Stochastic approach for modeling the effects of intersection geometry on turning vehicle paths

Wael K.M. Alhajyaseen, Miho Asano, Hideki Nakamura, Dang Minh Tan

Abstract

Analytical evaluation techniques for the safety performance of signalized intersections are applicable to limited scenarios and conditions, whereas simulation-based analysis tools are very flexible and promising. This study is part of intensive efforts to develop a microscopic simulation model for the safety assessment of signalized intersections. One important aspect of analyzing driver maneuver is vehicle paths. Broadly varying paths may result in widely distributed potential conflict points with other movements, which may affect the occurrence probability of severe conflicts. Therefore, this study aims to develop a technique to reproduce the variations in the paths of turning vehicles, considering intersection geometry, vehicle type, and speed. Several signalized intersections in Nagoya City, Japan, with various traffic and geometric characteristics were videotaped. The analysis revealed that the paths of right-turning vehicles (left-hand traffic) are more sensitive to the vehicle speed and turning angle whereas those of left-turning vehicles are more sensitive to the intersection corner radius, turning angle, and vehicle speed. For modeling individual vehicle paths, this study applies the Euler-spiral-based approximation methodology where each trajectory is fitted by an entering Euler spiral curve followed by a circular curve and an exit Euler spiral curve. The proposed models are unique since they provide a realistic and rational representation of the variations in turning vehicles’ paths inside intersections.

Article Info

Article history:
Received 21 November 2011
Received in revised form 29 May 2012
Accepted 6 September 2012

Keywords:
Intersection geometry
Turning traffic
Vehicle path
Euler spiral curve
Simulation

1. Introduction

Depending on cultural customs, prevailing traffic characteristics, and locally available technologies, various safety improvements are implemented at intersections; however, they are, thus far, based on experience and post-implementation assessments. A major achievement would be to enable engineers to conduct pre-implementation assessments at the planning stage. Simulation tools are the means to realize such an assessment. Existing simulation software, however, simplifies the traffic flow inside intersections to an extent that safety assessments are not reliable.

This study is part of continuous efforts to develop a microscopic simulation model for the safety assessment of signalized intersections. This model would allow practitioners to evaluate the effects of various improvements in the layout and operation of signalized intersections on the overall safety performance. For instance, through this simulation, it would be possible
to predict the impacts of adding channelization or adjusting the positions of crosswalks or intersection corner radii. The simulation can further be applied to modify the signal timing parameters such as all-red intervals. However, in order to develop a rational simulation model, driver behavior must be realistically represented.

One important aspect in analyzing driver behavior, which is a vital element in the safety performance of signalized intersections, is vehicle paths (spatial trajectories). Several previous studies found that there are significant variations in trajectories of turning vehicles. It is rational to assume that such variations might result in specific unfavorable conditions, which might lead to collisions. In reality, road users behave by anticipating other users’ behavior in order to avoid collisions. Broadly varying road user behavior and trajectories may lead to misunderstandings of other users’ decisions, which might result in safety problems. Large variations in vehicle paths lead to widely distributed potential conflict points with other users. This indicates that users must pay attention to a broader area where conflicts may occur. Therefore, it is quite important to consider not only an average vehicle maneuver but also its variations, which are affected by the geometric layout of intersections and interaction with other users.

The paths of left-turning and right-turning vehicles have different characteristics and influencing factors. It is important to mention that this study is based on the traffic system in Japan where vehicles are driven on the left side of the road and most drivers sit in the right side of vehicles. Thus, the definition of turning movements throughout this paper is based on left-hand traffic system. However since the positions of the driver relative to road curb in right-hand and left-hand traffic systems are symmetric, it is expected under the same operation and driver characteristics, the traffic systems do not leave significant impacts on vehicle paths.

In common signal phasing plans, pedestrians and through-left turning traffic of the same direction share the same phase; thus, left-turners will have frequent conflicts with pedestrians and cyclists, which might affect their paths. In contrast, right-turning vehicles are often provided with exclusive phases, which significantly limit the effects of pedestrians and cyclists. Furthermore, since right-turning vehicles cross from the middle of an intersection, their paths might get affected by various geometric elements such as the turning angle (i.e., the angle between entering and exit approaches) and median configuration at the entering and exiting approaches. Left-turning vehicles are affected by the corner radius as well as the turning angle.

The objective of this study is to investigate factors influencing turning vehicle paths and their variations. Therefore, a model representing variations in the turning vehicle paths (left-turning and right-turning) under different geometric conditions is proposed. The spatial distributions of vehicle paths are analyzed considering the intersection geometry, vehicle type, and the presence of pedestrians or cyclists. Finally, conclusions and future works are discussed.

2. Literature review

In general, intersection layout is the main factor affecting vehicle-turning maneuver (Japan Society of Traffic Engineers, 2002). For instance, the manual suggests modifying intersection corner geometry or crosswalk position to improve the safety performance regarding accidents between left-turning vehicles (left-hand traffic) and pedestrians. Therefore, understanding the effects of intersection layouts on driver turning maneuvers is essential.

