Lecture 12

Retirement/Replacement Decisions
Review

Last time, we learned how to evaluate retirement and replacement when:

- Both options had same lifetime
- Optimal life for each option was *known!*
Complicating factors

- Alternatives may have different lifetimes
- Optimal lifetime may not be known:
  - Given present and future salvage values,
  - Consider multiple possible salvage dates
    - To determine the optimal lifetime
Example

- Equipment is 2 years old
  - Purchased for $25,000
- Assume $i^* = 15\%$ (before income tax)
- Resale value = $13,000 today
  - $10,000 in 1 year
  - $7,500 in 2 years
  - $5,500 in 3 or more years
Example

- Based on the *outsider viewpoint* (i.e., opportunity cost, *not cash flow!*),
  keeping the equipment is the same as buying it used for $13,000

- Assuming operating costs are the same as for new equipment:
  - Can safely ignore in comparison!
Example

- Compare alternative lifetimes
- Cost of keeping equipment for 1 year:
  - $13,000 (A/P, 15%, 1) = $14,950
    - This is like a *cost* of keeping the equipment
    - You don’t get it unless you sell!
  - Year 1 salvage value = -$10,000
  - Total = $4,950
    - (in year 1 dollars!)
Example

- Cost of keeping equipment for 2 years (given that I am keeping it for 1 year):
  - $10,000 (A/P, 15%, 1) = $11,500
  - Year 2 salvage value = -$7,500
  - Total = $4,000
  - (in year 2 dollars!)
Example

- Cost of keeping equipment for 3 years (given that I am keeping it for 2 years):
  - $7,500 (A/P, 15%, 1) = $8,625
  - Year 3 salvage value = -$5,500
  - Total = $3,125
    - (in year 3 dollars!)
### Example

<table>
<thead>
<tr>
<th>Year</th>
<th>&quot;Cash flow&quot;</th>
<th>Discounted</th>
<th>Discount rate</th>
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<tbody>
<tr>
<td>1</td>
<td>4950</td>
<td>4304</td>
<td>0.15</td>
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<tr>
<td>2</td>
<td>4000</td>
<td>3025</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3125</td>
<td>2055</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Present</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Year 1</td>
<td>4304</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Years 1 and 2</td>
<td>7329</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Years 1, 2, 3</td>
<td>9384</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Annual (already in year 1 dollars)</td>
<td>4950</td>
<td></td>
</tr>
<tr>
<td></td>
<td>equivalent</td>
<td>7329 (A/P, 15%, 2)= 4508</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>9384 (A/P, 15%, 3)= 4110</td>
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</table>

- What is the best option?
  - Keeping for 3 years has *lowest annual cost*
General formulation

- Equivalent annual cost of an asset:
  - Bought for $B in year 0
    - (or with current value of $B in year 0)
  - Sold for salvage value $S_n$ in year $n$

  equals $B \left(\frac{A}{P}, i^*, n\right) - S_n \left(\frac{A}{F}, i^*, n\right)$

- Can solve this for lots of values of $n$
  - To find optimum lifetime
    - (Minimum annual cost!)
General formulation

- If alternatives have same lifetime, then can treat year 0 salvage value as either:
  - A *cost* of keeping the current equipment or
  - A *benefit* of selling it
- If the alternatives have different lives, this is a problem
  - Why??
General formulation

- Assumption:
  - Each option is repeated over and over
  - If we use the old equipment for 3 years and repeat over and over
    - (e.g., by buying similar used equipment), salvage value will be paid every 3 years
    - So *should be annualized over 3 years*!
General formulation

- If alternatives have different lives, then:
  - Current salvage value *must* be considered a cost of keeping the equipment!
- If we subtract it from the cost of the new equipment:
  - It will be annualized over the wrong period
Realistic example

- In the previous example,
  - Optimum lifetime was determined *only* by declining salvage values over time

- In practice, equipment will have
  - Declining salvage values,
  - but also
    - *Increasing annual expenses!*
Realistic example

- Current machine is 1 year old, i*=15%
  - Resale value = $12,500 today
    - $8,000 in 1 year
    - $5,000 in 2 years
    - $2,200 in 3 years
  - Annual expenses = $8,900 in year 1
    - $10,500 in year 2
    - $12,500 in year 3
Realistic example

- Compare based on annual equivalent

If we keep equipment for 1 year, cost =

- $12,500 \(\text{A/P, 15\%, 1}\) = $14,375
  - Annualized opportunity cost
- -$8,000 \(\text{A/F, 15\%, 1}\) = -$8,000
  - Year 1 salvage value
- $8,900 (already in year 1) = $8,900

Total annual cost = $15,275
Realistic example

If we keep for 2 years, annual cost =

- $12,500 \(A/P, 15\%, 2\) = $7,689
- $5,000 \(A/F, 15\%, 2\) = -$2,326
  - Year 2 salvage value
  - $8,900 \(P/F, 15\%, 1\)(A/P, 15\%, 2) = $4,760
  - $10,500 \(A/F, 15\%, 2\) = $4,884
- Total annual cost = $15,008

Is this better or worse?
Realistic example

- If we keep for 3 years, annual cost =
  - $12,500 \text{ (A/P, 15\%, 3)} = $5,475
  - -$2,200 \text{ (A/F, 15\%, 3)} = -$634
  - $8,900 \text{ (P/F,15\%,1)(A/P,15\%,3)} = $3,390
  - $10,500 \text{ (P/F,15\%,2)(A/P,15\%,3)} = $3,477
  - $12,500 \text{ (A/F, 15\%, 3)} = $3,600
- Total annual cost = $15,278
  - This is worse again! (optimal lifetime=2 years)
Realistic example

- Lowest possible annual cost = $15,008

- Optimal decision:
  - If the best replacement costs < $15,008,
    - Then replace now!
    - (The best you can do by keeping it is $15,008)
  - Otherwise, replace in 2 years
    - (Assuming cost of the replacement is the same)
  - Could even keep for 3 years--When???
Realistic example

- Information gathered in next 2 years may change that decision:
  - *Actual* optimal lifetime may be more or less than predicted!
Judging proposed investments

- Annualized capital recovery cost (based on capital cost minus salvage cost) usually decreases with longer lifetime:
  - *Can amortize capital cost over more years!*

- Annualized operation/maintenance cost usually increases with longer lifetime:
  - *Equipment wears out!*
- Maintenance cost is increasing
- Capital recovery cost is decreasing
Judging proposed investments

- Minimum annual cost is often fairly flat:
  - Can choose any one of several lifetimes with only a modest difference in total cost

- In this example,
  - Cost of keeping equipment for 3 or 5 years is only slightly higher than optimum
Judging proposed investments

- Minimum annualized cost occurs when
  - Equivalent annual cost
    - = Marginal cost of an additional year
- If another year has total cost < average
  - Then keep for one more year!
- If another year has total cost > average
  - Then don’t keep equipment for that year!
Judging proposed investments

- Different rules depending on whether we will replace with:
  - Similar equipment
  or
  - *Different equipment*!
Replace with similar item

- When annualized cost is increasing:
  - Then the best option is to replace *now!*
When annualized cost is increasing,
  It may still be worth keeping equipment:
    Depends on whether cost of current equipment is still less than cost of replacement!

Future options must also be considered:
  If development will yield a better option,
    It might be worth waiting, even if the current option is more costly than the replacement that’s currently available--Example?
Review

- We learned how to choose between
  - Life extension
  - Replacement

- When options have different lifetimes:
  - Annualize, treat current salvage as a *cost*

- When optimal lifetime is unknown:
  - Compute minimum annual cost, compare!