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Technical note

Transient analysis of a PV power generator charging a capacitor for measurement of the *I–V* characteristics

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Abstract

Measuring the *I–V* characteristics is of high importance since it can be considered as a quality and performance certificate for each PV generator. The most precise and inexpensive measuring method is represented in capacitor charging by the PV generator. Using the equivalent circuit of the PV generator with a capacitor as load and applying transient analysis on the circuit, we obtain the capacitor charging voltage and current as a function of time, as well as their differentials as a function of short circuit current and capacitor size. The derived equations facilitate the calculation of proper capacitance size for measuring the *I–V* characteristics, and considers the acquisition speed of the measuring system as demonstrated through two measurement samples in this paper. The capacitor size is directly and indirectly proportional to the short circuit current and open circuit voltage of the PV generator, respectively. Accordingly, the paper presents a capacitance calculation chart, which enables selecting the correct capacitance for measuring the *I–V* characteristics by a computerized data acquisition system.

Keywords: Photovoltaic generators; *I–V* characteristics measurements; Capacitor charge technique; Transient analysis

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1. Introduction

The I– V characteristics (or I– V curve) of a PV cell, module or array (PV generator) is the important key for identifying its quality and performance as a function of varying environmental parameters such as solar radiation and ambient temperature [1]. The curve indicates the characteristic parameters of the PV generator represented in short circuit current, open circuit voltage and the point of maximum power at which the generator would work at its peak efficiency. These parameters are indispensable for designing any small or large PV power system. Moreover, the curve render determining the equivalent circuit components of the PV generator represented in the series resistance and shunt resistance, which are disclosure parameters for classifying the quality of the generator substrate material [1,2].

Therefore, it is of prime importance to measure the I– V characteristics with high accuracy under natural environmental conditions. The easiest method is to use variable resistor as a load to the PV generator and measure the stepwise voltage and current. The I– V curve obtained by this method is deficient in accuracy, uniformity and smoothness, due to manual change of the load resistor and slowness of the measuring process. [3,4].

The second measuring method is to load the PV generator by a capacitor and to charge it fully from short circuit to open circuit, and to record the respective voltage and current by X– Y recorder or a computerized data acquisition system (CDAS). The I– V curve obtained by this method is much more accurate and uniform since it is measured in a very short time. In addition, it surpasses the first method by enabling the measurement of the I– V curve of PV generators of higher power with reasonable capacitor values [4]. The purpose of this paper is to develop a mathematical model enabling the measurement of I– V characteristic by capacitor charge technique while respecting the dynamic limits of the measuring equipment.

2. Deriving the equations of charging a capacitor by PV generator

The equivalent circuit of a poly- or mono-crystalline PV cell, PV module or PV array (PV generator) with its output terminals (a, b) and the capacitor (C) as a load and a shorting switch (s), is presented in Fig. 1 [2,4].

Considering that $R_{sh} \gg R_s$ which means that $I_{sh} \ll I$ [2] and applying the Kirchoffs current and voltage laws to the above circuit we obtain the following equations:

$$I_D + I = I_{ph}, \tag{1}$$

$$R_S I + V = V_D, \tag{2}$$

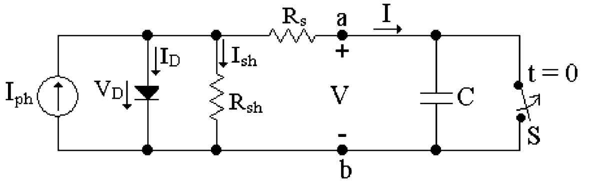

Fig. 1. Equivalent circuit of a PV cell, module or array (PV generator) with a capacitor as load.

