Design of reinforced concrete structures I (61390)
Homework No. 1

Q1: Determine the service moment capacity for a rectangular beam section that has a width of 350mm and a depth of 500mm and reinforced with top bars of 5Φ25, using the allowable strength method. The concrete compressive strength, f'_c= 32MPa and the steel yield strength, f_y= 400 MPa.

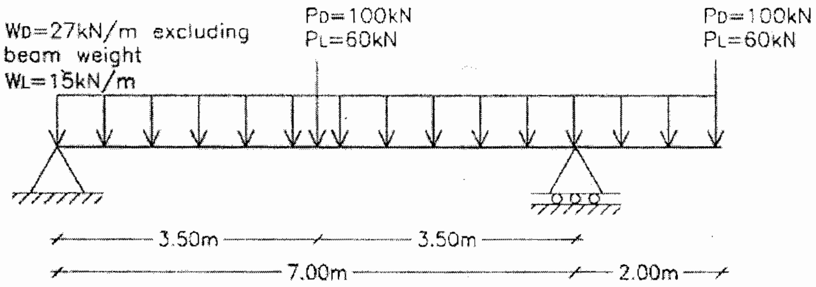
Q2: A rectangular beam section of 400mm width and 600mm depth (d= 540mm) and reinforced with bottom bars of 6Φ20 is to be converted to a beam of 400mm depth (d= 350mm). Determine the needed beam width and reinforcement using the same steel ratio. The concrete compressive strength, f'_c= 32MPa and the steel yield strength, f_y= 420 MPa.

Q3: A beam section of 350mm width is to be designed to resist a moment of 400kN.m. The concrete compressive strength, f'_c= 24MPa and the steel yield strength, f_y= 420 MPa. Determine the needed section depth and reinforcement using steel ratio varies from ρ= 0.003 to ρ= 0.015 considering steps of 0.001. Determine the concrete volume and the steel weight for each steel ratio considering one meter length of beam. If the concrete cost is 320 INS/m^3 and the steel cost is 3200 INS/ton, calculate the cost for the design for each steel ratio. Make comments.

Q4: A. Derive a formula for the maximum steel ratio allowed by the code (ACI 318-08) to have singly reinforced concrete rectangular beam section and calculate the corresponding steel reduction factor, Φ. (In ACI 318-08, the maximum allowed strain in tensile steel is 0.004). Determine the design moment capacity for a rectangular beam section of 450mm width and 600mm depth (d= 540mm) that have this steel ratio. The concrete compressive strength, f'_c= 28MPa and the steel yield strength, f_y= 420 MPa.

B. Derive a formula for the steel ratio to have failure in concrete and yielding of steel bars at the same time for a steel have yield strength, f_y for a rectangular singly reinforced beam section. (This steel ratio is called balanced steel ratio)

Q5: Design a rectangular beam section that has a width equal to half the thickness for the beam shown in the figure below. f'c= 24MPa and the steel yield strength, fy= 420MPa. Sketch the reinforcing layout of the beam. Assume maximum steel ratio, ρ=0.012

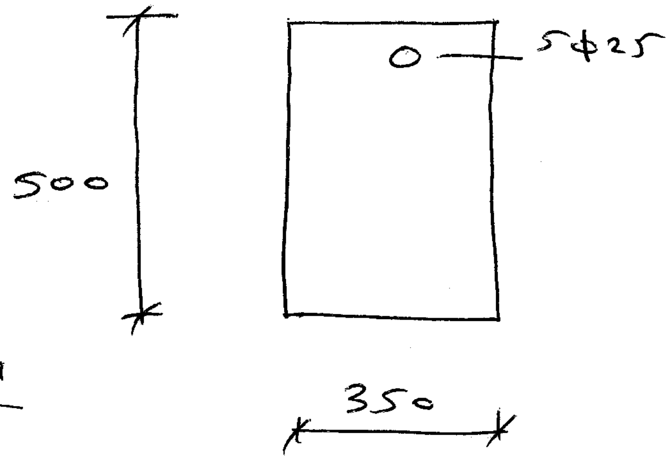


BEAM STRUCTURAL MODEL

Design of reinforced concrete structures 1
Homework 1

Y

Q1:



