Lecture 2

Classifications of Cost, and Financial Mathematics

What Do Organizations Produce?

- Physical output (products)
- Monetary income (profits)

What Inputs Do They Use?

- **People's services (labor)**
- **Materials and supplies**
  - Raw materials used to make their final products
  - Indirect materials (lubrication oil, etc.)
  - Electric power and other energy inputs
- **Capital (money), which is used to pay for:**
  - Land and buildings
  - Producer goods (e.g., tools, equipment)
  - Taxes (why are taxes an "input"?)

What Inputs Do We Need to Consider?

- **Direct costs:**
  - Costs associated with providing a particular product or service
  - Typically materials and labor (wages and salaries)
- **Overhead (or indirect costs):**
  - Costs that cannot be traced directly to a particular product/service, because they help support multiple products or services
  - Examples: Depreciation, taxes, insurance, maintenance
  - Supervisors, engineers, and other administrative/clerical personnel
  - Can also include materials and labor for inspection, testing, etc.
- **Operating expenses:**
  - The costs of doing business (typically not including depreciation)
  - Includes both direct and indirect costs, but not capital
  - Examples: Materials and supplies, wages and salaries, fuel, water, electric power, taxes, insurance

First Cost

- **The cost or total amount of investment required for getting an activity started:**
  - Occurs only once for any given activity
  - Typically assumed to be paid in year 0
  - Typically used for capital (land, buildings, tools, equipment), not operating expenses

Fixed Costs

- **Costs that remain constant:**
  - Don't vary with *level of production*
- **Examples:**
  - Depreciation, maintenance, taxes, insurance, lease rentals, interest, sales programs, administrative expenses, research, heat, light, janitorial services
- **Fixed costs are only *relatively* fixed**

Variable Costs

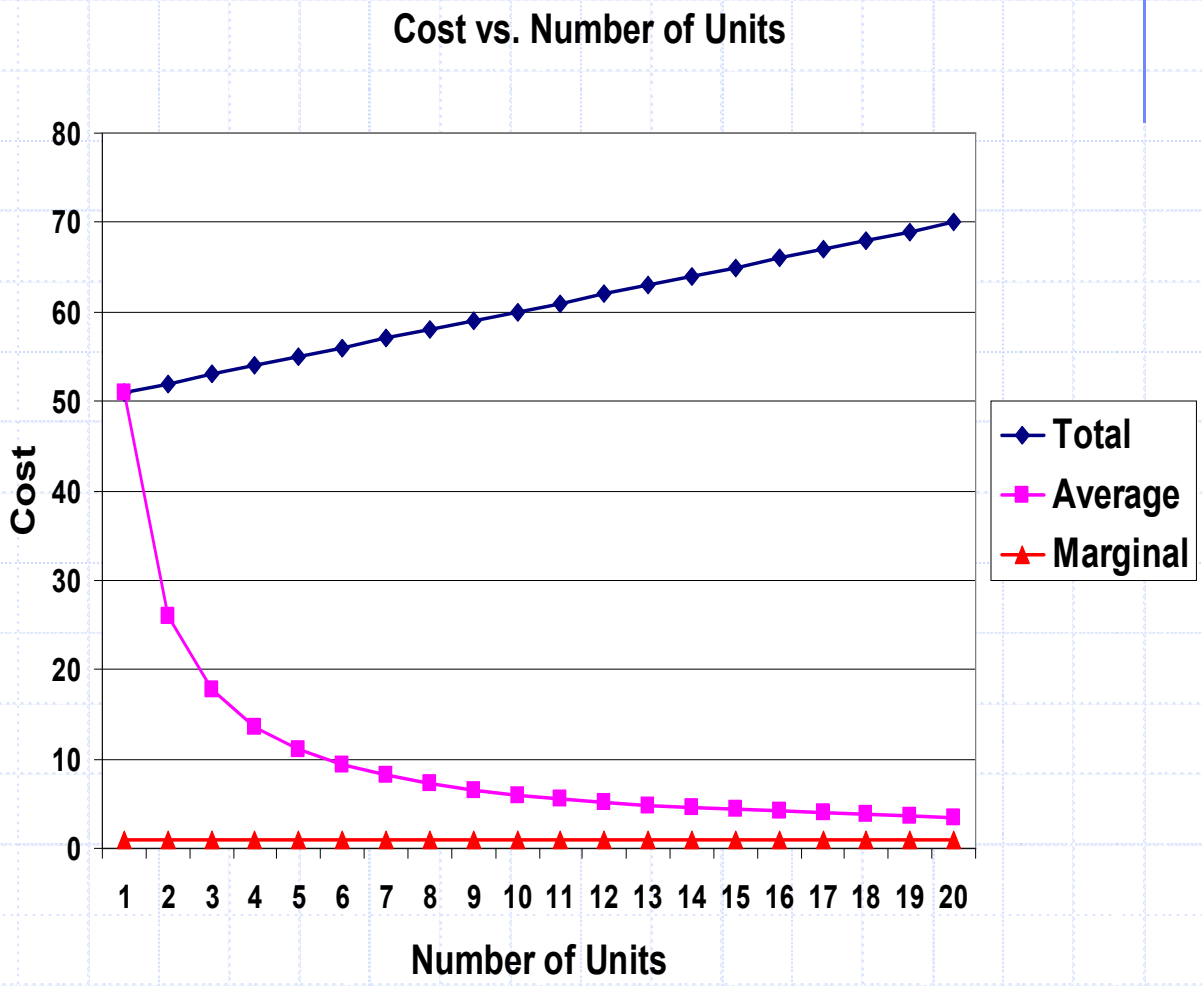
- **Costs that vary with activity level:**
  - E.g., with number of units produced
  - Typically only direct costs
  - May (or may not) remain constant *per unit of product*
- **Examples:**
  - Materials costs, direct labor, direct electric power