where I_{ph} is the generated photocurrent, I_D is the diode current, I is the load current (= capacitor charging current), V_D is the diode voltage, R_S is the series resistance and V is the voltage at the load. Differentiating Eqs. (1) and (2) with respect to time, we obtain the following first-order differential equations:

$$\frac{dI_D}{dt} + \frac{dI}{dt} = \frac{dI_{ph}}{dt} = 0, \tag{3}$$

$$R_S \frac{dI}{dt} + \frac{dV}{dt} = \frac{dV_D}{dt}. \tag{4}$$

The characteristic equation of the diode is given by [5]

$$I_D = I_0 \left[\exp\left(\frac{V_D}{AV_T}\right) - 1\right], \tag{5}$$

$$V_D = V_T \ln\left(\frac{I_D + I_0}{I_0}\right), \tag{6}$$

where V_T is thermal voltage, A is the ideality factor ($A \approx 1$) and I_0 is the reverse saturation current of diode (for Si diode $I_0 = 10^{-6}\,\text{A/cm}^2$). In addition, it applies for the capacitor:

$$\frac{dV}{dt} = \frac{I}{C}, \tag{7}$$

$$V_D = V_T \ln\left(\frac{I_{ph} - I + I_0}{I_0}\right), \tag{8}$$

$$\frac{dV_D}{dt} = -V_T \left(\frac{I_0}{I_{ph} - I + I_0}\right) \frac{dI}{dt}. \tag{9}$$

Substituting (7) and (9) in Eq. (4) we obtain the first-order differential equation

$$\left[R_S + \frac{V_T}{(I_{ph} - I + I_0)}\right] \frac{dI}{dt} + \frac{I}{C} = 0. \tag{10}$$

In order to obtain the highest speed of change for $I(t)$ and $V(t)$ we should set $R_S = 0$ and then integrate Eq. (10) [6]:

$$\int \frac{CV_T}{I(I_{ph} - I + I_0)}\, dI + \int dt = A,$$

$$\frac{CV_T}{(I_{ph} + I_0)} \ln I - \frac{CV_T}{(I_{ph} + I_0)} \ln(I_{ph} + I_0 - I) + t = A.$$

The integration constant A is obtainable from the initial conditions

$$I(t = 0) = I_{ph},$$

$$I(t) = \frac{(I_{ph} + I_0)}{1 + (I_0/I_{ph}) \exp\left(t(I_{ph} + I_0)/CV_T\right)}. \tag{11}$$

With $R_S = 0$ and Eqs. (1), (2), (5), and (11) we obtain $V(t)$:

$$V(t) = V_T \ln\left[\left(\frac{I_{ph}+I_0}{I_0}\right)\left(1-\frac{I_{ph}}{I_{ph}+I_0\ \exp\big(t(I_{ph}+I_0)/CV_T\big)}\right)\right]. \tag{12}$$

The pen-speeds in the Y and X directions of the X– Y recorder are proportional to $\mathrm{d}I/\mathrm{d}t$ and $\mathrm{d}V/\mathrm{d}t$, respectively,:

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -\frac{I_0 I_{ph}(I_{ph}+I_0)^2}{CV_T}\frac{\exp(t(I_{ph}+I_0)/CV_T)}{[I_{ph}+I_0\exp(t(I_{ph}+I_0)/CV_T)]^2}. \tag{13}$$

Considering the capacitor Eq. (7) with Eq. (11) we obtain $\mathrm{d}V/\mathrm{d}t$:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{I_{ph}}{C}\frac{(I_{ph}+I_0)}{I_{ph}+I_0\ \exp(t(I_{ph}+I_0)/CV_T)}. \tag{14}$$

The maximum pen-speed in I-direction at the X– Y recorder is given at time t_M. The time t_M is determined by the following equations:

$$\left.\frac{\mathrm{d}^2 I(t)}{\mathrm{d}t^2}\right|_{(t=t_M)} = 0, \tag{15}$$

$$t_M = \frac{CV_T}{I_{ph}+I_0}\ln\frac{I_{ph}}{I_0}. \tag{16}$$

Substituting Eq. (16) in (11) then in (13) we obtain:

$$I(t_M) = \frac{I_{ph}+I_0}{2}, \tag{17}$$

$$\left.\frac{\mathrm{d}I}{\mathrm{d}t}\right|_{(t=t_M)} = -\frac{(I_{ph}+I_0)^2}{4CV_T}. \tag{18}$$

The maximum pen-speed in V-direction at the X– Y recorder is given at time $t = 0$. Substituting this value in Eqs. (12) and (14).

$$V(0) = 0, \tag{19}$$

$$\left.\frac{\mathrm{d}V}{\mathrm{d}t}\right|_{t=0} = \frac{I_{ph}}{C}. \tag{20}$$

