Lecture 11

Retirement/Replacement Decisions
Reasons for retirement

- A better alternative exists
- Needs have changed
- The equipment has deteriorated
- The equipment has been damaged
Retirement or replacement

- An asset that is “retired” from one application may be used elsewhere
  - (Either sold to another business, or kept)
- Replacement may not mean retirement:
  - Old equipment may be kept for other uses
Complicating factors

- Extending the life of an existing asset is different from replacing it
- Retirement and replacement may have implications for income taxes
  - (Will be addressed in more detail later)
- The existing asset and the replacement may have different lifetimes
Example

- Should we sell our old warehouse
  - And rent space?
- Warehouse is 10 years old
  - Purchased for:
    - $40,000 land
    - $160,000 building
  - Is this relevant???
- Assume $i^* = 20\%$ (before income tax)
Example

- Current annual expenses:
  - $14,000/year operations and maintenance
  - $4,600/year property tax
  - $1,500/year insurance on warehouse
  - $3,000/year insurance on inventory
  - $23,100/year total

(Ignore income tax for now)
Example

- Plan was to sell 10 years from now:
  - For $250,000
- Just received an offer today:
  - For $350,000
Example

New expenses if we rent space:

- $65,000/year rent
- $5,200/year operations and maintenance
- $1,600/year insurance on inventory
- $71,800/year total
Example

- Compare based on annual equivalent

Annual equivalent of current option:

- Annual expenses $23,100
- $350,000 \((A/P, 20\%, 10)\) = $83,500
  - This is like a *cost* of keeping the warehouse
  - You don’t get it unless you sell!
- -$250,000 \((A/F, 20\%, 10)\) = -$9,600
- Total = $97,000
Example

- Annual equivalent of renting:
  - Annual expenses $71,800

- Annual equivalent of current option:
  - Annual expenses $97,000

- Is renting better?
  - Yes, it’s cheaper to rent!
    - In practice, might want to do after-tax analysis with lower i*
Example

- Compare based on IRR
- Keeping warehouse has higher first cost
  - Because we forego the current sale price
- Cost of renting - cost of owning:
  - Year 0                - $350,000
  - Years 1-10 ($71,800-$23,100) = $48,700
  - Year 10 salvage value $250,000
Example

First try: discount rate $i^* = 20\%$

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>Discounted</th>
<th>Discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-350.0</td>
<td>-350.00</td>
<td>0.2</td>
</tr>
<tr>
<td>1</td>
<td>48.7</td>
<td>40.58</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>48.7</td>
<td>33.82</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>48.7</td>
<td>28.18</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>48.7</td>
<td>23.49</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>48.7</td>
<td>19.57</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>48.7</td>
<td>16.31</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>48.7</td>
<td>13.59</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>48.7</td>
<td>11.33</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>48.7</td>
<td>9.44</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>48.7</td>
<td>7.87</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>250.0</td>
<td>40.38</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>-105.45</td>
<td></td>
</tr>
</tbody>
</table>

Present worth: -105.45
### Example

#### Second try: discount rate $i^* = 10\%$

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>Discounted</th>
<th>Discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-350.0</td>
<td>-350.00</td>
<td>0.1</td>
</tr>
<tr>
<td>1</td>
<td>48.7</td>
<td>44.27</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>48.7</td>
<td>40.25</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>48.7</td>
<td>36.59</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>48.7</td>
<td>33.26</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>48.7</td>
<td>30.24</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>48.7</td>
<td>27.49</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>48.7</td>
<td>24.99</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>48.7</td>
<td>22.72</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>48.7</td>
<td>20.65</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>48.7</td>
<td>18.78</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>250.0</td>
<td>96.39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Present worth</td>
<td>45.63</td>
<td></td>
</tr>
</tbody>
</table>
### Example

<table>
<thead>
<tr>
<th>Year</th>
<th>Cash flow</th>
<th>Discounted</th>
<th>Discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-350.0</td>
<td>-350.00</td>
<td>0.123</td>
</tr>
<tr>
<td>1</td>
<td>48.7</td>
<td>43.37</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>48.7</td>
<td>38.62</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>48.7</td>
<td>34.39</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>48.7</td>
<td>30.62</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>48.7</td>
<td>27.27</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>48.7</td>
<td>24.28</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>48.7</td>
<td>21.62</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>48.7</td>
<td>19.25</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>48.7</td>
<td>17.14</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>48.7</td>
<td>15.27</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>250.0</td>
<td>78.37</td>
<td></td>
</tr>
</tbody>
</table>

**Present worth**: 0.19

- **Converged**: discount rate $i^* = 12.3\%$
Example

- Analysis is easy:
  - Because both options have the same lives (10 years)

- Keeping the old warehouse is like investing money at 12.3%:
  - Since our i* is 20%, this is not good!
Judging proposed investments

- In this example, we used annual equivalent and internal rate of return interchangeably
  - This is OK, because options had same life
- With different lives:
  - *Use annual equivalent!*
Judging proposed investments

- Decisions about life extension involve both present and future **salvage values**
  - Forego **present** value to get **future** one *(opportunity cost!)*

- Present salvage value can be **either**
  - Added to the life extension cost, **or**
  - Subtracted from the replacement cost,
  - But not both!
Judging proposed investments

- When lifetimes are different,
  - Present salvage value *must be* added to the life extension cost!
  - Or else it won’t recur with right frequency!