The design vehicle turn templates, such as the template shown in AASHTO (2004), are commonly assumed to provide actual turning paths. However, the variation in driver-vehicle interaction at intersections and the resulting paths have not been studied nor analyzed intensively.

Stover (2008) identified the issue of variations in vehicle turning trajectories as one of the main concerns that should be carefully studied in the design of intersections, since a significant variation in driver paths was found empirically. Stover and Koepke (2002) presented a number of figures illustrating the dispersion of right-front wheel paths of right-turning vehicles (right-hand traffic) for various combinations of curb radii and roadway widths. These figures clearly showed a considerable variability in vehicle trajectories. However, they did not provide any quantification for such variations.

Decabooter and Solberg (1988) collected data for several trucks making a right turn (right-hand traffic) at two signalized intersections. A camera system was used to obtain the observed paths of the left-front overhangs and the right-rear wheels of individual vehicles. They found significant variations in the paths of trucks in the two sites, which reflect the idea that drivers behave differently at intersections with identical geometric characteristics. However, the study did not provide any methods to predict such variations under various geometric and traffic conditions.

Read (2008) analyzed the effect of a vehicle’s body (A-pillar) on driver visibility and its effect on the path variation of right- and left-turning vehicles. However, since the objective of the study focused on vehicle design issues, the factors that affect driver behavior while turning were not modeled or analyzed in depth.

Although a few studies analyzed the path variations of vehicles, these studies did not quantitatively analyze the effect of intersection geometric characteristics on paths and their variations. This study proposes an empirical model to explain this relationship.

Asano et al. (2011) analyzed the paths of left-turning vehicles (left-hand traffic) at several signalized intersections in Nagoya City, Japan. They analyzed the effect of intersection geometry on the variations of vehicle paths and proposed a procedure to model the variations in left-turning vehicle paths. The modeling method is based on approximating the curvature of each individual vehicle using a polygonal line. The results showed that left-turning vehicle paths can be well fitted by using a polygonal line, which is a combination of straight segments, Euler spiral curve segments, and circular curve segments. One of
the shortcomings of their proposed models is that they do not consider the effect of left-turning vehicle speed. Furthermore, their analysis is limited to left-turning vehicles.

This paper is an extension to the previous works done by Asano et al. (2011), aiming to refine the developed empirical models to consider vehicle speed and to extend the analysis to right-turning vehicles, as well.

3. Study sites and data observation

3.1. Study sites

In order to analyze the significance of various factors influencing the maneuvers of turning vehicles (left-hand traffic), video data was collected at several signalized intersections with different geometric and traffic conditions. Tables 1 and 2 present the observation dates and geometric characteristics of the observed sites for the analysis of left-turning vehicles and right-turning vehicles, respectively. All sites are located in Nagoya City, Japan. The definitions of the parameters in Tables 1 and 2 are illustrated in Fig. 1. The observed sites have significantly different geometric layouts, such as curb radii, intersection angles, crosswalk setback distances, and median configurations. Such a wide range of layouts is necessary to rationally study the variations in the maneuvers of turning vehicles. All observed sites in Table 2 are operated with an exclusive right-turning phase. Thus, observed right-turning vehicles theoretically have no conflict with opposite through traffic or crossing pedestrians.

The average turning vehicle, pedestrian, and bicycle demands during the observation periods are presented in Table 3. Pedestrian and left-turning vehicle demands are quite high at the Imaike Intersection. Furthermore, according to the observations, all sites have very low left-turning heavy vehicle demands, except the Nishi-osu Intersection where almost 14% of

<table>
<thead>
<tr>
<th>Intersection name</th>
<th>Approach</th>
<th>Survey date</th>
<th>Radius of the corner $R_c$ (m)</th>
<th>Intersection angle $h_i$ (°)</th>
<th>Number of exit lanes $N_e$</th>
<th>Crosswalk setback distance $D_s$ (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Suemori-dori 2</td>
<td>East</td>
<td>11/18/2008 9:00–12:00</td>
<td>9.7</td>
<td>88.3</td>
<td>2</td>
<td>4.8</td>
</tr>
<tr>
<td></td>
<td>West</td>
<td>19</td>
<td>65.4</td>
<td></td>
<td>2</td>
<td>8.3</td>
</tr>
<tr>
<td></td>
<td>North</td>
<td>17</td>
<td>117</td>
<td></td>
<td>3</td>
<td>13.1</td>
</tr>
<tr>
<td>Chikatetsu Horita</td>
<td>East</td>
<td>06/18/2009 9:00–10:30</td>
<td>14</td>
<td>94.1</td>
<td>3</td>
<td>3.5</td>
</tr>
<tr>
<td></td>
<td>South</td>
<td>12</td>
<td>88.3</td>
<td></td>
<td>3</td>
<td>13.7</td>
</tr>
<tr>
<td>Taiko-dori 3</td>
<td>West</td>
<td>10/13/2009 7:30–10:30</td>
<td>17</td>
<td>94.1</td>
<td>3</td>
<td>12.4</td>
</tr>
<tr>
<td></td>
<td>South</td>
<td>17</td>
<td>88.3</td>
<td></td>
<td>2</td>
<td>14.6</td>
</tr>
<tr>
<td>Nishi-osu</td>
<td>West</td>
<td>01/18/2009 9:00–12:30</td>
<td>17</td>
<td>76.9</td>
<td>3</td>
<td>17.8</td>
</tr>
<tr>
<td>Kawana</td>
<td>West</td>
<td>12/01/2008 7:30–10:30</td>
<td>21</td>
<td>106</td>
<td>2</td>
<td>22.0</td>
</tr>
<tr>
<td>Imaike</td>
<td>East</td>
<td>10/15/2005 13:00–15:00</td>
<td>16</td>
<td>91.2</td>
<td>3</td>
<td>18.5</td>
</tr>
</tbody>
</table>

