Lecture 10
Interest and equivalence

- Know how to obtain one value
  - P, F, or A
  - Given the interest rate and the number of time periods

- Know how to use:
  - \((P/A, i, n), P/F, i, n\), \((F/P, i, n), (A/F, i, n)\), etc.
Unknown interest rates

- Know how to get the interest rate in closed form:
  - When given P, F, and n
- Know how to get the interest rate using interpolation:
  - In more complex problems
Compounding

Know how to use the formula:

\[ F = P \left(1 + \frac{r}{m}\right)^{# \text{ of periods}} \]

What is \( r \)?

What is \( m \)?

- \( m \) and \( # \text{ of periods} \) must be measured in the same units
Compounding (continued)

- Know how to convert a nominal annual rate to the corresponding effective rate when compounded:
  - Monthly, quarterly, semiannually, etc.
  - and vice versa
- Know when to use:
  - \( F = P \ (1+i)^n \)
- and when to use:
  - \( F = P \ (1+r/m)^{# \ of \ periods} \)
Present worth analysis

- Know how to select the best alternative based on present worth:
  - Know how to use (P/A, I, n)
  - Know how to use (P/F, I, n)
  - Remember to bring everything back to the present
  - Be aware of “gaps” between annual streams

- Know how to handle perpetual lifetimes

- What are the most important things to know and pitfalls to avoid in present worth?
Annual equivalent cash flow

- Know how to use:
  - \((A/P, i, n)\), \((A/F, i, n)\), \((P/A, i, n)\), \((F/A, i, n)\)
  - Be aware of “gaps” between annual streams

- Know how to handle problems with different lifetimes:
  - Perpetual lifetimes
  - Why you don’t need to convert to a common lifetime
Internal rate of return

- Know what the concept means:
  - Internal rate of return is the value of i for which the present worth equals zero

- Know how to compare the internal rate of return to \( i^* \):
  - When is an alternative desirable?
  - Remember that \( i^* \) is equal to the minimum acceptable rate of return

- Know how to obtain the internal rate of return from a set of costs and benefits
Internal rate of return (cont.)

- What are the most important things to know and pitfalls to avoid in internal rate of return?
Lecture 10

Payback Period
Definitions

- Payback period:
  - Estimated time to recover initial investment

- Crude (or “no-return”) payback period:
  - Recover initial investment with no return

- Payback period with interest:
  - Recover initial investment plus return
    - (Time at which discounted future cash flow equals initial investment)
Payback period

- Payback period is not an appropriate way to compare investments:
  - All cash flows after the payback period are neglected!
  - Not mathematically equivalent to present worth, annual equivalent, rate of return, and benefit/cost ratio
  - May neglect required return on investment
Payback period

- This is a bad method!
  - Not equivalent to discounted cash flow methods
  - Ignores important information

- Nevertheless, it’s commonly found:
  - No need for discounted cash flow calculations, minimum acceptable rate of return
  - Easy to understand
  - Favors events nearer in time (less uncertain)

- So we need to know about it
Crude (simple) payback

- Look for *the first time at which the sum of cash flows up to then is nonnegative*:
  - This is the “payback period”

Example:

- Cash flows of -1000, 500, 500
- Payback period is 2
  - All dollars recovered by end of period 2
More on crude payback

- Now consider another investment:
  - Cash flows of -1000, 0, 0, 10,000
- Discounted cash flow methods prefer this cash flow to the previous one at any MARR up to 300%:
  - Yet payback period is 3,
  - So crude payback test would prefer previous cash flow to this one!
Discounted payback

- Also known as “payback with time value of money”

- Now we change payback rule to look for:
  - The first time at which the discounted present value of cash flows up to then is nonnegative
  - Use minimum acceptable rate of return for discounting

- This is used in place of crude payback period
More on discounted payback

- This ignores any cash flows beyond the payback period

Example:
- Cash flow 1: -1500, 2000
- Cash flow 2: -1500, 1000, 10,000
  - Crude payback test prefers cash flow 1
  - Discounted payback test prefers cash flow 1 at any rate of return between –33% and +33%
Payback period

- Biases of payback period:
  - Tends to favor short-term projects
  - Can minimize beneficial investments
My advice