Incremental or Marginal Cost

- **Refer to essentially the same concept:**
  - Additional cost of making one more unit
- **Let's say:**
  - Fixed cost $50, variable cost $1 per unit
- **If we make 10 units:**
  - *Total* cost is $60
  - *Average* cost is total cost/number of units:
    - $60/10 = $6 per unit
  - *Marginal* cost is the extra cost of increasing production by 1 unit: $1 per unit

Average Versus Marginal Cost

Number	Total	Average	Marginal
1	51	51.00	1
2	52	26.00	1
3	53	17.67	1
4	54	13.50	1
5	55	11.00	1
6	56	9.33	1
7	57	8.14	1
8	58	7.25	1
9	59	6.56	1
10	60	6.00	1
11	61	5.55	1
12	62	5.17	1
13	63	4.85	1
14	64	4.57	1
15	65	4.33	1
16	66	4.13	1
17	67	3.94	1
18	68	3.78	1
19	69	3.63	1
20	70	3.50	1



Incremental or Marginal Cost

- **This is the correct value to look at in deciding whether to increase production**
  - It's the extra cost we would have to pay!
- **In our example:**
  - Marginal cost << average cost
  - High fixed cost creates *economies of scale*
- **Marginal cost can be > average cost:**
  - _____?
  - _____?

Sunk Costs

- **Sunk cost is any cost that occurred in the past:**
  - Cannot be changed by a future decision or action
- **Examples:**
  - _____?
  - _____?
- **Sunk costs are irrelevant for making decisions:**
  - Sunk costs should be ignored in your choice!
  - (Except if they affect tax liability and depreciation)

Sunk Costs

- **Why are sunk costs irrelevant?**
  - Decisions should be made on the basis of differences between choices
  - Identical factors can be canceled out
- **Sunk costs are already spent:**
  - Remain constant regardless of what you do
  - Should be ignored in making decisions
- **This principle is difficult to apply:**
  - Why: _____?

Sunk Costs

- **How should you consider sunk costs?**
  - *To learn what went wrong!*
  - So you can avoid that in future decisions
- **Example:**
  - Ignore sunk costs in deciding whether to finish a half-completed project
  - Study them to learn:
    - Why your project went wrong
    - How to avoid similar problems in the future

Financial Mathematics

Note!

