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Stochastic vibration of laminated composite coated beam traversed by a random moving load

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Abstract

This paper deals with the random vibration of a simply-supported laminated composite coated beam traversed by a random moving load. The moving load is assumed to move with accelerating, decelerating and constant velocity types of motion. Using basic analytical techniques, this investigation aims at improving the random response of beams made from isotropic materials by using composite coats. Closed form solutions for the variance of the response is obtained. The results arrived at in this paper discuss the random vibration characteristics of the response of the aforementioned beam taking into account the speed effect, lamina thickness and orientation of the coat effects.

1. Introduction

The problem of vibrations of beams resulting from the passage of different types of loads is of great practical importance. In this paper the random response of a simply supported isotropic beam coated with different lamina orientation of composite coats subjected to a single constant random force moving with different types of motion is investigated. Composite materials are widely used in different engineering applications. Their use has increased tremendously in recent years due to their high specific strength, stiffness, and favourable fatigue characteristics. Furthermore, laminated composites may be utilized as a coating to improve the behaviour of dynamical systems such as reduction of vibrations and noise and increase of total internal damping of the system [1–5]. Based on the excellent book by Frýba [6] and the references therein, different papers that deal with various aspects of the moving load problem have emerged. Perhaps the most important aspect of these is the one that deals with the problem from the random or stochastic point of view due to inherent randomness in material properties, and nature and speed of the load [6–17]. The problem of dynamic response of composite structures due to different types of loads has been discussed by several authors [1,3–4]. This paper is considered as a continuation of the work presented in references [14–17]. The beam considered, and as mentioned earlier, is multi-layered where the top and bottom layers consist of composite material. The approach used here is based on the work presented in references [1,3–5]. It is assumed that mid-plane symmetry exists, that is, the bending stretching coupling and traverse shear are ignored. These assumptions serve the purpose of introducing a simple model to study the effect of random moving load on composite beams.

2. Theoretical formulation

A laminated composite coated beam with its physical dimensions and coordinate axes is shown in Fig. 1. The central laminate or the core is made from an isotropic material, steel in this case, where its modulus of elasticity is designated as E_c. The top and bottom lamina or the faces are made from composite material where its modulus of elasticity E_f along the axis of the beam is written [1]

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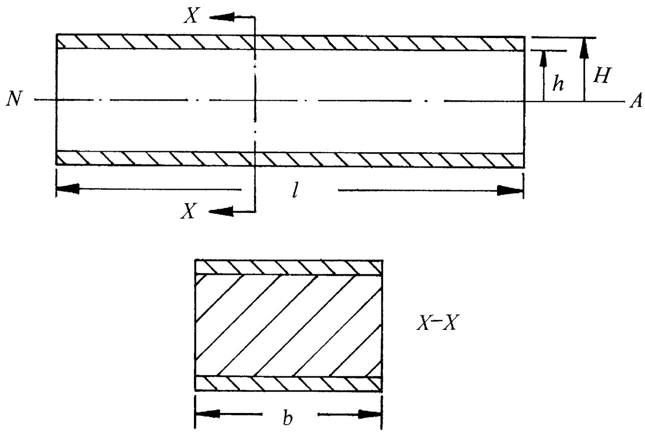

Fig. 1. Geometry of a laminated composite beam.

$$\frac{1}{E_f} = \frac{1}{E_{11}}\cos^4\theta + \left(\frac{1}{G_{12}} - \frac{2v_{12}}{E_{11}}\right)\cos^2\theta\sin^2\theta + \frac{1}{E_{22}}\sin^4\theta \tag{1}$$

where E_{11}, E_{22}, G_{12} and v_{12} are the mechanical properties along the laminate's directions. The angle θ is the angle between the principal laminate' s direction and the axis of the beam.

The problem to be considered is that of transverse vibrations of a simply supported uniform elastic sandwich beam of finite length l originally at rest. The beam is acted upon by a random force which moves from left to right in a uniform accelerating, decelerating or uniform velocity types of motion. This problem is described by the following differential equation

$$(EI)_{equ}\frac{\partial^4 v(x,t)}{\partial x^4} - N\frac{\partial^2 v(x,t)}{\partial x^2} + \mu_{equ}\frac{\partial^2 v(x,t)}{\partial t^2} + 2\mu_{equ}\omega_b\frac{\partial v(x,t)}{\partial t} = p(x,t) \tag{2}$$

