CSSU Math Frameworks

The CSSU Math Frameworks is based on the National Council for Teachers of Mathematics (NCTM) Standards and is aligned with the *Vermont Expectations for Mathematics* (Vermont Institutes.)

What are the NCTM Standards? Descriptions of the mathematical understanding, knowledge, and skills that students should acquire from pre-kindergarten through grade 12

- **Content Standards** explicitly describe the content students should learn.
- **Process Standards** highlight ways of acquiring and using content knowledge.

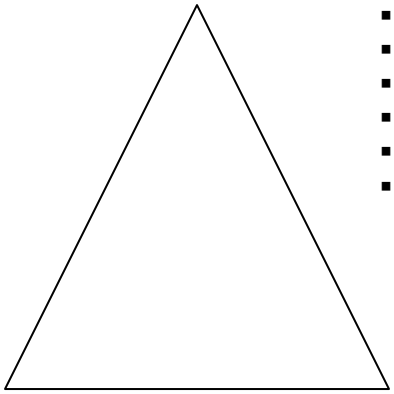
Goals for students:

- learn to value mathematics;
- become confident in their ability to do mathematics;
- learn to communicate mathematics; and
- learn to reason mathematically.

CURRICULUM

NCTM Content & Process Standards

- Number & Operations
- Algebra
- Geometry
- Measurement
- Data Analysis & Probability
- Process Standards (Problem Solving, Reasoning & Communication)



INSTRUCTION

- Instructional elements: NCTM Principles and Teaching Standards
- Instructional Programs: (*Everyday Math, Connected Math,* Algebra courses, etc.)

ASSESSMENT

- *Vermont Grade Level Expectations for Math*
- State assessments
- Local assessments

CSSU Math Curriculum Committee

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NCTM PRINCIPLES

EQUITY: ***Excellence in mathematics education requires equity and high expectations and strong support for all students.***

- ♦ High expectations and worthwhile opportunities for all
- ♦ Accommodating differences to help everyone learn mathematics
- ♦ Resources and support for all classrooms and all students

Equity does not mean that every student should receive identical instruction; instead, it demands that reasonable and appropriate accommodations be made as needed to promote access and attainment for all students.

CURRICULUM: ***A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.***

- ♦ Seeing how ideas build and connect with other ideas
- ♦ Mathematics that will prepare students for continued study and for solving problems in a variety of school, home, and work settings
- ♦ Opportunities to learn increasingly more sophisticated mathematical ideas as they progress through the grades

The strands are highly interconnected. A coherent curriculum effectively organizes and integrates concepts so that students can see how the ideas connect… enabling them to develop new understandings.

TEACHING: ***Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.***

- ♦ Knowing and understanding mathematics, students as learners, and pedagogical strategies
- ♦ A challenging and supporting classroom learning environment
- ♦ Opportunities to reflect on and refine instructional practices

To be effective, teachers must know and understand deeply the mathematics they are teaching and be able to draw on that knowledge with flexibility in their teaching tasks.

LEARNING: ***Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.***

- ♦ The vision of *Principles and Standards* is based on students learning mathematics with understanding.
- ♦ Conceptual understanding allows students to use their knowledge flexibly and solve new problems.
- ♦ When students learn mathematics, they combine factual knowledge, procedural facility, and conceptual understanding in powerful ways.

Learning with understanding is essential to enable students to solve the new kinds of problems they will eventually face in the future. Effective learners reflect on their thinking and learn from their mistakes. Classroom discourse and social interaction can be used to promote the recognition of connections...

ASSESSMENT: ***Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.***

- ♦ Assessment should be an on-going process in which teachers measure student understanding in order to guide their instruction and enhance student learning.
- ♦ Three types of assessment are pre-assessment, formative assessment, and summative assessment.
- ♦ Assembling evidence from a variety of sources such as open-ended questions, constructed-response tasks, selected-response items, performance tasks, observations, conversations, journals, and portfolios.

Assessment is an integral part of instruction that informs and guides teachers as they make instructional decisions. Assessment should become a routine part of the ongoing classroom rather than an interruption… assembling evidence from a variety of sources is more likely to yield an accurate picture.