- **We will *assume no inflation!***
  - In the discussion that follows
  - (And for the next several weeks)

Notation

- **i = interest rate (per time period)**
- **n = # of time periods**
- **P = money at *present***
- **F = money in *future***
  - After n time periods
  - Equivalent to P now, at interest rate i
- **A = payment at end of each time period**
  - E.g., *annual*

Assumptions

- **Assume all cash flow occurs at the *end* of each time period**
  - For example, all year 1 payments are due on December 31 of year 1
- **The present is the end of period 0**

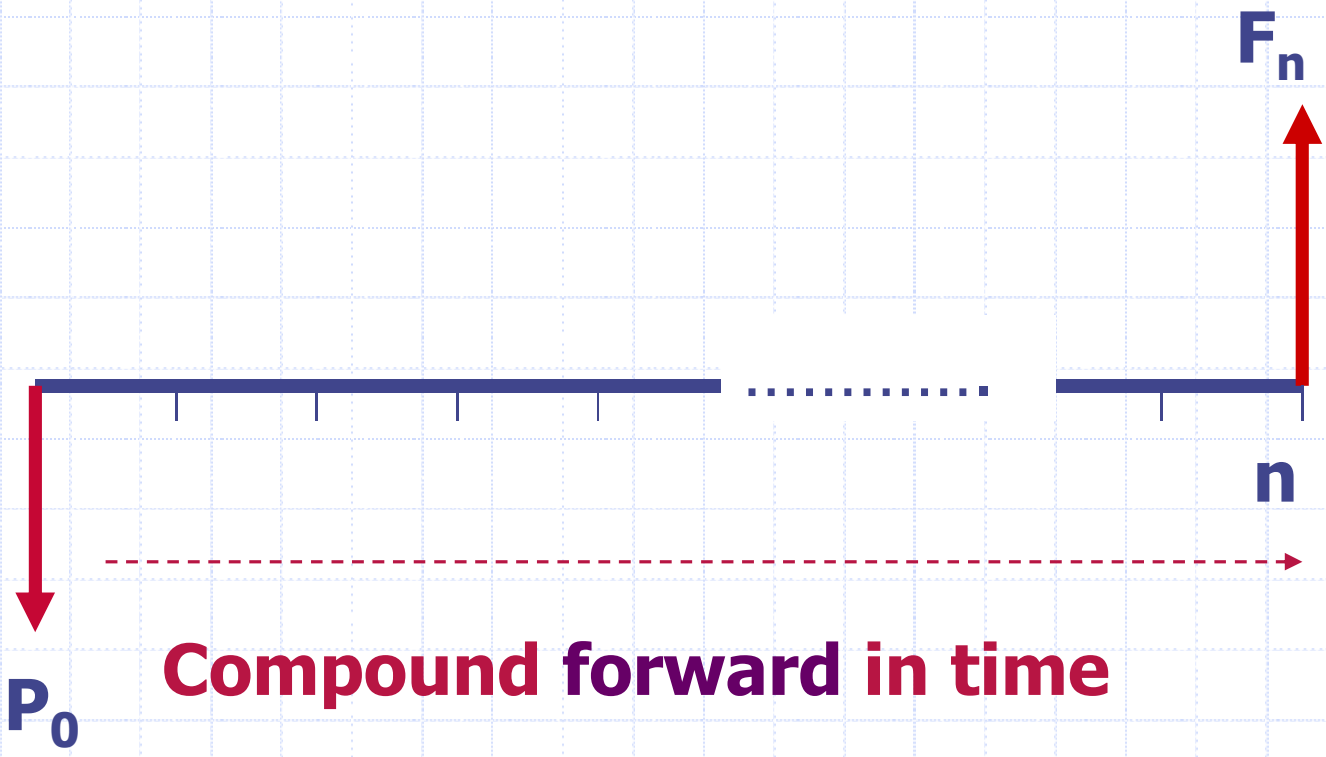
Overview

- Converting from P to F, and from F to P
- Converting from A to P, and from P to A
- Converting from F to A, and from A to F