with boundary and initial conditions

$$v(0,t) = 0,\quad v(l,t) = 0,\quad \frac{\partial^2 v(x,t)}{\partial x^2}\Big|_{x=0} = 0,\quad \frac{\partial^2 v(x,t)}{\partial x^2}\Big|_{l=0} = 0,$$
$$v(x,0) = 0,\quad \frac{\partial v(x,t)}{\partial t}\Big|_{t=0} = 0 \tag{3}$$

where $(EI)_{equ}$, N, μ_{equ}, ω_b, l, and $v(x,t)$ denote, respectively, the equivalent flexural rigidity of the beam, the axial force applied at the ends of the beam, the equivalent mass per unit length of the beam, the circular frequency of damping of the beam, the length of the beam, and the vertical deflection of the beam at point x and time t. Considering Eq. (1), the equivalent flexural rigidity of the beam under investigation is expressed as

$$(EI)_{equ} = \frac{2b}{3}[E_c h^3 + E_f(H^3 - h^3)], \tag{4}$$

where b is the width of the beam; and h and H are related to the thickness of the beam as shown in Fig. 1. The equivalent mass per unit length of the beam can be written as

$$\mu_{equ} = 2b[\rho_c h + \rho_f(H - h)], \tag{5}$$

where ρ_c and ρ_f are the densities of the core and faces of the beam, respectively. The load $p(x,t)$ is written as [6]

$$p(x,t) = \delta[x - f(t)]P(t), \tag{6}$$

where $\delta(.)$ denotes the familiar Dirac delta function. The function $f(t)$ describes the motion of the load along the beam at time t defined as

$$f(t) = x_0 + ct + \frac{1}{2}at^2, \tag{7}$$

where x_0 is the point of application of the force, c is the initial speed, and a is the constant acceleration. This function describes a uniform decelerating or accelerating motion. The uniform velocity type of motion, of course, is given by

$$f(t) = ct \tag{8}$$

the force $P(t)$ is defined as

$$P(t) = P_0 + \tilde{P}(t), \tag{9}$$

where P_0 is the constant mean value representing the deterministic part of the force and $\tilde{P}(t)$ is the random part of the force whose covariance is written as [7,15]

$$C_{pp}(t_1,t_2) = E[\tilde{P}(t_1)\tilde{P}(t_2)], \tag{10}$$

where $E[.]$ denotes expectations. For a random force of white noise type, the covariance becomes

$$C_{PP}(t_1,t_2) = S_P\delta(t_2 - t_1), \tag{11}$$

where S_P is the constant value of the spectral density function of the white noise force. It follows that the covariance of the moving force can be written as

$$C_{pp}(x_1,x_2,t_1,t_2) = \delta[x_1 - f(t_1)]\delta[x_2 - f(t_2)]S_P\delta(t_2 = t_1). \tag{12}$$

In modal form, the transverse deflection of the beam is written as

$$v(x,t) = \sum_{k=1}^{\infty} X_k(x)Y_k(t), \tag{13}$$

where $X_k(x)$ are the normal modes of free vibration, and $Y_k(t)$ are the generalized deflections or modal responses. For a simply supported beam $X_k(x) = \sin(\kappa_k x)$ where

$\kappa_k = k\pi/l$. Carrying out the familiar operations, the differential equation of the kth mode of the generalized deflection or the modal response is written as

$$\ddot{Y}_k(t) + 2\omega_k\xi_k\dot{Y}_k(t) + \omega_k^2 Y_k(t) = \frac{2}{\mu_{equ}l}P(t)\sin[\kappa_k f(t)]. \tag{14}$$

The natural frequency of the undamped beam ω_k is defined as

$$\omega_k^2 = \omega_{1,0}^2 k^2(k^2 \pm \psi), \tag{15}$$

where

$$\omega_{1,0}^2 = \frac{(EI)_{equ}}{\mu_{equ}}\kappa_1^4,\ \psi = \frac{N}{N_{cr}},\ N_{cr} = \frac{\pi^2(EI)_{equ}}{l^2}. \tag{16a - c}$$