At $t = 0$, $V(0) = 0$ and the short circuit current (I_{sc}) of the PV generator flows in the capacitor; setting $t = 0$ in Eq. (11) we obtain

$$I(0) = I_{ph} \approx I_{sc}. \tag{21}$$

At $t = \infty$ the capacitor C will be fully charged. Substituting $t = \infty$ in Eq. (12), we obtain the open circuit voltage (V_{oc}) of the PV generator:

$$V_{oc} = V_T \ln\frac{I_{ph}+I_0}{I_0}. \tag{22}$$

Since $I_{ph} \gg I_0$ and $I_{ph} \approx I_{sc}$, Eq. (22) will be

$$V_T = \frac{V_{oc}}{\ln(I_{sc}/I_0)}. \tag{23}$$

Setting this value in Eq. (16), with $I_{ph} = I_{sc}$ in Eq. (17), (18) and (20) we obtain

$$t_M = C\,\frac{V_{oc}}{I_{sc}}, \tag{24}$$

$$I(t_M) = \frac{I_{sc}}{2}, \tag{25}$$

$$\left|\frac{dI}{dt}\right|_{(t=t_M)} = -\frac{I_{sc}^2}{4CV_T}, \tag{26}$$

$$\left.\frac{dV}{dt}\right|_{t=0,\ V(0)=0} = \frac{I_{sc}}{C}. \tag{27}$$

The charging process of the capacitor is settled at t_S (settling time) when $I(t) \approx I_0$ and, with Eq. (23), we obtain t_S:

$$t_S = 2C\,\frac{V_{oc}}{I_{sc}} \tag{28}$$

or

$$C = \frac{t_S}{2}\,\frac{I_{sc}}{V_{oc}}. \tag{29}$$

Therefore, the capacitor size is directly and indirectly proportional to the short circuit and open circuit voltage of the PV generator, respectively. Furthermore, the higher the speed of the measuring system, the smaller will be the size of the required capacitor [7].

3. Measuring the *I– V* characteristics

3.1. Using a rheostat

The most simple and inexpensive way to measure the *I– V* curve of a PV generator of small peak power is to use a variable resistor R_L (rheostat) as shown in Fig. 2. The value of R_L will be varied in steps from 0 to ∞ to measure the points of the *I– V* curve from short circuit to open circuit [7]. However, the accuracy of this method is not high, since it is

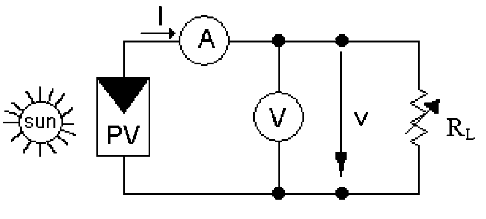

Fig. 2. Schematic circuit for measuring the *I– V* curve of a PV generator using a rheostat.

susceptible to varying solar radiation and thermal conditions during the measurement. The measured *I–V* curve lacks uniformity and smoothness. Furthermore, this method is applicable only for PV generators of very low peak power (<1 kW) since rheostats for higher power are mostly not available.

3.2. Using an X–Y recorder

Respecting the characteristic parameters of the PV generator and the speed of the measuring equipment to be used, the derived Eqs. (24)–(29) enable determining the proper capacitor size for measuring the *I–V* characteristics.

Fig. 3 illustrates the circuit for measuring the *I–V* characteristics of a PV generator by capacitor charging. Voltage and current (V,I) of the PV generator and the voltage signals representing the solar radiation intensity on the PV generator surface (G), the PV cell temperature (T_C), the ambient temperature (T_A) and the wind velocity (V_W) have to be measured contemporarily, while sweeping the *I–V* characteristics from short circuit to open circuit. The switchable capacitor bank allows switching the proper capacitor value (C) while the load resistor (R_L) is used for discharging the capacitor via S_3. The switches S_1, S_2 and S_3 can be either manual switched or are automatic switches (controlled transistor—relay switches) as it is in the modern CDAS.