TECHNOLOGY: ***Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.***

- ♦ Students can learn more mathematics more deeply with the appropriate and responsible use of technology.
- ♦ Technology cannot replace the mathematics teacher, nor can it be used as a replacement for basic understandings and intuitions.
- ♦ When technology tools are available, students can focus on decision making, reflection, reasoning, and problem solving.

Technology supports effective teaching, but should not be used as a replacement for basic understandings and intuitions.

TEACHING STANDARDS

Choosing an excellent mathematics series that meets the goals of NCTM Standards can be the first step toward revitalizing our mathematics classes. But materials alone do not make the program. Another key to success is the teacher's willingness to increase the effectiveness of the materials by spending considerable time planning the lesson, listen carefully to what the students are saying in the classroom, analyzing what they are learning, and consequently adjusting the mathematical tasks and the questions asked. In other words, teaching matters. --Glenda Lappan, President, NCTM

WORTHWHILE MATHEMATICAL TASKS: ***The teacher of mathematics should pose tasks that are based on-***

- Knowledge of the range of ways that diverse students learn mathematics
- Develop students' mathematical understandings and skills
- Stimulate students to make connections
- Call for problem formulation, problem solving, and mathematical reasoning
- Promote communications about mathematics
- Represent mathematics as an ongoing human activity

TEACHER'S ROLE IN DISCOURSE: ***The teacher of mathematics should orchestrate discourse by-***

- Posing questions and tasks that elicit, engage, and challenge each student's thinking
- Listening carefully to student's ideas
- Asking students to clarify and justify their ideas orally and in writing

STUDENTS' ROLE IN DISCOURSE: ***The teacher of mathematics should promote discourse in which students-***

- Listen to, respond to, and question the teacher and one another
- Use a variety of tools to reason, make connections, solve problems, and communicate
- Try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers
- Rely on mathematical evidence to determine validity

TOOLS FOR ENHANCING DISCOURSE: ***The teacher of mathematics, in order to enhance discourse, should encourage and accept the use of-***

- Computers, calculators, and other technology
- Concrete materials used as models
- Pictures, diagrams, tables, and graphs
- Metaphors, analogies, and stories

LEARNING ENVIRONMENT: ***The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by-***

- Providing structure and time necessary to explore and grapple with significant ideas and problems
- Using the physical space and materials to facilitate student learning
- Respecting and valuing students' ideas, ways of thinking, and mathematical dispositions ***and by consistently expecting and encouraging students to-***
- Work independently and collaboratively to make sense of mathematics
- Take intellectual risks by raising questions and posing conjectures

ANALYSIS OF TEACHING AND LEARNING: ***the teacher of mathematics should engage in ongoing analysis of teaching and learning by-***

- Observing, listening to, and gathering information about students to assess what they are learning
- Examining effects of tasks, discourse, and learning environment on students' mathematical knowledge, skills, and dispositions

In order to-

Ensure that every student is learning
Challenge and extend students' thinking
Adapt or change activities while teaching
Report to students, parents, and administrators

Sources:

- *Professional Standards for Teaching Mathematics*. Copyright 1991 by National Council of Teachers of Mathematics.
- *Principles and Standards for Teaching Mathematics*. Copyright 2000 by National Council of Teachers of Mathematics.
- *Principles and Standards for Teaching Mathematics – Outreach CD, 2nd Edition*. Copyright 2001 by National Council of Teachers of Mathematics.

- C. Hopkinson, 3/28/04

Everyday Math (EDM)

Everyday Mathematics is a research-based curriculum developed by the University of Chicago School Mathematics Project. UCSMP was founded in 1983 during a time of growing consensus that our nation was failing to provide its students with an adequate mathematical education. The goal of this on-going project is to significantly improve the mathematics curriculum and instruction for all school children in the U.S. Based on their findings, the authors established several basic principles that have guided the development of Everyday Mathematics. These principles are:

- Students acquire knowledge and skills, and develop an understanding of mathematics from their own experience. Mathematics is more meaningful when it is rooted in real life contexts and situations, and when children are given the opportunity to become actively involved in learning. Teachers and other adults play a very important role in providing children with rich and meaningful mathematical experiences.
- Children begin school with more mathematical knowledge and intuition than previously believed. A K-6 curriculum should build on this intuitive and concrete foundation, gradually helping children gain an understanding of the abstract and symbolic.
- Teachers, and their ability to provide excellent instruction, are the key factors in the success of any program. Previous efforts to reform mathematics instruction failed because they did not adequately consider the working lives of teachers.