$\omega_{1,0}$ is the first natural frequency of the beam without axial force, N_{cr} is the Euler buckling force, and ψ is the axial force ratio. The positive sign is used for tension and the negative sign is used for compression. The coefficient of damping ξ_k is defined as

$$\xi_k = \frac{\omega_b}{\omega_k}. \tag{17}$$

The solution of Eq. (14) is written as

$$Y_k(t) = \frac{2}{\mu_{equ}l}\int_0^t h_k(t-\tau)P(\tau)\sin[\kappa_k f(\tau)]d\tau \tag{18}$$

where $h_k(t)$ is the impulse response function defined as

$$h_k(t) = \begin{cases} \frac{1}{\omega_{dk}}e^{-\xi_k\omega_k t}\sin\omega_{dk}t & t\geq 0 \\ 0 & t < 0 \end{cases} \tag{19}$$

where ω_{dk} is defined as

$$\omega_{dk}^2 - \omega_k^2 = \omega_b^2. \tag{20}$$

Using Eq. (18) to solve Eq. (14) and substituting the result into Eq. (13), the transverse deflection of the beam is obtained as

$$v(x,t) = \frac{2}{\mu_{equ}l}\sum_{k=1}^{\infty}\frac{1}{\omega_{dk}}\sin\kappa_k x\int_0^t \sin\omega_{dk}(t-\tau)e^{-\xi_k\omega_k(t-\tau)}\sin[\kappa_k f(\tau)]P(\tau)d\tau. \tag{21}$$

To obtain the random part of the response, the covariance of the generalized deflection can be written by means of Eq. (12) as

$$C_{Y_kY_r}(t_1,t_2) = \frac{4S_P}{\mu_{equ}^2 l^2}\int_{-\infty}^{\infty} h_k(t_1-\tau_1)h_r(t_2-\tau_1)\sin[\kappa_k(\tau_1)]\sin[\kappa_r f(\tau_1)]d\tau_1. \tag{22}$$

In light of Eq. (13), the covariance of the deflection is written as

$$C_{vv}(x_1,x_2,t_1,t_2) = \sum_{k=1}^{\infty}\sum_{r=1}^{\infty}\sin(\kappa_k x_2)\sin(\kappa_r x_2)C_{Y_kY_r}(t_1,t_2). \tag{23}$$

It follows that the variance of deflection is written as

$$\sigma_v^2(x,t) = C_{vv}(x,x,t,t). \tag{24}$$

Neglecting the cross-covariance terms, i.e. $C_{Y_kY_r}(t_1,t_2) = 0$ for $k \neq r$ [6,7] and substituting Eqs. (22) and (23) into Eq. (24), the variance of deflection is obtained as [18]

$$\sigma_v^2(x,t) = \frac{S_P}{(2\mu_{equ}l)^2}\sum_{k=1}^{\infty}\frac{1}{\omega_{dk}^2}\sin^2(\kappa_k x)R_1^2(t), \tag{25}$$

where

$$\begin{aligned} R_1^2(t) = {} & \left(\frac{2(1-\xi_k^2)}{\xi_k\omega_k} + \frac{2\xi_k\omega_k}{q_1} - \frac{2}{\xi_k\omega_k}\right)e^{-2\xi_k\omega_k t} + \left(\frac{1}{q_2} + \frac{1}{q_3} - \frac{1}{q_1}\right)\xi_k\omega_k\cos 2\kappa_k ct + \left(\frac{\kappa_k c-\omega_{dk}}{q_2} \right. \\ & \left. + \frac{\kappa_k c + \omega_{dk}}{q_3} - \frac{2\kappa_k c}{q_1}\right)\sin 2\kappa_k ct + \left(\frac{2}{\omega_k} - \frac{\omega_k}{q_3} - \frac{\omega_k}{q_2}\right)\xi_k e^{-2\xi_k\omega_k t}\cos 2\omega_{dk}t + \left(\frac{k_k c + \omega_{dk}}{q3} - \frac{\kappa_k c-\omega_{dk}}{q_2} \right. \\ & \left. - \frac{2\omega_{dk}}{\omega_k^2}\right)e^{-2\xi_k\omega_k t}\sin 2\omega_{dk}t, \end{aligned} \tag{26}$$

in which

$$\begin{aligned} q_1 &= \xi_k^2\omega_k^2 + \kappa_k^2 c^2 \\ q_2 &= \xi_k^2\omega_k^2 + (\kappa_k c-\omega_{dk})^2 \\ q_3 &= \xi_k^2\omega_k^2 + (\kappa_k c + \omega_{dk})^2. \end{aligned} \tag{27a - c}$$

