Q1: Determine the service moment capacity for a rectangular beam section that has a width of 350mm and a depth of 500mm and reinforced with top bars of 5\(\phi 25\), using the allowable strength method. The concrete compressive strength, \(f'_c\) = 32MPa and the steel yield strength, \(f_y\) = 400 MPa.

Q2: A rectangular beam section of 400mm width and 600mm depth (d= 540mm) and reinforced with bottom bars of 6\(\phi 20\) is to be converted to a beam of 400mm depth (d= 350mm). Determine the needed beam width and reinforcement using the same steel ratio. The concrete compressive strength, \(f'_c\) = 32MPa and the steel yield strength, \(f_y\) = 420 MPa.

Q3: A beam section of 350mm width is to be designed to resist a moment of 400kN.m. The concrete compressive strength, \(f'_c\) = 24MPa and the steel yield strength, \(f_y\) = 420 MPa. Determine the needed section depth and reinforcement using steel ratio varies from \(p=0.003\) to \(p=0.015\) considering steps of 0.001. Determine the concrete volume and the steel weight for each steel ratio considering one meter length of beam. If the concrete cost is 320 INS/m³ and the steel cost is 3200 INS/ton, calculate the cost for the design for each steel ratio. Make comments.

Q4: A. Derive a formula for the maximum steel ratio allowed by the code (ACI 318-08) to have singly reinforced concrete rectangular beam section and calculate the corresponding steel reduction factor, \(\Phi\). (In ACI 318-08, the maximum allowed strain in tensile steel is 0.004). Determine the design moment capacity for a rectangular beam section of 450mm width and 600mm depth (d= 540mm) that have this steel ratio. The concrete compressive strength, \(f'_c\) = 28MPa and the steel yield strength, \(f_y\) = 420 MPa.

B. Derive a formula for the steel ratio to have failure in concrete and yielding of steel bars at the same time for a steel have yield strength, \(f_y\) for a rectangular singly reinforced beam section. (This steel ratio is called balanced steel ratio)

Q5: Design a rectangular beam section that has a width equal to half the thickness for the beam shown in the figure below. \(f'_c\) = 24MPa and the steel yield strength, \(f_y\) = 420MPa. Sketch the reinforcing layout of the beam. Assume maximum steel ratio, \(p=0.012\)
Design of reinforced concrete structures I
Homework I

Q1:

\[ d = 500 - 65 = 435 \text{ mm} \]

\[ f_c^0 = 32 \text{ MPa} \]

\[ f_{\text{calc}} = 0.45 \times 32 = 14.4 \text{ MPa} \]

\[ f_y = 400 \text{ MPa} \]

\[ f_{\text{sec}} = 0.4 \times f_y = 0.4 \times 400 = 160 \text{ MPa} \]

\[ f_s = \frac{M}{A_s \cdot d} \quad f_c = \frac{2M}{b d^2 k_f} \]

\[ E_c = 47000 \sqrt{321} = 26587 \text{ MPa} \]

\[ E_s = 200000 \text{ MPa} \]

\[ n = \frac{E_s}{E_c} = \frac{200000}{26587} = 7.52 \]

\[ A_s = 5 \times 491 = 2455 \text{ mm}^2 \]

\[ \rho = \frac{A_s}{b d} = \frac{2455}{350 \times 435} = 0.0161 \]

\[ k = -\rho + \sqrt{(\rho)^2 + 2 \rho} = 0.386 \]

\[ j = 1 - k(3) = 0.821 \]

\[ \Rightarrow 160 = \frac{M \times 10^6}{2455 \times 0.821 \times 435} \quad \Rightarrow M = 148.8 \text{ kN.m} \]

\[ \Rightarrow 14.4 = \frac{2M \times 10^6}{350 \times 435^2 \times 0.306 \times 0.821} \quad \Rightarrow M = 160.3 \text{ kN.m} \]

\[ \Rightarrow M = 148.8 \text{ kN.m} \]
Q2:

$f_c = 32 \text{ MPa}$

$f_y = 420 \text{ MPa}$

Moment capacity of section 1:

$A_s = 6 \times 314$

$= 1884 \text{ mm}^2$

$q = \frac{A_s f_y}{0.85 f_c^2}$

$= \frac{1884 \times 420}{0.85 \times 32 \times 400}$

$= 72.73 \text{ mm}$

$M_u = A_s f_y (d - a/2)$

$= 1884 \times 420 \times (540 - 72.73) / 400 = 398.5 \text{ kN.m}$

$P_{0.005} = 0.325 \beta_1 \cdot 0.85 f_y^2$

$\beta_1 = 0.85 - 0.05 (32 - 28) = 0.82$

$\rightarrow P_{0.005} = 0.325 \times 0.82 \times 0.85 \times 32 / 420$

$= 0.0199$

$\rho = \frac{1884}{400 \times 540} = 0.0082^2 < 0.0199$
\[ \phi = 0.9 \]

\[ 8/M_n = 0.9 \times 398.5 = 358.7 \text{ Nm} \]