What are some key features of Everyday Math?

- Problem solving for everyday situations
- Developing readiness through hands-on activities
- Establishing links between past experiences and explorations of new concepts
- Sharing ideas through discussion
- Cooperative learning
- Practice through games
- Ongoing review throughout the year
- Daily routines
- Ongoing assessment
- Home and school partnership

When should I expect mastery of a skill?

- A concept builds within a school year and within the K-5 experience
- Concept is introduced and then revisited in a new way asking children to apply concepts over time, deepening their understanding
- 5 times in 2 years
- Proficiency levels – Beginning, Developing, Secure

What are the components of Everyday Math?

- Journal
- Mental Math and Reflexes
- Math Boxes
- Student Reference Book
- Homelinks
- Games
- Checking Progress and Profiles of Progress

Connected Math Project (CMP)

The Connected Mathematics Project (CMP) was funded by the National Science Foundation between 1991 and 1997 to develop a mathematics curriculum for grades 6, 7, and 8. The result was Connected Mathematics, a complete mathematics curriculum that helps students develop understanding of important concepts, skills, procedures, and ways of thinking and reasoning in number, geometry, measurement, algebra, probability, and statistics. Listed here are some key features of Connected Mathematics:

- It is problem-centered. Important mathematical concepts are embedded in engaging problems. Students develop understanding and skill as they explore the problems individually, in a group, or with the class.
- It provides skills practice. The in-class problems and homework questions give students practice with important concepts, skills, and algorithms.
- It is complete. The Connected Mathematics units-multiple units for each grade-form a complete middle school curriculum that develops mathematical skills and conceptual understanding across mathematical strands. In addition, the program provides a complete assessment package that includes quizzes, tests, and projects.
- It is for teachers as well as students. The Connected Mathematics materials were written so teachers can learn from them too. The Teacher's Guides include extensive notes regarding mathematics, pedagogy, and assessment.
- It is research based. Each Connected Mathematics unit has been field tested, evaluated, and revised over a three- to four-year period. Approximately 160 teachers and 45,000 students in diverse school settings across the United States participated in the development of the curriculum.
- It is effective. Research results consistently show that CMP students outperform other students on tests of problem-solving ability, conceptual understanding, and proportional reasoning. And CMP students do as well as, or better than, other students on tests of basic skills.

Source: Connected Mathematics Project Website (http://www.math.msu.edu/cmp)

What is Computational Fluency?

"Children should master the basic facts of arithmetic that are essential components of fluency with paper-and-pencil and mental computation and with estimation…. It is important for children to learn the sequence of steps - and the reasons for them - in the paper-and-pencil algorithms used widely in our culture."
Curriculum and Evaluation Standards for School Mathematics, 1989, p. 47

- ***Efficiency*** implies that the student does not get bogged down in many steps or lose track of the logic of the strategy. An efficient strategy is one that the student can carry out easily, keeping track of sub-problems and using intermediate results to solve the problem.
- ***Accuracy*** means getting a correct answer. It depends on several things, including knowledge of basic number combinations and number relationships, checking for reasonableness, and, in some cases, careful recording of work.
- ***Flexibility*** means that students understand the underlying properties of the strategies they are using. It means using approaches that are appropriate for the particular problem. It generally means that a students uses multiple approaches to compute.

Students who demonstrate computational fluency:

- demonstrate flexibility in choosing computational methods;
- understand and can explain these methods;
- produce accurate answers efficiently;
- can represent their thinking and work;
- exhibit number sense.

Principles of computational fluency:

- Computational fluency is an essential goal for school mathematics.
- The methods a student uses to compute should be grounded in understanding.
- Students should know the basic number combinations for addition and subtraction by the end of grade 2 and for multiplication and division by the end of grade 4.
- Students should be computing fluently with whole numbers by the end of grade 5 and with fractions and decimals by the end of grade 8.
- Students can achieve computational fluency using a variety of methods and should, in fact, be comfortable with more than one approach.
- Students should have opportunities to invent strategies for computing on the basis of their knowledge of place value, number properties and the operations.
- Students should investigate conventional algorithms for computing whole numbers.
- Students should be encouraged to use computational methods that are appropriate for the context and purpose, including mental computation, estimation, calculator, or paper-and-pencil.