- Don’t use payback in any form:
  - Crude or discounted

- Know what it is and how to use it:
  - In case your organization requires it

- Be prepared to argue for use of correct discounted cash flow methods:
  - If you have an opportunity to do so
Payback period

- May be acceptable as a secondary criterion:
  - Especially if funds are very limited!
    - (E.g., in small businesses)
  - Shorter payback period frees up funds sooner for other investments
Breakeven Analysis
Breakeven analysis

- If all of the parameters for a problem are known except one:
  - Then the unknown parameter can be calculated or approximated
- Set present worth, future worth, or annual worth equal to zero:
  - And solve for (or approximate) the unknown parameter
Example

- Your company is preparing a quote to machine 500,000 castings per year for 5 years
- The company will need to:
  - Purchase machining centers
  - Purchase or lease space
  - Hire labor
- **What price should the company charge to earn 15% rate of return?**
Example (continued)

- You could also specify fixed price per casting:
  - And determine how many castings your company would need to sell
  - To earn 15% rate of return
Solving for a breakeven value

Ways of solving for an unknown parameter:

1. Direct solution (manually) – simple problems (e.g., just P and F)
2. Trial and error (manually) – slightly more complex problems
3. Spreadsheet model – can use Excel financial functions (PV, FV, RATE, IRR, NPV, PMT, NPER)
Problems with costs and revenues

- Breakeven analysis is commonly used to study relationships among costs, revenue, and **volume**:
  - Define cost and revenue functions
  - Linear (or non-linear) functions of volume, price, etc.

- Objective: Find the value (volume, price, etc.) that maximizes profits
Fixed costs

- Do not vary with production or activity levels, price, etc.

Examples:
- Buildings
- Insurance
- Fixed overhead
- Equipment
- Etc.
Fixed costs

- Constant for all values of the variable in question
  - Even if no level of activity:
    - Fixed costs still continue!
  - Must **shut down** the activity before fixed costs can be eliminated
- To compensate for fixed costs:
  - *You need a minimum volume or price!*
Variable costs

- Vary with the level of activity
- Examples:
  - Direct labor (wages)
  - Materials
  - Indirect costs (e.g., fringe benefits)
  - Marketing
  - Advertising
  - Warranty
  - Etc.
Variable costs

- More activity (volume):
  - Greater variable costs
- Less activity (volume):
  - Lower variable costs
- Variable costs can also be affected by higher sales volume:
  - If non-linear function of volume
  - (E.g., economies of scale)
Total costs

- **Total cost:**
  - Fixed cost plus variable cost

- **Profit:**
  - Revenue minus total cost
Cost and revenue relationships

- The relationship of cost and revenue to volume may be:
  - Linear, or
  - Non-linear (e.g., economies of scale)
- Both are just approximations
Linear cost relationship
Non-linear cost relationship

- **Fixed costs (level)**
- **Variable costs**
- **Total costs**

Level of activity vs. Cost
Breakeven

- The breakeven point is the point where the revenue and total cost relationships intersect

- For non-linear functions:
  - It is possible to have more than one breakeven point!
Breakeven

- Assumed revenue and cost relationships tend to be static:
  - May not reflect the reality of a dynamic firm
  - (E.g., reductions in variable cost, to improve efficiency)
- However, the breakeven point can still be useful for planning purposes
Non-linear functions

Multiple breakeven points for a non-linear model!
Non-linear functions

For non-linear functions:

- There may be multiple breakeven points
- Simply being above a breakeven point may not guarantee a positive profit
- We want to find the level of volume or price that yields the maximum profit!
The breakeven point for a problem can be expressed as:

- Units per time period
- Hours per month
- Price per unit
- Etc.

At breakeven, you are indifferent about whether to do the project.
Summary

- Revenue and cost can be:
  - Linear, or
  - Non-linear

- Breakeven analysis is a form of *sensitivity analysis*

- Complex models can be evaluated using Excel
Concept Quiz

- Can payback period be made consistent with present worth if implemented correctly?
- Name two important factors that are ignored by non-discounted payback period
- Why is payback period commonly used in practice?
- What can go wrong if decisions are made based on payback period?