Measuring the *I–V* characteristics by an *X–Y* recorder is limited by its low speed which forces the use of high capacitor values. The input *X* and *Y* channels of the recorders will be connected to terminals a, b and c, d, respectively (Fig. 3). The other parameters will be measured with external measuring equipment. From Eq. (29) it is obvious that the capacitor size (C) is directly and indirectly proportional to I_{sc} and V_{oc}. According to Eqs. (26) and (27), the capacitor should have a value so that $\mathrm{d}I/\mathrm{d}t$ and $\mathrm{d}V/\mathrm{d}t$ remain below the corresponding maximum pen-speeds $\mathrm{d}Y/\mathrm{d}t$ and $\mathrm{d}X/\mathrm{d}t$ of the *X–Y* recorder.

Nevertheless, the *X–Y* recorders are practically applicable for measuring the *I–V* curves of PV generators of peak power up to 10 kW where switchable capacitor banks (as shown in Fig. 3) and transistor power switches are required.

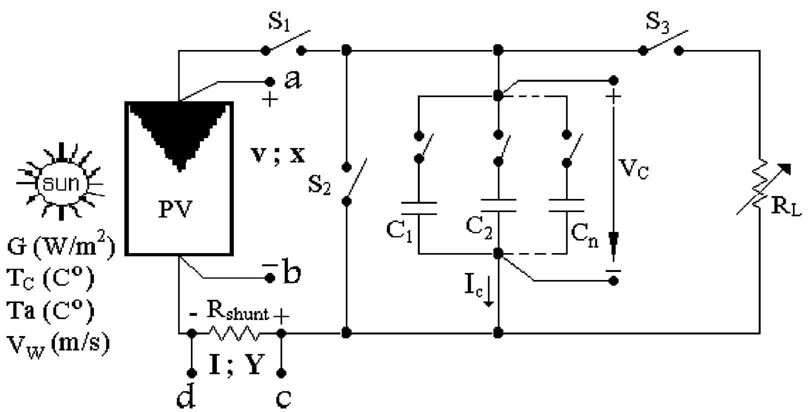

Fig. 3. Circuit for measuring the *I–V* characteristics by *X–Y* reorder and data acquisition system.

3.2.1. Measuring example

The I–V curve of a PV module has been measured by X–Y recorder. Module specifications, the measuring results and evaluation are presented hereafter:

PV module type: AEG—Germany PQ 10/40/01.

PV module area: 0.495 m^2; 40 cells connected in series.

- V_{oc} at standard conditions: 22 V.
- I_{sc} at standard conditions: 2.37 A.
- $I_0 = 10^{-6} \times 40 \times 100 = 4\,\text{mA}$.
- $$V_T = \frac{22}{\ln(2.37/4 \times 10^{-3})} = 3.45\,\text{V}.$$
- $C = 100\,\text{mF}$, $10 \times (C = 10\,\text{mF}/100\,\text{V})$ connected in parallel.

Substituting these values in Eqs. (24)–(28) we obtain

$$t_M = 0.93\,\text{s}, \quad I(t_M) = 1.185\,\text{A}, \quad t_S = 1.86\,\text{s},$$

$$\left(\frac{dI}{dt}\right)_{t=t_M} = -4.07\,\frac{A}{s}, \quad \left(\frac{dV}{dt}\right)_{t=0} = 23.7\,\text{V/s}.$$

By selection of the above capacitor value and the proper recorder scales, the corresponding speeds dY/dt and dX/dt remain below the maximum speeds of the X–Y recorder.

The I–V characteristics of the PV module illustrated in Fig. 4 had been measured at a solar radiation intensity (G) on the module surface of 859 W/m^2 and at an ambient temperature of 35.1 °C, which makes the result deviate a little from the I–V characteristics measured under standard conditions.

3.3. Using a CDAS

Using this system allows to sweep the PV generator operating voltage and current from short circuit to open circuit with the other environmental parameters indicated in Fig. 3, in a short time of 200 ms. The high speed of this system enables to measure the I–V

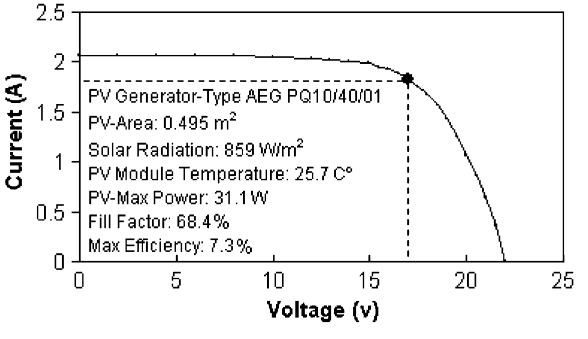

Fig. 4. I–V characteristics of a polycrystalline silicon PV module (40 cells connected in series) measured by the X–Y recorder.

characteristics for a PV generator of relatively high peak power with a small capacitor value [7]. For instance, the I–V curve of a PV generator of 20 kW peak power, with a maximum open circuit voltage of 300 V and a maximum short circuit current of 85 A can be measured by a capacitor bank of 28 mF/300 V.