NCTM standard N1: Understand numbers, ways of representing numbers, relationships among numbers, and number systems.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Count with understanding and recognize "how many" in sets of objects.	g. Understand the place value structure of the base-ten number system and be able to represent and compare whole numbers and decimals.	n. Work flexibly with fractions, decimals, and percents to solve problems.	u. Develop a deeper understanding of very large and very small numbers and of various representations of them.
b. Use multiple models to develop initial understandings of place value and the base-ten number system.	h. Recognize equivalent representations for the same number and generate them by decomposing and composing numbers.	o. Compare and order fractions, decimals, and percents efficiently and find their approximate locations on a number line.	v. Compare and contrast the properties of numbers and numbers systems, including the rational and real numbers, and understand complex numbers as solutions to quadratic equations that do not have real solutions.
c. Develop understanding of the relative position and magnitude of whole numbers and of ordinal and cardinal numbers and their connections.	i. Develop understanding of fractions as parts of unit wholes, as parts of a collection, as locations on number lines, and as divisions of whole numbers.	p. Develop meaning for percents greater than 100 and less than 1.	w. Understand vectors and matrices as systems that have some of the properties of the real-number system.
d. Develop a sense of whole numbers and represent them in flexible ways, including relating, composing, and decomposing numbers.	j. Use models, benchmarks, and equivalent forms to judge the size of fractions.	q. Understand and use ratios and proportions to represent quantitative relationships.	x. Use number-theory arguments to justify relationships involving whole numbers.
e. Connect number words and numerals to the quantities they represent, using various physical models and representations.	k. Recognize and generate equivalent forms of commonly used fractions, decimals, and percents.	r. Develop an understanding of large numbers and recognize and appropriately use exponential, scientific, and calculator notation.	
f. Understand and represent commonly used fractions, such as 1/4, 1/3, and 1/2.	l. Explore numbers less than 0 by extending the number line and through familiar applications.	s. Use factors, multiples, prime factorization, and relatively prime numbers to solve problems.	
	m. Describe classes of numbers according to characteristics such as the nature of their factors.	t. Develop meaning for integers and represent and compare quantities with them.	

NCTM standard N2: Understand meanings of operations and how they relate to one another.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Understand various meanings of addition and subtraction of whole numbers and the relationship between the two operations.	d. Understand various meanings of multiplication and division.	h. Understand the meaning and effects of arithmetic operations with fractions, decimals, and integers.	k. Judge the effects of such operations as multiplication, division, and computing powers and roots on the magnitudes of quantities.
b. Understand the effects of adding and subtracting whole numbers.	e. Understand the effects of multiplying and dividing whole numbers.	i. Use the associative and commutative properties of addition and multiplication and the distributive property of multiplication over addition to simplify computations with integers, fractions, and decimals.	l. Develop an understanding of properties of, and representations for, the addition and multiplication of vectors and matrices.
c. Understand situations that entail multiplication and division, such as equal groupings of objects and sharing equally.	f. Identify and use relationships between operations, such as division as the inverse of multiplication, to solve problems. g. Understand and use properties of operations, such as the distributivity of multiplication over addition.	j. Understand and use the inverse relationships of addition and subtraction, multiplication and division, and squaring and finding square roots to simplify computations and solve problems.	m. Develop an understanding of permutations and combinations as counting techniques.

NCTM standard N3: Compute fluently and make reasonable estimates.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Develop and use strategies for whole-number computations, with a focus on addition and subtraction.	d. Develop fluency with basic number combinations for multiplication and division and use these combinations to mentally compute related problems, such as 30 x 50.	j. Select appropriate methods and tools for computing with fractions and decimals from among mental computation, estimation, calculators or computers, and paper and pencil, depending on the situation, and apply the selected methods.	n. Develop fluency in operations with real numbers, vectors, and matrices, using mental computation or pencil-and-paper calculations for simple cases and technology for more-complicated cases.
b. Develop fluency with basic number combinations for addition and subtraction.	e. Develop fluency in adding, subtracting, multiplying, and dividing whole numbers.	k. Develop and analyze algorithms for computing with fractions, decimals, and integers and develop fluency in their use.	o. Judge the reasonableness of numerical computations and their results.
c. Use a variety of methods and tools to compute, including objects, mental computation, estimation, paper and pencil, and calculators.	f. Develop and use strategies to estimate the results of whole-number computations and to judge the reasonableness of such results.	l. Develop and use strategies to estimate the results of rational-number computations and judge the reasonableness of the results.	
	g. Develop and use strategies to estimate computations involving fractions and decimals in situations relevant to students' experience.	m. Develop, analyze, and explain methods for solving problems involving proportions, such as scaling and finding equivalent ratios.	
	h. Use visual models, benchmarks, and equivalent forms to add and subtract commonly used fractions and decimals.		
	i. Select appropriate methods and tools for computing with whole numbers from among mental computation, estimation, calculators, and paper and pencil according to the context and nature of the computation and use the selected method or tool.		