Present to Future, and Future to Present

Converting from Present to Future

- **To find F given P:**



Derive by Recursion

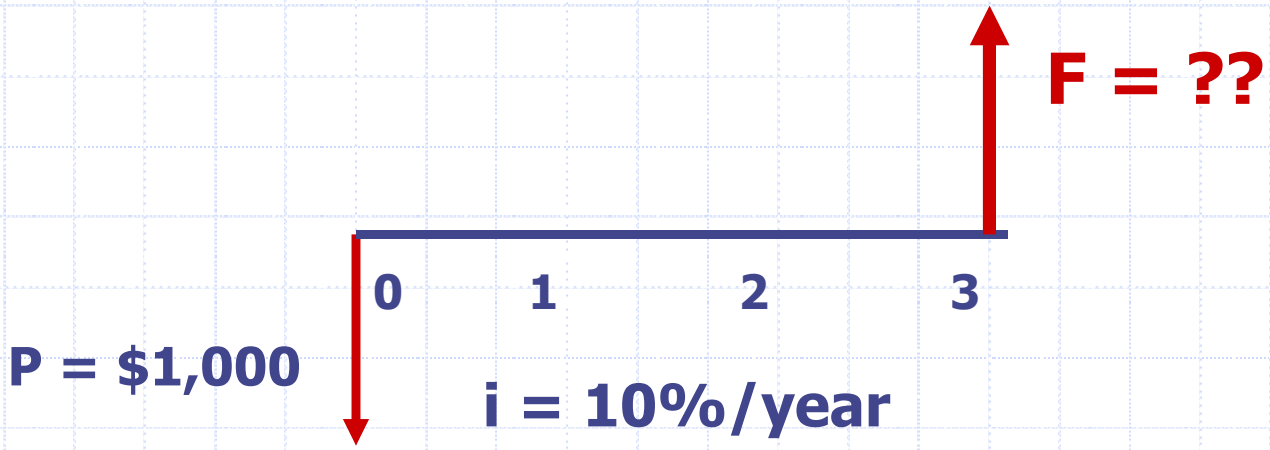
- **Invest an amount P at rate i:**
  - Amount at time 1 = $P(1+i)$
  - Amount at time 2 = $P(1+i)^2$
  - Amount at time n = $P(1+i)^n$
- **So we know that $F = P(1+i)^n$**
  - $(F/P, i\%, n) = (1+i)^n$
  - Single payment compound amount factor

$$F_n = P(1+i)^n$$

$$F_n = P(F/P, i\%, n)$$

Example—Present to Future

- Invest P=$1,000, n=3, i=10%
- What is the future value, F?