The random dynamic coefficient of variation of the deflection at the midspan of the beam is written as

$$V_{vP}\left(\frac{l}{2},t\right) = \frac{\sigma_v\left(\frac{l}{2},t\right)}{v_0 V_P} \tag{28}$$

where v_0 is the static deflection of the composite beam at its midspan produced by a concentrated force P acting at the midspan written as

$$v_0 = \frac{Pl^3}{48(EI)_{equ}} \tag{29}$$

and V_P is the coefficient of variation of the force written as

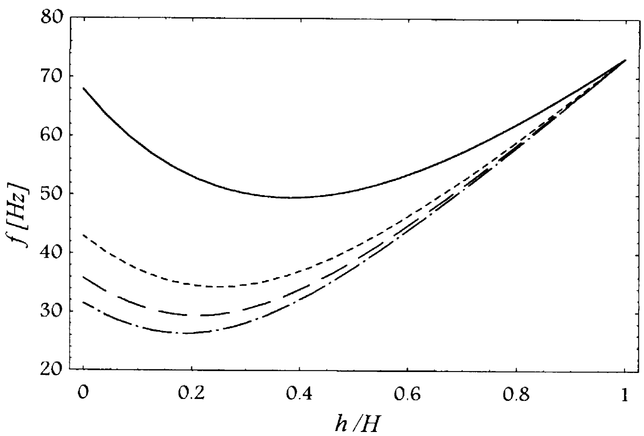

Fig. 2. Variation of the natural frequency of the laminated composite coated beam versus thickness ratio h/H.

$$V_P = \frac{\sqrt{S_P \omega_1}}{P}. \tag{30}$$

Substituting Eqs. (29) and (30) into Eq. (28) and considering Eq. (25), the random dynamic coefficient of variation of the deflection, Eq. (28), is rewritten as