$d = 500 - 65$
$= 435$ mm

$f'_c = 32$ MPa
$f_{call} = 0.45 \times 32 = 14.4$ MPa

$f_y = 400$ MPa
$f_{sall} = 0.4\, f_y = 0.4 \times 400 = 160$ MPa

$f_s = \dfrac{M}{A_s\, j\, d}$ $\qquad f_c = \dfrac{2M}{b d^2 k j}$

$E_c = 4700\sqrt{32} = 26\,587$ MPa
$E_s = 200\,000$ MPa
$\rightarrow n = \dfrac{E_s}{E_c} = \dfrac{200\,000}{26\,587} = 7.52$

$A_s = 5 \times 491 = 2455$ mm^2
$\rho = \dfrac{A_s}{b d} = \dfrac{2455}{350 \times 435} = 0.0161$
$k = -n\rho + \sqrt{(n\rho)^2 + 2n\rho} = 0.386$
$j = 1 - k/3 = 0.871$

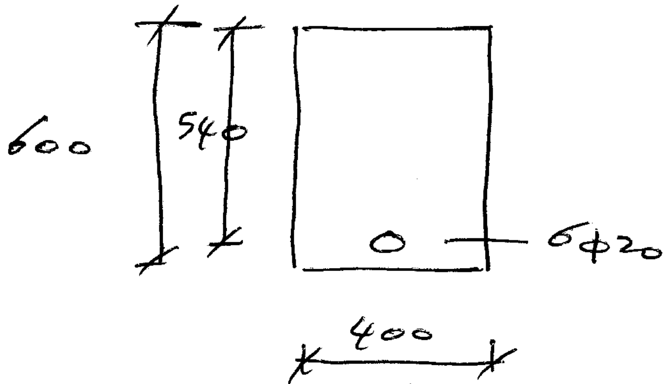
$\rightarrow 160 = \dfrac{M \times 10^6}{2455 \times 0.871 \times 435} \rightarrow M = 148.8$ kN.m

$\rightarrow 14.4 = \dfrac{2M \times 10^6}{350 \times 435^2 \times 0.386 \times 0.871} \rightarrow M = 160.3$ kN.m

$\rightarrow M = 148.8$ kN.m

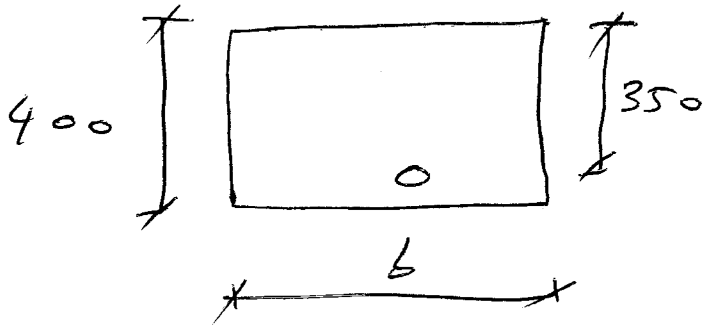
Q2:

$f'_c = 32$ MPa
$f_y = 420$ MPa



section1

Moment capacity of section1:



section2

$A_s = 6 \times 314$
$= 1884\ mm^2$

$a = \dfrac{A_s f_y}{.85 f'_c b}$

$= \dfrac{1884 \times 420}{.85 \times 32 \times 400} = 72.73\ mm$

$M_n = A_s f_y (d - a/2)$

$= 1884 (420) (540 - \frac{72.73}{2}) / 10^6 = 398.5\ kN.m$

$\rho_{0.005} = 0.375 \beta_1 \ \ .85 f'_c / f_y$

$\beta_1 = 0.85 - \dfrac{0.05 (32 - 28)}{7} = 0.82$

$\rightarrow \rho_{0.005} = 0.375 \times 0.82 \times 0.85 \times 32 / 420$
$= 0.0199$