NCTM standard A1: Understand patterns, relations, and functions.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Sort, classify, and order objects by size, number, and other properties.	d. Describe, extend, and make generalizations about geometric and numeric patterns.	f. Represent, analyze, and generalize a variety of patterns with tables, graphs, words, and, when possible, symbolic rules.	i. Generalize patterns using explicitly defined and recursively defined functions.
b. Recognize, describe, and extend patterns such as sequences of sounds and shapes or simple numeric patterns and translate from one representation to another.	e. Represent and analyze patterns and functions, using words, tables, and graphs.	g. Relate and compare different forms of representation for a relationship.	j. Understand relations and functions and select, convert flexibly among, and use various representations for them.
c. Analyze how both repeating and growing patterns are generated.		h. Identify functions as linear or nonlinear and contrast their properties from tables, graphs, or equations.	k. Analyze functions of one variable by investigating rates of change, intercepts, zeros, asymptotes, and local and global behavior. l. Understand and perform transformations such as arithmetically combining, composing, and inverting commonly used functions, using technology to perform such operations on more-complicated symbolic expressions. m. Understand and compare the properties of classes of functions, including exponential, polynomial, rational, logarithmic, and periodic functions. n. Interpret representations of functions of two variables.

NCTM standard A2: Represent and analyze mathematical situations and structures using algebraic symbols.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Illustrate general principles and properties of operations, such as commutativity, using specific numbers.	c. Identify such properties as commutativity, associativity, and distributivity and use them to compute with whole numbers.	f. Develop an initial conceptual understanding of different uses of variables.	j. Understand the meaning of equivalent forms of expressions, equations, inequalities, and relations.
b. Use concrete, pictorial, and verbal representations to develop an understanding of invented and conventional symbolic notations.	d. Represent the idea of a variable as an unknown quantity using a letter or a symbol.	g. Explore relationships between symbolic expressions and graphs of lines, paying particular attention to the meaning of intercept and slope.	k. Write equivalent forms of equations, inequalities, and systems of equations and solve them with fluency—mentally or with paper and pencil in simple cases and using technology in all cases.
	e. Express mathematical relationships using equations.	h. Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.	l. Use symbolic algebra to represent and explain mathematical relationships.
		i. Recognize and generate equivalent forms for simple algebraic expressions and solve linear equations.	m. Use a variety of symbolic representations, including recursive and parametric equations, for functions and relations.
			n. Judge the meaning, utility, and reasonableness of the results of symbol manipulations, including those carried out by technology

NCTM standard A3: Use mathematical models to represent and understand quantitative relationships.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Model situations that involve the addition and subtraction of whole numbers, using objects, pictures, and symbols.	b. Model problem situations with objects and use representations such as graphs, tables, and equations to draw conclusions.	c. Model and solve contextualized problems using various representations, such as graphs, tables, and equations.	d. Identify essential quantitative relationships in a situation and determine the class or classes of functions that might model the relationships. e. Use symbolic expressions, including iterative and recursive forms, to represent relationships arising from various contexts. f. Draw reasonable conclusions about a situation being modeled.

NCTM standard A4: Analyze change in various contexts.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Describe qualitative change, such as a student's growing taller. b. Describe quantitative change, such as a student's growing two inches in one year.	c. Investigate how a change in one variable relates to a change in a second variable. d. Identify and describe situations with constant or varying rates of change and compare them.	e. Use graphs to analyze the nature of changes in quantities in linear relationships.	f. Approximate and interpret rates of change from graphical and numerical data.