$$V_{vP}\left(\frac{l}{2},t\right) = \beta R_1(t) \tag{31}$$

where β is defined as

$$\beta = \frac{24\sqrt{\omega_1}}{\pi^4\sqrt{1-\xi^2}}. \tag{32}$$

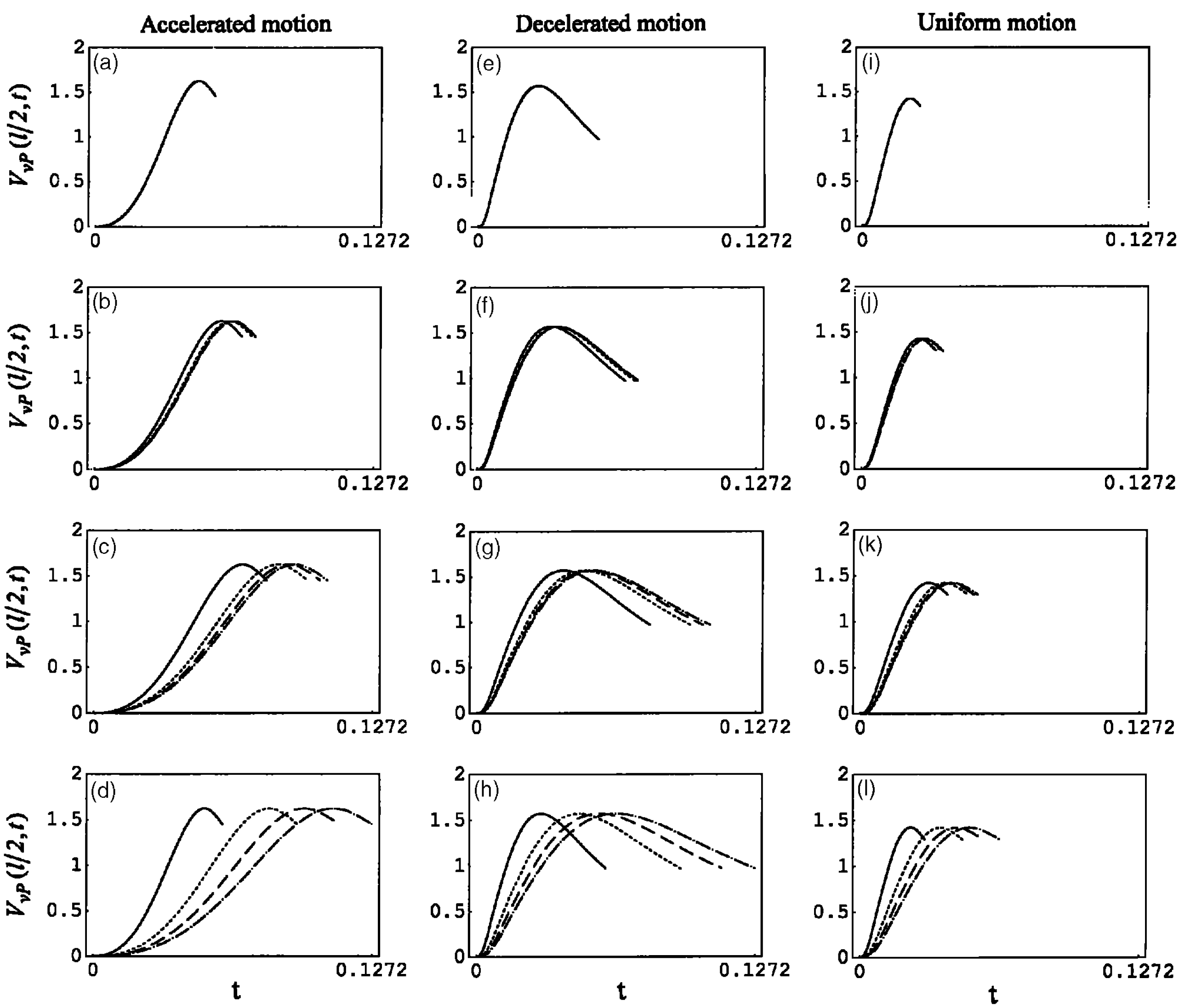

Fig. 3. Random dynamic effect versus time; α=0.25; (a–d) accelerated motion, (a) h=4 mm, (b) h=3 mm, (c) h=2 mm, (d) h=0 mm; (e–h) decelerated motion, (e) h=4 mm, (f) h=3 mm, (g) h=2 mm, (h) h=0 mm; (i–l) uniform velocity, (i) h=4 mm, (j) h=3 mm, (k) h=2 mm, (l) h=0 mm; (–)θ=0°, (...)θ=30°, (—)θ=60°, (–—)θ=90°.