$\rho = \dfrac{1884}{400 \times 540} = 0.00872 < 0.0199$

$\rightarrow \phi = 0.9$

$\phi M_n = 0.9 \times 398.5 = 358.7$ kN.m

for section 2:

$$bd^2 = \frac{M_u}{\phi \rho f_y \left(1 - \frac{\rho f_y}{1.7 f'_c}\right)}$$

$$= \frac{358.7 \times 10^6}{0.9 \times 0.00872 \times 420 \left(1 - \frac{0.00872 \times 420}{1.7 \times 32}\right)}$$

$$= 116.7 \times 10^6 \text{ mm}^3$$

$$b = \frac{116.7 \times 10^6}{(350)^2} = 953 \text{ mm}$$

so, $b = 950$ mm

$h = 400$ mm

$A_s = 0.00872 \times 950 \times 350$

$= 2899 \text{ mm}^2$

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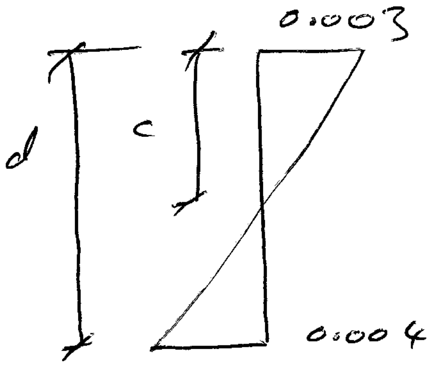
Q4: A



$C = T$

$.85 f'_c \, b a = A_s f_y$

$.85 f'_c \, b \, \beta_1 \, 0.43 d = A_s f_y$

$$\rho_{0.004} = 0.43 \beta_1 \frac{.85 f'_c}{f_y}$$

$$\frac{0.003}{c} = \frac{0.004}{d-c}$$

~~c = 0.6d~~

$0.004c = 0.003d - 0.003c$

$0.007c = 0.003d$

$\rightarrow c = \frac{3}{7} d$

$= 0.43 d$

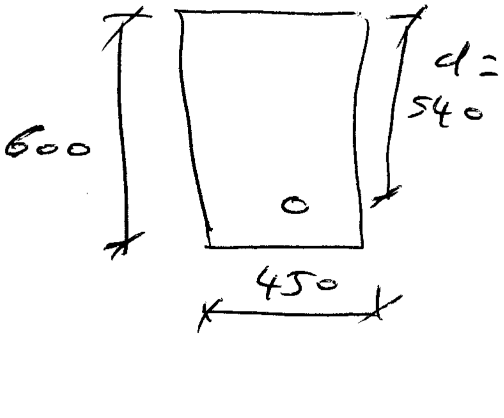
$a = \beta_1 c$

$$\phi = 0.65 + (\epsilon_t - 0.002)\left(\frac{250}{3}\right)$$
$$= 0.817$$

section capacity:

$A_s = \rho b d$

$$\rho = \frac{0.43 (0.85)(0.85)(28)}{420}$$
$$= 0.0207$$



$A_s = 0.0207 \times 450 \times 540$
$= 5030 \text{ mm}^2$

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$$a = \frac{A_s f_y}{.85 f'_c b} = \frac{5030\,(420)}{.85 \times 28 \times 450}$$