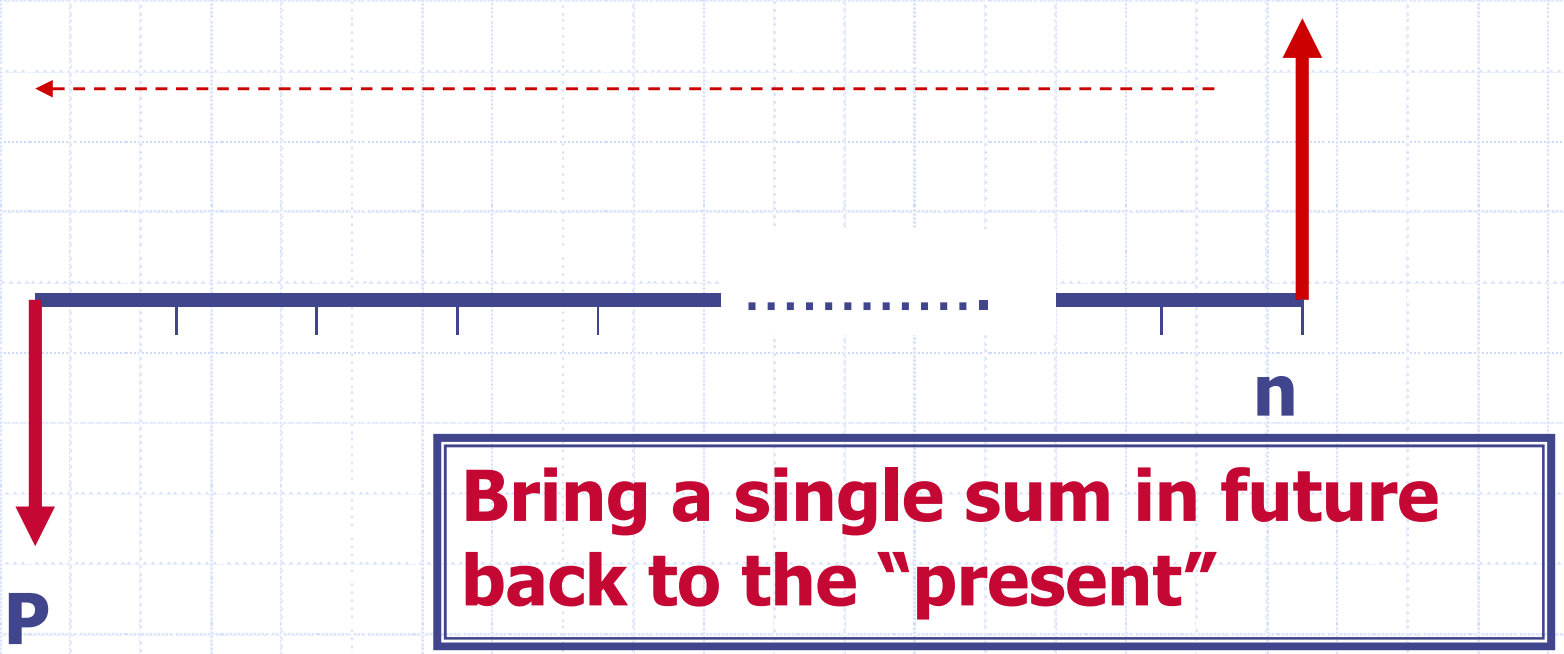


$F_3 = \$1{,}000\ (F/P, 10\%, 3) = \$1{,}000\ (1.10)^3$

$= \$1{,}000\ (1.3310) = \underline{\$1{,}331.00}$

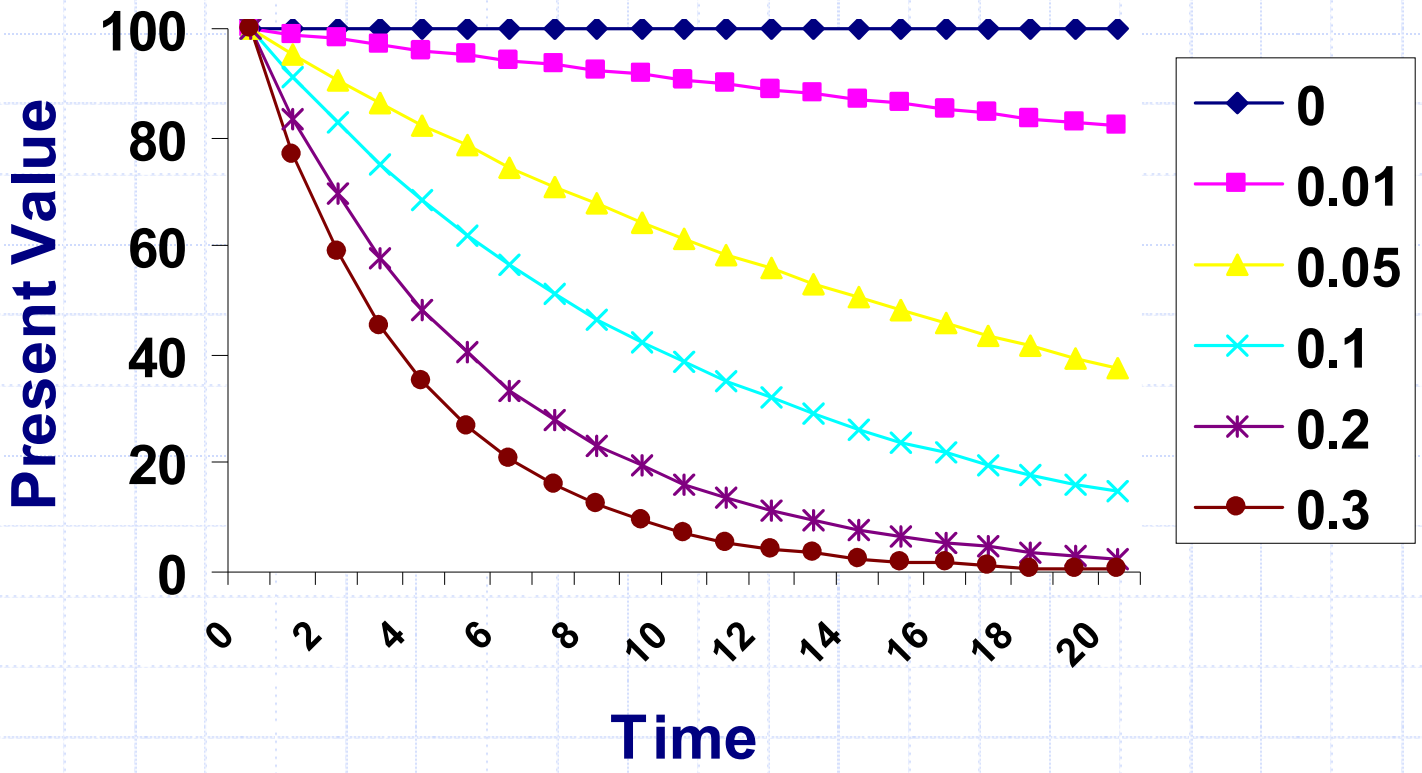
Converting from Future to Present

- **To find P given F:**
  - Discount back from the future F_n



Bring a single sum in future back to the "present"