For section 2:

\[ \frac{6d^2}{b} = \frac{M}{y} \]

\[ \phi \rho f y \left( 1 - \frac{\rho f y}{1 + f \phi} \right) \]

\[ = \frac{358.7 \times 10^6}{0.9 \times 0.000072 \times 920 \left( 1 - \frac{0.000072 \times 920}{1.7 \times 32} \right)} \]

\[ = 116.7 \times 10^6 \text{ mm}^3 \]

\[ b = \frac{116.7 \times 10^6}{(350)^2} = 953 \text{ mm} \]

So, \[ b = 950 \text{ mm} \]

\[ h = 400 \text{ mm} \]

\[ A_S = 0.000072 \times 950 \times 350 = 2899 \text{ mm}^2 \]
A4: A

C = T

0.85 \cdot b \cdot a = A_3 f_y

\[ \frac{0.85}{0.004} \cdot 0.43 = A_3 f_y \]

\[ \rho = 0.43 \cdot 0.85 \cdot \frac{0.004}{f_y} \]

\[ 0.003 = 0.004 \cdot \frac{c}{d - c} \]

\[ c = 0.6 \cdot d \]

\[ 0.1004c = 0.1003d \]

\[ \Rightarrow c = \frac{3}{7} d \]

\[ a = b \cdot c \]

\[ \phi = 0.65 + \left( \varepsilon - 0.002 \right) \left( \frac{250}{3} \right) \]

\[ = 0.817 \]

Section capacity:

\[ A_s = \rho \cdot f_y \]

\[ \rho = 0.43 \cdot 0.85 \cdot 0.85 \cdot 28 \]

\[ = 0.0207 \]

\[ A_s = 0.0207 \times 450 \times 540 \]

\[ = 5030 \text{ mm}^2 \]
\[
a = A_s f_y = \frac{5030 \times 420}{0.85 \times 20 \times 450} = 197.3 \text{ mm}
\]

\[
\phi Mu = 0.817 A_s f_y (d - a/2)
\]

\[
= 0.817 (5030)(420)(540 - \frac{197.3}{2})/106
\]

\[
= 762 \text{ kN.m}
\]

\[
0.85 f_c' b_b c = A_s f_y
\]

\[
\frac{0.003}{c} = \frac{\epsilon_y}{d - c}
\]

\[
0.003 d - 0.003 c = c \epsilon_y
\]

\[
0.003 d = (0.003 + \epsilon_y) c
\]

\[
c = \frac{0.003 d}{0.003 + \epsilon_y}
\]

\[
\rightarrow 0.85 f_c' b_b \left[ \frac{0.003}{0.003 + \epsilon_y} \right] d = A_s f_y
\]
Divide both sides of equation by \( bd \) to get:

\[
P = \frac{0.003}{0.003 + ey} \beta_1 \frac{0.85 f_c}{fy}
\]

A5:

Preliminary size of beam:

- \( bw = \) 
  - \( h = 2.6w \)

\[
h_{\text{min}} = \frac{3}{18.5} = 0.16 m
\]

Let 
- \( bw = 0.35 \)
- \( h = 0.70 \)

So, weight of beam = \( 0.35 \times 0.7 \times 25 \) 

\[
= 6.125 \text{ kN/m}
\]

\[
\Rightarrow \ W_u = 1.2(27 + 6.125) + 1.6(15) = 63.75 \text{ kN/m}
\]
\[ bd^2 = \frac{559.5 \times 10^6}{0.012 \times 0.9 \times 420 (1 - \frac{0.012 \times 420}{1.7 \times 24})} \]

\[ bd^2 = 140.73 \times 10^6 \text{ mm}^3 \]

\[ b (1.2b)^2 = 140.73 \times 10^6 \]

\[ d = 0.9b = 0.9(2b) = 1.8b \]

\[ \rightarrow b = 352 \text{ mm} \]

Try

\[ b = 350 \]
\[ h = 700 \]
\[ d = 630 \text{ mm} \]

\[ \rightarrow \rho = 0.01216 \text{ mm} \]

\[ A_s = 2681 \text{ mm}^2 \]

\[ (4 \phi 32) \text{ or } (6 \phi 25) \]

Use

\[ b = 350 \text{ mm} \]
\[ h = 700 \text{ mm} \]

For \[ Mu = 468 \text{ kN.m} \]

\[ \rho = 0.00943 \]

\[ A_s = 2679 \text{ mm}^2 \]

\[ (3 \phi 32) \text{ or } (5 \phi 25) \]

\[ 4 \phi 32 \]