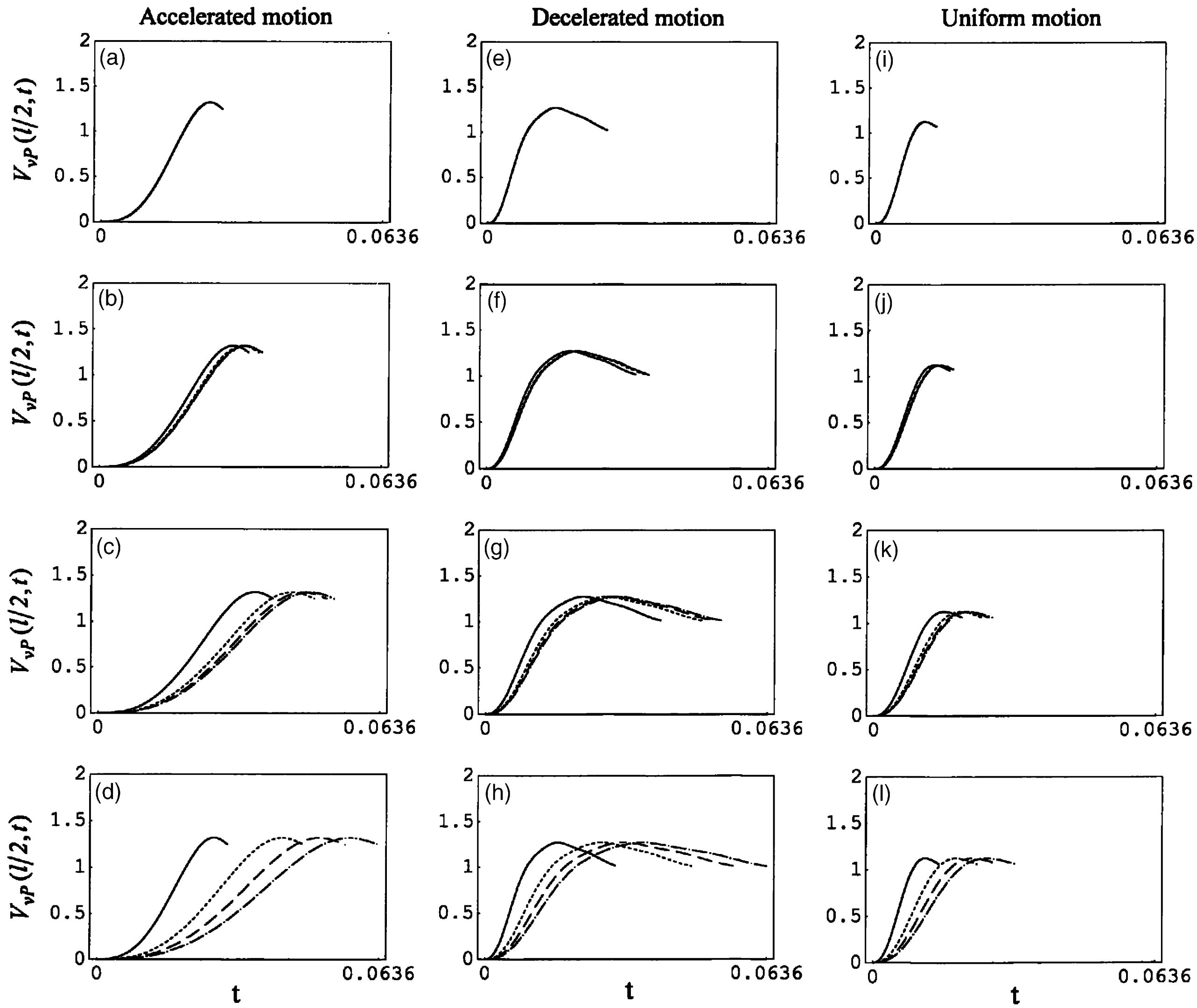

Fig. 4. Random dynamic effect versus time; α=0.25; (a–d) accelerated motion, (a) h=4 mm, (b) h=3 mm, (c) h=2 mm, (d) h=0 mm; (e–h) decelerated motion, (e) h=4 mm, (f) h=3 mm, (g) h=2 mm, (h) h=0 mm; (i–l) uniform velocity, (i) h=4 mm, (j) h=3 mm, (k) h=2 mm, (l) h=0 mm; (–)θ=0°, (...)θ=30°, (—)θ=60°, (—–)θ=90°.

Similarly, the random dynamic coefficient of variation of the deflection at the midspan of the beam due to the passage of accelerating and/or decelerating random moving force is written as

$$V_{vP}(\frac{l}{2},t) = \beta R_2(t) \tag{33}$$

where $R_2(t)$ is obtained in a similar fashion as $R_1(t)$ and is written as [18]

$$\begin{aligned} R_2^2(t) = {} & \frac{2}{\xi_k\omega_k}[1-e^{-2\xi_k\omega_k t}]-\frac{2}{\omega_k^2}[\xi_k\omega_k \\ & + e^{-2\xi_k\omega_k t}(\omega_{dk}\sin 2\omega_{dk}t-\xi_k\omega_k\cos 2\omega_{dk}t)] \\ & -2\mathrm{Re}\{r_1e^{z_1}[\mathrm{erf}(r_2t + z_2) = \mathrm{erf}(z_2)]\} \\ & + \mathrm{Re}\{r_1e^{z_3}[\mathrm{erf}(r_2t + z_4)-\mathrm{erf}(z_4)]\} \\ & + \mathrm{Re}\{r_1e^{z_5}[\mathrm{erf}(r_2t + z_6)-\mathrm{erf}(z_6)]\} \end{aligned} \tag{34}$$

where

$$\begin{aligned} r_1 &= \sqrt{\frac{\pi}{2\kappa_k a}}(1 + i) \\ r_2 &= \sqrt{\frac{\kappa_k a}{2}}(1-i) \\ r_3 &= \sqrt{\frac{-1}{2\kappa_k a}} \\ r_4 &= \sqrt{\frac{1}{\kappa_k a}} \end{aligned} \tag{35a − j}$$