3.3.1. Measuring example

The CDAS allows also the analysis of the I–V curve and to compute the important quality parameters such as the maximum power point, the PV efficiency and the fill factor. Fig. 5 illustrates the I–V curve of a mono-crystalline silicon module with the quality parameter measured by the CDAS using a capacitor charging technique according to Fig. 3.

The type of this PV module is Siemens M55 which has, at standard conditions, the characteristic parameters:

$$I_{sc} = 3.25\,\text{A} \quad \text{and} \quad V_{oc} = 21.7\,\text{V}.$$

According to Eq. (29) the proper capacitor used for measurement is

$$C = \frac{200}{2}\,\frac{3.25}{21.7} = 15\,\text{mF}.$$

Due to the high speed of CDAS, the size of this capacitor is only 15% of that necessary for measuring the I–V curve by an X–Y recorder.

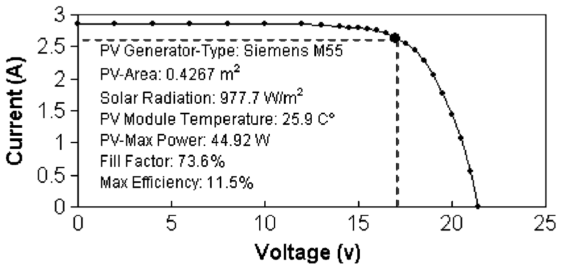

Fig. 5. I–V characteristics of a mono-crystalline silicon PV module (36 cells in series) measured by CDAS.

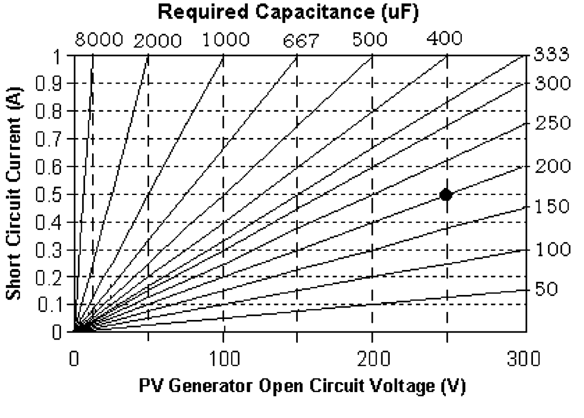

Fig. 6. Capacitance calculation chart for measuring the I–V curve of PV generators.

3.3.2. Capacitance calculation chart

The relationship between the short circuit current I_{sc}, the open circuit voltage V_{oc} of the PV generator and the required capacitance value (C), according to Eq. (29), enables to construct the capacitance calculation chart illustrated in Fig. 6. Since the required capacitance is a linear function of I_{sc}, the current scale may be changed by a scale factor, which would change the required capacitance by the same factor. For example, consider the marked point in the chart (Fig. 6). For an open circuit voltage of 250 V and a short circuit current of 0.5 A, the required capacitance is 200 μF. If the short circuit current were 5 A instead of 0.5 A the required capacitance would be 2000 μF. Again, if the short circuit current were 50 A, the required capacitance would be 20000 μF.

4. Conclusions

The equations derived through transient analysis of capacitor charging by a PV generator had proved their practical applicability in calculating the proper capacitance value for measuring the I–V characteristics of PV generators by X–Y recorders and computerized data acquisition systems (CDAS). The higher the speed of a measuring system, the lower will be the required capacitor size. Respecting the measuring speed limits of this equipment, and complying with reasonable capacitor size, the X–Y recorders can be used for PV generators of peak power up to 10 kW, with the CDAS for a PV generator of higher power.

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