Amount at time $n$ = $P(1+i)^n$

So we know that $F = P(1+i)^n$
- $(F/P, i\%, n) = (1+i)^n$
- Single payment compound amount factor

\[ F_n = P(1+i)^n \]
\[ F_n = P(F/P, i\%, n) \]
Example—Present to Future

Invest $P=1,000$, $n=3$, $i=10\%$

What is the future value, $F$?

\[
F_3 = $1,000 \times (F/P, 10\%, 3) = $1,000 \times (1.10)^3$
\[
\begin{align*}
&= $1,000 \times (1.3310) \\
&= $1,331.00
\end{align*}
\]
Converting from Future to Present

To find P given F:
- Discount back from the future

\[ F_n \]

Bring a single sum in future back to the “present”
Illustration of Discounting
Converting from Future to Present

**Amount F at time n:**
- Amount at time n-1 = $\frac{F}{(1+i)}$
- Amount at time n-2 = $\frac{F}{(1+i)^2}$
- Amount at time 0 = $\frac{F}{(1+i)^n}$

**So we know that** $P = \frac{F}{(1+i)^n}$

- $(P/F, \ i\%, \ n) = \frac{1}{(1+i)^n}$
- Single payment present worth factor
Example—Future to Present

Assume we want $F = $100,000 in 9 years. How much do we need to invest now, if the interest rate $i = 15\%$?

\[ P = \frac{F}{(1 + i)^n} \]

\[ P = \frac{100,000}{(1.15)^9} = 100,000 \left[ \frac{1}{(1.15)^9} \right] \]

\[ P = 100,000 \times 0.1111 = $11,110 \] at time $t = 0$
Review

Categories of cost:
- Capital costs, operating expenses
- Direct and indirect costs
- First cost, fixed cost, variable cost
- Sunk cost!

Ways to measure the cost of an activity:
- Total cost
- Average cost (total cost/number of units)
- Incremental or marginal cost

We learned how to convert present to future, and vice versa