Illustration of Discounting

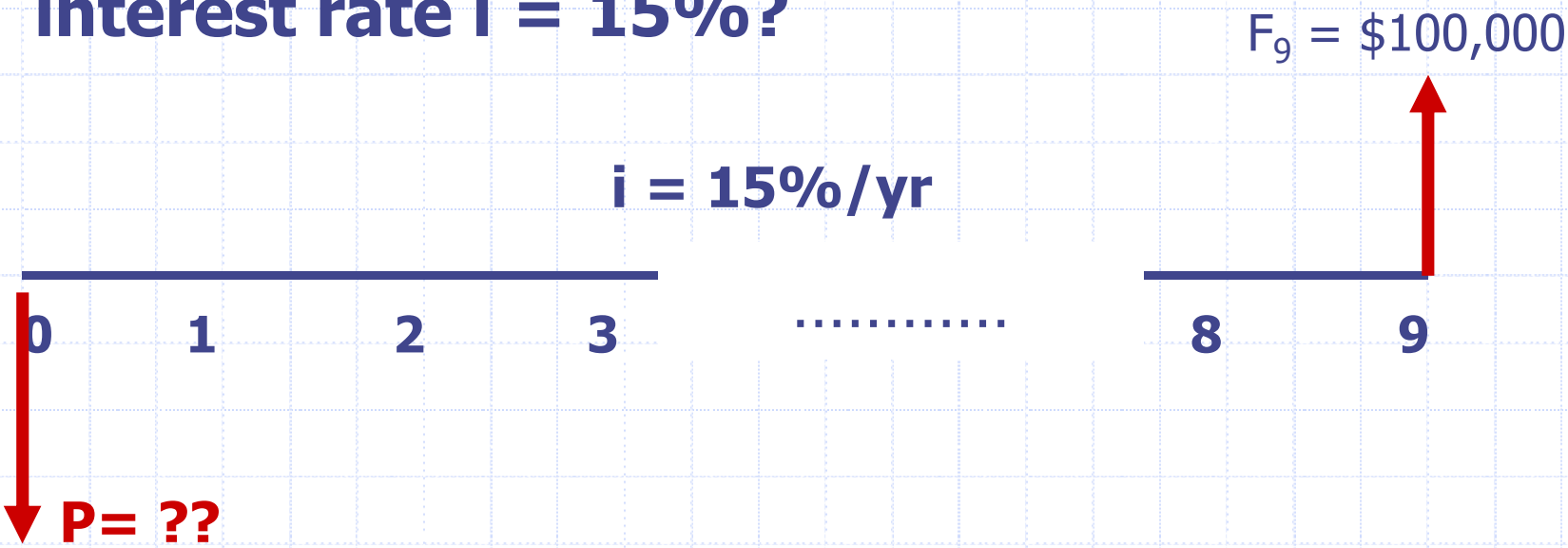


Converting from Future to Present

- **Amount F at time n:**
  - Amount at time n-1 = $F/(1+i)$
  - Amount at time n-2 = $F/(1+i)^2$
  - Amount at time 0 = $F/(1+i)^n$
- **So we know that $P = F/(1+i)^n$**
  - $(P/F, i\%, n) = 1/(1+i)^n$
  - Single payment present worth factor

Example—Future to Present

- Assume we want F = $100,000 in 9 years.
- How much do we need to invest now, if the interest rate i = 15%?



P = $100,000 (P/F, 15%, 9) = $100,000 [1/(1.15)^9]

= $100,000 (0.1111) = $11,110 at time t = 0

Review

- ***Categories* of cost:**
  - Capital costs, operating expenses
  - Direct and indirect costs
  - First cost, fixed cost, variable cost
  - *Sunk cost!*
- **Ways to *measure* the cost of an activity:**
  - Total cost
  - Average cost (total cost/number of units)
  - Incremental or *marginal cost*
- **We learned how to convert present to future, and vice versa**