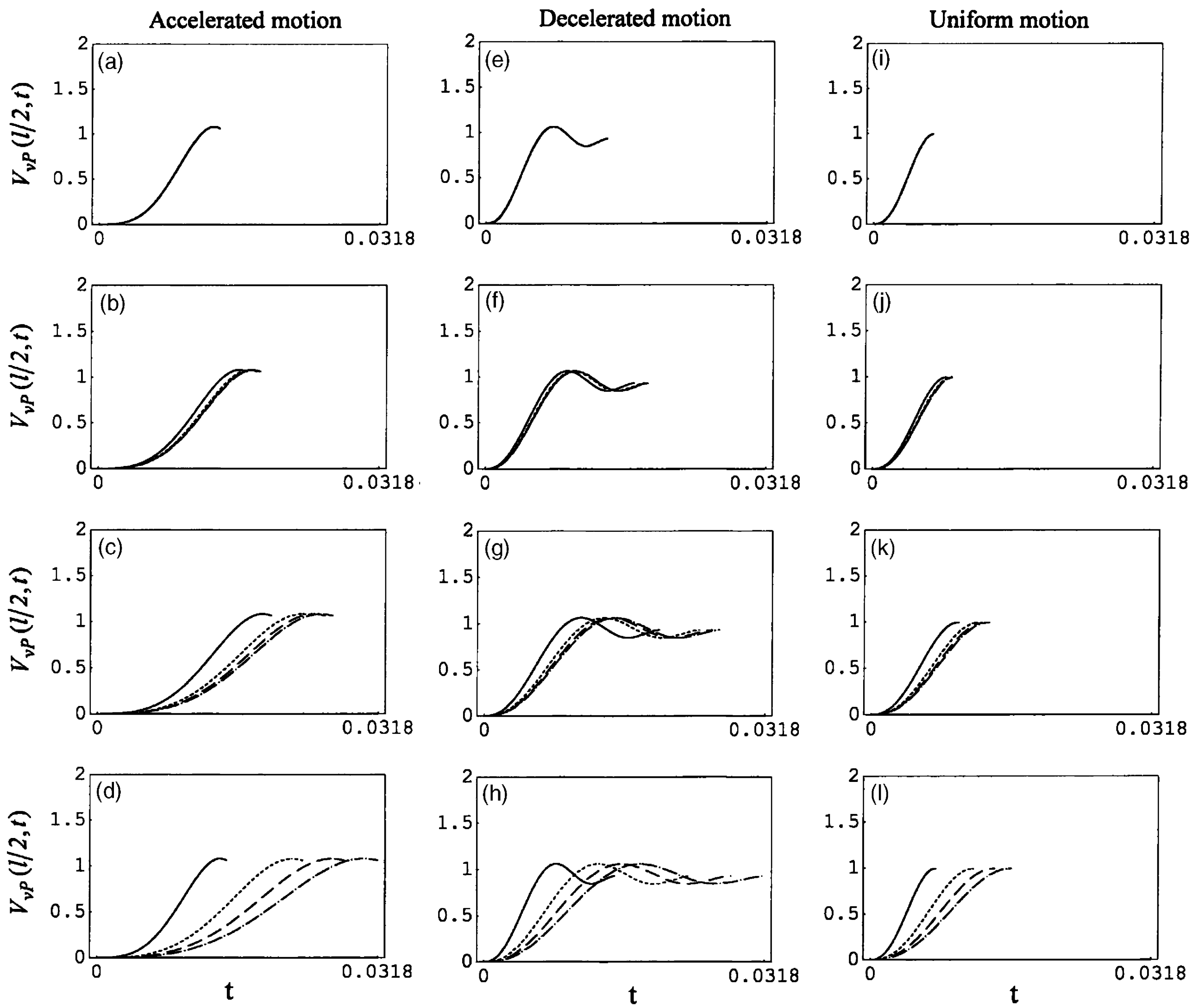

Fig. 5. Random dynamic effect versus time; α=0.25; (a–d) accelerated motion, (a) h=4 mm, (b) h=3 mm, (c) h=2 mm, (d) h=0 mm; (e–h) decelerated motion, (e) h=4 mm, (f) h=3 mm, (g) h=2 mm, (h) h=0 mm; (i–l) uniform velocity, (i) h=4 mm, (j) h=3 mm, (k) h=2 mm, (l) h=0 mm; (–)θ=0°, (...)θ=30°, (—)θ=60°, (——)θ=90°.

$$z_1 = -2\xi_k\omega_k\left(t+\frac{c}{a}\right) + ir_4[\xi_k^2\omega_k^2 + \kappa_k^2(2ac-c^2)]$$

$$z_2 = r_3[\xi_k\omega_k-\kappa_k c + i(\xi_k\omega_k + \kappa_k c)]$$

$$z_3 = -2\xi_k\omega_k\left(t+\frac{c}{a}-r_4\omega_{dk}\right) + ir_4[\xi_k^2\omega_k^2-\omega_{dk}^2 + \kappa_k^2(2ab-c^2) + 2\kappa_k\omega_{dk}(c+at)]$$

$$z_4 = r_3[\xi_k\omega_k-\kappa_k c + \omega_{dk} + i(\xi_k\omega_k + \kappa_k c-\omega_{dk})]$$

$$z_5 = -2\xi_k\omega_k\left(t+\frac{c}{a}+r_4\omega_{dk}\right) + ir_4[\xi_k^2\omega_k^2-\omega_{dk}^2 + \kappa_k^2(2ab-c^2)-2\kappa_k\omega_{dk}(c+at)]$$

$$z_6 = r_3[\xi_k\omega_k-\kappa_k c-\omega_{dk} + i(\xi_k\omega_k + \kappa_k c + \omega_{dk})].$$