NCTM standard G1: Analyze characteristics and properties of 2- and 3-dimensional geometric shapes and develop mathematical arguments about geometric relationships.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Recognize, name, build, draw, compare, and sort two- and three-dimensional shapes.	d. Identify, compare, and analyze attributes of two- and three-dimensional shapes and develop vocabulary to describe the attributes.	i. Precisely describe, classify, and understand relationships among types of two- and three-dimensional objects using their defining properties.	l. Analyze properties and determine attributes of two- and three-dimensional objects.
b. Describe attributes and parts of two- and three-dimensional shapes.	e. Classify two- and three-dimensional shapes according to their properties and develop definitions of classes of shapes such as triangles and pyramids.	j. Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects.	m. Explore relationships (including congruence and similarity) among classes of two- and three-dimensional geometric objects, make and test conjectures about them, and solve problems involving them.
c. Investigate and predict the results of putting together and taking apart two- and three-dimensional shapes.	f. Investigate, describe, and reason about the results of subdividing, combining, and transforming shapes.	k. Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship.	n. Establish the validity of geometric conjectures using deduction, prove theorems, and critique arguments made by others.
	g. Explore congruence and similarity.		o. Use trigonometric relationships to determine lengths and angle measures.
	h. Make and test conjectures about geometric properties and relationships and develop logical arguments to justify conclusions.		

NCTM standard G2: Specify locations and describe spatial relationships using coordinate geometry and other representational systems.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Describe, name, and interpret relative positions in space and apply ideas about relative position.	d. Describe location and movement using common language and geometric vocabulary.	g. Use coordinate geometry to represent and examine the properties of geometric shapes.	i. Use Cartesian coordinates and other coordinate systems, such as navigational, polar, or spherical systems, to analyze geometric situations.
b. Describe, name, and interpret direction and distance in navigating space and apply ideas about direction and distance.	e. Make and use coordinate systems to specify locations and to describe paths.	h. Use coordinate geometry to examine special geometric shapes, such as regular polygons or those with pairs of parallel or perpendicular sides.	j. Investigate conjectures and solve problems involving two- and three-dimensional objects represented with Cartesian coordinates.
c. Find and name locations with simple relationships such as “near to” and in coordinate systems such as maps.	f. Find the distance between points along horizontal and vertical lines of a coordinate system.		

NCTM standard G3: Apply transformations and use symmetry to analyze mathematical situations.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Recognize and apply slides, flips, and turns.	c. Predict and describe the results of sliding, flipping, and turning two-dimensional shapes.	f. Describe sizes, positions, and orientations of shapes under informal transformations such as flips, turns, slides, and scaling.	h. Understand and represent translations, reflections, rotations, and dilations of objects in the plane using sketches, coordinates, vectors, function notation, and matrices.
b. Recognize and create shapes that have symmetry.	d. Describe a motion or a series of motions that will show that two shapes are congruent. e. Identify and describe line and rotational symmetry in two- and three-dimensional shapes and designs.	g. Examine the congruence, similarity, and line or rotational symmetry of objects using transformations.	i. Use various representations to help understand the effects of simple transformations and their compositions.

Geometry (G)

NCTM standard G4: Use visualization, spatial reasoning, and geometric modeling to solve problems.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Create mental images of geometric shapes using spatial memory and spatial visualization.	e. Build and draw geometric objects.	k. Draw geometric objects with specified properties, such as side lengths or angle measurements.	p. Draw and construct representations of two- and three-dimensional geometric objects using a variety of tools.
b. Recognize and represent shapes from different perspectives.	f. Create and describe mental images of objects, patterns, and paths.	l. Use two-dimensional representations of three-dimensional objects to visualize and solve problems such as those involving surface area and volume.	q. Visualize three-dimensional objects from different perspectives and analyze their cross sections.
c. Relate ideas in geometry to ideas in number and measurement.	g. Identify and build a three-dimensional object from two-dimensional representations of that object.	m. Use visual tools such as networks to represent and solve problems.	r. Use vertex-edge graphs to model and solve problems.
d. Recognize geometric shapes and structures in the environment and specify their location.	h. Identify and build a two-dimensional representation of a three-dimensional object.	n. Use geometric models to represent and explain numerical and algebraic relationships.	s. Use geometric models to gain insights into, and answer questions in, other areas of mathematics.
	i. Use geometric models to solve problems in other areas of mathematics, such as number and measurement. j. Recognize geometric ideas and relationships and apply them to other disciplines and to problems that arise in the classroom or in everyday life.	o. Recognize and apply geometric ideas and relationships in areas outside the mathematics classroom, such as art, science, and everyday life.	t. Use geometric ideas to solve problems in, and gain insights into, other disciplines and other areas of interest such as art and architecture.