- This seems unintuitive:
  - You don’t actually *get* the salvage value unless you *replace* the equipment!
Judging proposed investments

- Think of it as “buying a used item of equipment” from scratch:
  - You are choosing the best *policy*,
  - Not making a one-time choice

- If keeping your used equipment is best,
  - Assume that you will eventually replace it with a *similar* used item of equipment
  - Take an outsider’s viewpoint!
Another example

- Wrong equipment bought:
  - Pump cost $3,600 one year ago
  - Power cost = $2,000/year
    - Because of poor pump choice
  - Is this relevant???

- Assume $i^* = 18\%$
  - (before income tax)
Another example

- New expenses if we replace pump:
  - $3,400 cost of new pump
  - $1,100/year power cost
  - $700 salvage value of old pump
Another example

- Should we replace the pump?
- Assume a 10-year remaining life
  - (For both new pump and existing pump)
Another example

- Compare based on annual equivalent

Annual equivalent of current pump:

- Annual expenses $2,000/year
- $700 (A/P, 18%, 10) = $156/year
  - This is like a *cost* of keeping the current pump,
  - because you don’t get it unless you sell!

- Total = $2,156/year
Another example

- Annual equivalent of replacing pump:
  - Annual expenses $1,100/year
  - $3,400 (A/P, 18%, 10) = $757/year
  - Total = $1,857/year

- Is it better to replace the pump?
Note

- Sunk cost
  - E.g., sell asset before its expected lifetime
- Does this show that the past decision was “bad”?
Review

- We learned how to choose between
  - Life extension
    - (Keeping old item of equipment)
  - Replacement

- So far:
  - Both options had same lifetime
  - Optimal life for each option was *known!*
Dynamic example (study on your own?)

- Should we replace leaking gas mains?
  - Gas costs $5/thousand feet$^3$
  - New pipe costs $40,000/mile
  - Old pipe has $0$ salvage value

- New pipe has no leaks for 15 years,
  - Then increases by 100,000 feet$^3$/mile/year

- Assume $i^* = 10\%$
Dynamic example

- Annual equivalent of replacing pipe:
  - Need annual equivalent of gas losses
  - This is complicated:
    - Non-equal (gradient) amounts by year
    - Doesn’t start for 15 years
  - Two ways to do this:
    - Convert gradient to annual (in several steps)
    - Trial and error in spreadsheet
Dynamic example

- Convert gradient to annual
  - Leaks in years 16-25 are equivalent to:
    - 11-year gradient starting in year 15
      - (1st year of gradient is always 0)
  - Convert to “present” value in year 14 (before start of gradient) according to:
    - $5/thousand ft\(^3\) (100,000 ft\(^3\)) (P\(_{14}/G\), 10\%, 11)
      - = $500 (26.4) = $13,200
Dynamic example

- Convert gradient to annual
  - Can we convert “present” value in year 14 directly to annual amount over years 1-25?
    - *No!* Two different time periods involved:
      - $P_{14}$ is 14 years into the future
      - But we want to annualize it over 25 years!
    - Convert to year 0 by $13,200 (P/F, 10\%, 14)$
      - $= 3475$
    - Annualize by $3475 (A/P, 10\%, 25) = 383$
<table>
<thead>
<tr>
<th>Year</th>
<th>Losses</th>
<th>Discounted</th>
<th>Annual</th>
<th>Discounted</th>
<th>Discount rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>348.09</td>
<td>0.1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>316.45</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>287.68</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>261.53</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>237.75</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>216.14</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>196.49</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>178.63</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>162.39</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>147.62</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>134.20</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>122.00</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>110.91</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>100.83</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0.00</td>
<td>382.9</td>
<td>91.66</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>500</td>
<td>108.81</td>
<td>382.9</td>
<td>83.33</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>1000</td>
<td>197.84</td>
<td>382.9</td>
<td>75.75</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>1500</td>
<td>269.79</td>
<td>382.9</td>
<td>68.87</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>2000</td>
<td>327.02</td>
<td>382.9</td>
<td>62.61</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>2500</td>
<td>371.61</td>
<td>382.9</td>
<td>56.92</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>3000</td>
<td>405.39</td>
<td>382.9</td>
<td>51.74</td>
<td></td>
</tr>
<tr>
<td>22</td>
<td>3500</td>
<td>429.96</td>
<td>382.9</td>
<td>47.04</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>4000</td>
<td>446.71</td>
<td>382.9</td>
<td>42.76</td>
<td></td>
</tr>
<tr>
<td>24</td>
<td>4500</td>
<td>456.87</td>
<td>382.9</td>
<td>38.87</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>5000</td>
<td>461.48</td>
<td>382.9</td>
<td>35.34</td>
<td></td>
</tr>
<tr>
<td><strong>Present worth</strong></td>
<td></td>
<td></td>
<td><strong>3475.48</strong></td>
<td><strong>3475.60</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Converged: annual equivalent = 383**
Dynamic example

- Compare based on annual equivalent

Annual equivalent of replacing pipe:

- Annualized gas losses $383
- $40,000 (A/P, 10%, 20) = $4,407
  - Annualized capital cost

- Total = $4,790
Dynamic example

- Replace current pipe if losses > $4,790
- Assumptions:
  - Current losses are only going to grow
  - Original cost of current pipe is a *sunk cost!*
  - Current pipe has 0 salvage value,
    - So there is no opportunity cost of keeping it
    - Only annualized losses (no capital cost)