3. Numerical examples and discussion of results

Different examples are presented to clarify the results arrived at in this paper. The physical dimensions of the beam are: $l = 500$ mm, $H = 4$ mm, and $b = 25$ mm. As mentioned earlier, the laminated composite coated beam consists of a core layer and top and bottom lamina that are made from composite material. The core is assumed to be steel for which $E_c = 200$ GPa and $\rho_c = 7850$ kg/m^3. The faces are made from glass/epoxy composite material for which $E_{11} = 38.6$ GPa, $E_{22} = 8.27$ GPa, $G_{12} = 4.14$ GPa, $\nu_{12} = 0.26$, and $\rho_f = 1759$ kg/m^3 [2]. Fig. 2 shows for different fiber orientation, the variation of the natural frequency versus the thickness ratio h/H where h and H are shown in Fig. 1. It is clear that the natural frequency of the composite beam can be controlled by choosing the proper fiber orientation or laminate thickness. It is noticed that the stiffness is higher when the fibers are oriented longitudinally along axis of the beam, $\theta = 0°$, than when the fibers are oriented transversely with the axis of the beam, $\theta = 90°$. In the accelerated motion, a beam at rest is entered from the left-hand side at point $x_0 = 0$ by a concentrated force P moving according to Eq. (7). The motion is assumed to be uniformly accelerated so that it reaches the speed c at point $x = l$. The instant t_1 at which the force arrives at the right-hand side of the beam and the acceleration a are written as

$$t_1 = \frac{2l}{c}, \quad a = \frac{c^2}{2l}. \tag{36,37}$$

In the decelerated motion, a beam also at rest is entered from the left-hand side at point $x_0 = 0$ by a concentrated force P moving according to Eq. (7). The motion is assumed to be uniformly decelerated so that it stops at the right-hand side of the beam at point $x = l$. the instant t_2 at which the force stops and the deceleration a are written as

$$t_2 = \frac{2l}{c}, \quad a = -\frac{c^2}{2l}. \tag{38,39}$$

In the uniform velocity case, a beam is also at rest is entered from the left-hand side at point $x_0 = 0$ by a concentrated force P moving according to Eq. (8). The instant at which the force arrives at the right-hand side of the beam is

$$t_3 = \frac{l}{c}. \tag{40}$$

The effect of speed is represented by the dimensionless speed parameter α, which is defined as

$$\alpha = \frac{c}{c_{cr}} \tag{41}$$

where c_{cr} is the critical speed, defined as [6,7]

$$c_{cr} = \frac{\omega_1 l}{\pi}. \tag{42}$$

In Figs. 3-5, the effect of damping is taken as =0.05. Fig. 3(a-l) show the variation of the random dynamic effect versus the time the force needs to cross the beam for $\alpha = 0.25$ for the three types of motion. The accelerated motion is shown in Fig. 3(a–d); the thickness of the steel core takes on the value of h = 4,3,2,0 mm, respectively. In Fig. 3(a) the beam is all made from steel; while in Fig. 3(d) the beam is all made from composite material. Fig. 3(e–h) and (i–l) are similar to Fig. 3(a–d) but for decelerated motion and uniform velocity types of motion, respectively. These figures show the effect of the type of material, fiber orientation, and laminate thickness on the random dynamic coefficient of the beam due to a random moving load. It is noticed that the effect of fiber orientation increases as the laminate thickness increases. The random dynamic coefficient is higher when the fibers are oriented longitudinally as opposed to transversely oriented fibers. It is also noticed, and as expected, that the random dynamic coefficient decreases as the thickness of the laminate increases. In other words, the damping effect increases as the thickness of the laminate increases. The accelerated motion is more sensitive to fiber orientation and to laminate thickness than the decelerated motion. It is also noticed that the random dynamic coefficient is higher when the force is on the first half of the beam for decelerated motion than for accelerated motion [14]. Fig. 4(a–l) and Fig. 5(a–l) are similar to Fig. 3(a–l) but for $\alpha = 0.5$ and $\alpha = 1.0$, respectively. In all figures, the random dynamic effect increases as α decreases.

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