NCTM standard M1: Understand measurable attributes of objects and the units, systems, and processes of measurement.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Recognize the attributes of length, volume, weight, area, and time.	e. Understand such attributes as length, area, weight, volume, and size of angle and select the appropriate type of unit for measuring each attribute.	j. Understand both metric and customary systems of measurement.	m. Make decisions about units and scales that are appropriate for problem situations involving measurement.
b. Compare and order objects according to these attributes.	f. Understand the need for measuring with NCTM standard units and become familiar with NCTM standard units in the customary and metric systems.	k. Understand relationships among units and convert from one unit to another within the same system.	
c. Understand how to measure using non-NCTM standard and NCTM standard units.	g. Carry out simple unit conversions, such as from centimeters to meters, within a system of measurement.	l. Understand, select, and use units of appropriate size and type to measure angles, perimeter, area, surface area, and volume.	
d. Select an appropriate unit and tool for the attribute being measured.	h. Understand that measurements are approximations and understand how differences in units affect precision.		
	i. Explore what happens to measurements of a two-dimensional shape such as its perimeter and area when the shape is changed in some way.		

NCTM standard M2: Apply appropriate techniques, tools, and formulas to determine measurements.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Measure with multiple copies of units of the same size, such as paper clips laid end to end.	e. Develop strategies for estimating the perimeters, areas, and volumes of irregular shapes.	j. Use common benchmarks to select appropriate methods for estimating measurements.	p. Analyze precision, accuracy, and approximate error in measurement situations.
b. Use repetition of a single unit to measure something larger than the unit, for instance, measuring the length of a room with a single meter stick.	f. Select and apply appropriate NCTM standard units and tools to measure length, area, and volume, weight, time, temperature, and the size of angles.	k. Select and apply techniques and tools to accurately find length, area, volume, and angle measures to appropriate levels of precision.	q. Understand and use formulas for the area, surface area, and volume of geometric figures, including cones, spheres, and cylinders.
c. Use tools to measure.	g. Select and use benchmarks to estimate measurements.	l. Develop and use formulas to determine the circumference of circles and the area of triangles, parallelograms, trapezoids, and circles and develop strategies to find the area of more-complex shapes.	r. Apply informal concepts of successive approximation, upper and lower bounds, and limit in measurement situations.
d. Develop common referents for measures to make comparisons and estimates.	h. Develop, understand, and use formulas to find the area of rectangles and related triangles and parallelograms.	m. Develop strategies to determine the surface area and volume of selected prisms, pyramids, and cylinders.	s. Use unit analysis to check measurement computations.
	i. Develop strategies to determine the surface areas and volumes of rectangular solids.	n. Solve problems involving scale factors, using ratio and proportion.	
		o. Solve simple problems involving rates and derived measurements for such attributes as velocity and density.	

NCTM standard D1: Formulate questions that can be addressed with data and collect, organize, and display relevant data to answer them.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Pose questions and gather data about themselves and their surroundings.	d. Design investigations to address a question and consider how data collection methods affect the nature of the data set.	h. Formulate questions, design studies, and collect data about a characteristic shared by two populations or different characteristics within one population.	j. Understand the differences among various kinds of studies and which types of inferences can legitimately be drawn from each.
b. Sort and classify objects according to their attributes and organize data about the objects.	e. Collect data using observations, surveys, and experiments.	i. Select, create, and use appropriate graphical representations of data, including histograms, box plots, and scatter plots.	k. Know the characteristics of well-designed studies, including the role of randomization in surveys and experiments.
c. Represent data using concrete objects, pictures, and graphs.	f. Represent data using tables and graphs such as line plots, bar graphs, and line graphs.		l. Understand the meaning of measurement data and categorical data, of univariate and bivariate data, and of the term variable.
	g. Recognize the differences in representing categorical and numerical data.		m. Understand histograms, parallel box plots, and scatterplots and use them to display data.
			n. Compute basic statistics and understand the distinction between a statistic and a parameter.