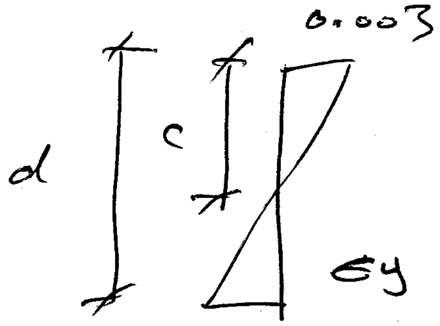
$$= 197.3 \text{ mm}$$

$$\phi M_u = 0.817\, A_s f_y (d - a/2)$$

$$= 0.817(5030)(420)\left(540 - \frac{197.3}{2}\right)/10^6$$

$$= 762 \text{ kN·m}$$

B



$$0.85 f'_c\, b\, \beta_1 c = A_s f_y$$

$$\frac{0.003}{c} = \frac{\epsilon_y}{d-c}$$

$$0.003 d - 0.003 c = c\, \epsilon_y$$

$$0.003 d = (0.003 + \epsilon_y) c$$

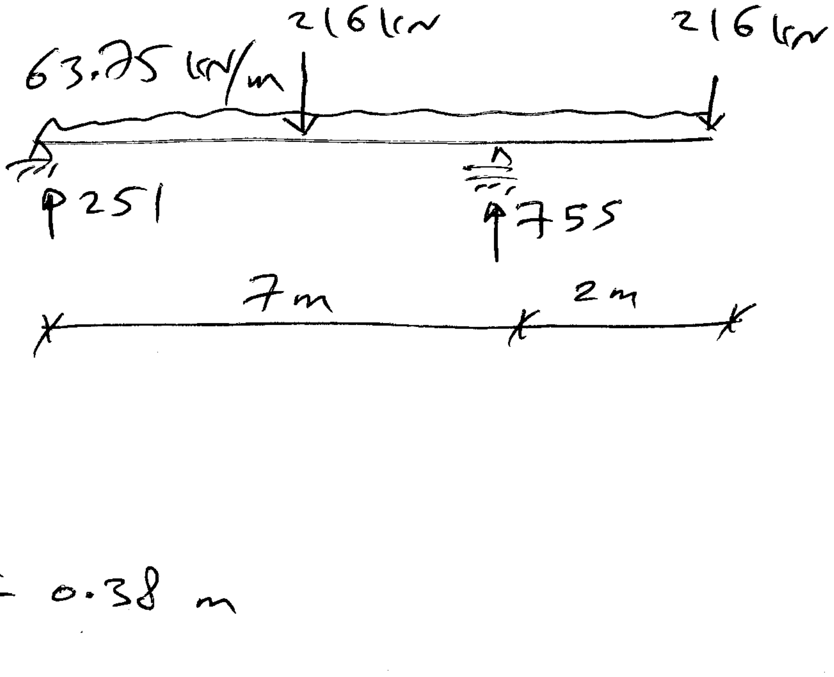
$$c = \frac{0.003}{0.003 + \epsilon_y} d$$

$$\rightarrow 0.85 f'_c\, b\, \beta_1 \left(\frac{0.003}{0.003 + \epsilon_y}\right) d = A_s f_y$$

divide both sides of equation by bd →

$$\rho_{bal} = \left(\frac{0.003}{0.003 + \epsilon_y}\right) \beta_1 \frac{0.85 f'_c}{f_y}$$

Q5:



preliminary size of beam:

$b_w =$

$h = 2 b_w$

$$h_{min} = \frac{7}{18.5} = 0.38 \text{ m}$$

let $b_w = 0.35$

$h = 0.70$

so, weight of beam $= 0.35 \times 0.7 \times 25$

$= 6.125$ kN/m

$\rightarrow w_u = 1.2(27 + 6.125) + 1.6(15) = 63.75$ kN/m

$$bd^2 = \frac{559.5 \times 10^6}{0.012 \times 0.9 \times 420 \left(1 - \frac{0.012 \times 420}{1.7 \times 24}\right)}$$

$bd^2 = 140.73 \times 10^6 \ mm^3$

$b(1.8b)^2 = 140.73 \times 10^6$ $\quad$ $d \approx 0.9h$

$\rightarrow b = 352 \ mm$ $\quad$ $= 0.9(2b)$

$\quad$ $= 1.8b$

Try $b = 350$

$h = 700$ $\quad$ $d = 630 \ mm$

$\rightarrow \rho_{559.5 \ kN \cdot m} = 0.01216 \approx 0.012$

$As = 2681 \ mm^2$

$(4\phi 32)$ or $(6\phi 25)$ ok

Use $b = 350 \ mm$ $\quad$ $h = 700 \ mm$

for $Mu = 488 \ kN \cdot m \rightarrow \rho = 0.00943$

$As = 2079 \ mm^2$

$(3\phi 32)$

or $(5\phi 25)$