NCTM standard D2: Select and use appropriate statistical methods to analyze data.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Describe parts of the data and the set of data as a whole to determine what the data show.	b. Describe the shape and important features of a set of data and compare related data sets, with an emphasis on how the data are distributed. c. Use measures of center, focusing on the median, and understand what each does and does not indicate about the data set. d. Compare different representations of the same data and evaluate how well each representation shows important aspects of the data.	e. Find, use, and interpret measures of center and spread, including mean and interquartile range. f. Discuss and understand the correspondence between data sets and their graphical representations, especially histograms, stem-and-leaf plots, box plots, and scatter plots.	g. For univariate measurement data, be able to display the distribution, describe its shape, and select and calculate summary statistics. h. For bivariate measurement data, be able to display a scatterplot, describe its shape, and determine regression coefficients, regression equations, and correlation coefficients using technological tools. i. Display and discuss bivariate data where at least one variable is categorical. j. Recognize how linear transformations of univariate data affect shape, center, and spread. k. Identify trends in bivariate data and find functions that model the data or transform the data so that they can be modeled.

NCTM standard D3: Develop and evaluate inferences and predictions that are based on data.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
a. Discuss events related to students' experiences as likely or unlikely.	b. Propose and justify conclusions and predictions that are based on data and design studies to further investigate the conclusions or predictions.	c. Use observations about differences between two or more samples to make conjectures about the populations from which the samples were taken. d. Make conjectures about possible relationships between two characteristics of a sample on the basis of scatterplots of the data and approximate lines of fit. e. Use conjectures to formulate new questions and plan new studies to answer them.	f. Use simulations to explore the variability of sample statistics from a known population and to construct sampling distributions. g. Understand how sample statistics reflect the values of population parameters and use sampling distributions as the basis for informal inference. h. Evaluate published reports that are based on data by examining the design of the study, the appropriateness of the data analysis, and the validity of conclusions. i. Understand how basic statistical techniques are used to monitor process characteristics in the workplace.

NCTM standard D4: Understand and apply basic concepts of probability.

Grades PreK-2	Grades 3-5	Grades 6-8	Grades 9-12
	a. Describe events as likely or unlikely and discuss the degree of likelihood using such words as certain, equally likely, and impossible. b. Predict the probability of outcomes of simple experiments and test the predictions. c. Understand that the measure of the likelihood of an event can be represented by a number from 0 to 1.	d. Understand and use appropriate terminology to describe complementary and mutually exclusive events. e. Use proportionality and a basic understanding of probability to make and test conjectures about the results of experiments and simulations. f. Compute probabilities for simple compound events, using such methods as organized lists, tree diagrams, and area models.	g. Understand the concepts of sample space and probability distribution and construct sample spaces and distributions in simple cases. h. Use simulations to construct empirical probability distributions. i. Compute and interpret the expected value of random variables in simple cases. j. Understand the concepts of conditional probability and independent events. k. Understand how to compute the probability of a compound event.

Pre-K-12 NCTM Process Standards

NCTM standard	Expectation
Problem Solving (PRS)	PRS1: Build new mathematical knowledge through problem solving PRS2: Solve problems that arise in mathematics and in other contexts PRS3: Apply and adapt a variety of appropriate strategies to solve problems PRS4: Monitor and reflect on the process of mathematical problem solving
Reasoning (RES)	RES1: Recognize reasoning and proof as fundamental aspects of mathematics RES2: Make and investigate mathematical conjectures RES3: Develop and evaluate mathematical arguments and proofs RES4: Select and use various types of reasoning and methods of proof
Communication (COM)	COM1: Organize and consolidate their mathematical thinking through communication COM2: Communicate their mathematical thinking coherently and clearly to peers, teachers, and others COM3: Analyze and evaluate the mathematical thinking and strategies of others COM4: Use the language of mathematics to express mathematical ideas precisely
Connections (CON)	CON1: Recognize and use connections among mathematical ideas CON2: Understand how mathematical ideas interconnect and build on one another to produce a coherent whole CON3: Recognize and apply mathematics in contexts outside of mathematics
Representation (REP)	REP1: Create and use representations to organize, record, and communicate mathematical ideas REP2: Select, apply, and translate among mathematical representations to solve problems REP3: Use representations to model and interpret physical, social, and mathematical phenomena