---

<table>
<thead>
<tr>
<th>Intersection name</th>
<th>Approach</th>
<th>Survey date</th>
<th>Number of exit lanes $N_e$</th>
<th>Intersection angle $h_i$ (°)</th>
<th>Distance from center of the intersection$^a$ to the median hard nose (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Entering hard nose</td>
</tr>
<tr>
<td>Suemori-dori 2</td>
<td>West</td>
<td>11/18/2008 9:00–12:00</td>
<td>2</td>
<td>88</td>
<td>15.3$^b$</td>
</tr>
<tr>
<td></td>
<td>North</td>
<td>3</td>
<td>67</td>
<td></td>
<td>23.0</td>
</tr>
<tr>
<td></td>
<td>South</td>
<td>3</td>
<td>93</td>
<td></td>
<td>17.2</td>
</tr>
<tr>
<td>Sunadabashi</td>
<td>West</td>
<td>6/27/2008 7:30–11:00</td>
<td>2</td>
<td>90</td>
<td>19.3</td>
</tr>
<tr>
<td></td>
<td>North</td>
<td>2</td>
<td>91</td>
<td></td>
<td>15.3</td>
</tr>
<tr>
<td>Nishi-osu</td>
<td>North</td>
<td>01/18/2009 9:00–12:30</td>
<td>3</td>
<td>74</td>
<td>31.1</td>
</tr>
<tr>
<td></td>
<td>East</td>
<td>3</td>
<td>106</td>
<td></td>
<td>27.2</td>
</tr>
<tr>
<td>Kawana</td>
<td>North</td>
<td>12/01/2008 7:30–10:30</td>
<td>2</td>
<td>106</td>
<td>35.4</td>
</tr>
<tr>
<td>Sakurayama</td>
<td>South</td>
<td>12/04/2008 7:20–10:20</td>
<td>2</td>
<td>90</td>
<td>22.9</td>
</tr>
<tr>
<td>Atsuta Shrine South</td>
<td>North</td>
<td>12/04/2008 7:00–12:00</td>
<td>3</td>
<td>61</td>
<td>21.1</td>
</tr>
<tr>
<td>Taiko-dori 3</td>
<td>West</td>
<td>10/31/2009 7:30–10:30</td>
<td>3</td>
<td>84</td>
<td>22.6</td>
</tr>
<tr>
<td></td>
<td>North</td>
<td>2</td>
<td>95</td>
<td></td>
<td>17.9</td>
</tr>
</tbody>
</table>

---

$^a$ Center of the intersection is defined as the crossing point of the median extension from the entering and exiting approaches.

$^b$ At the West approach of Suemori-dori 2 Intersection, there is a plastic pole in the zebra marking after the hard nose of the median. Thus, this pole is considered as the hard nose of the median.
the observed left-turning vehicles are heavy vehicles. Regarding right-turners, most are passenger cars at all observation sites.

3.2 Trajectory tracking

Vehicle trajectories, which include the positions and timings, were extracted from video data using the video image processing system TrafficAnalyzer (Suzuki and Nakamura, 2006). The positions of each vehicle were extracted every 0.5 s, and the corresponding video coordinates were converted to global coordinates by projective transformation. The point where the right-rear wheel is touching the ground is the reference observation point for all vehicles. All video observations were conducted from high buildings surrounding the intersections; thus, for all video tapes, the observation angle is large. This allows us to track the right rear wheel of all turning vehicles without issues such as occlusion or high coordinate transformation error. By considering the size of each turning vehicle, the observed path based on the right-rear wheel is transformed to the path that corresponds to the center-front of the vehicle. The transformed paths are smoothened by the Kalman smoothing method.
4. Data analysis on paths of turning vehicles

Several factors might affect the path variations of turning vehicles. According to the observations, the turning angle (i.e., the angle between entering and exit approaches), corner radius, number of exit lanes, vehicle type, and approaching speed are the most significant factors that affect the left-turning vehicle paths, as discussed by Asano et al. (2011). Regarding right-turning vehicles, observations revealed that the turning angle, the distances from the center of the intersection to the entering and to the exit hard noses, the number of exit lanes, and the approaching speed are the most significant parameters that affect the path distribution. The hard nose is defined as the end of the physical median either at the entering approach (entering hard nose) or at the exit approach (exit hard nose) as shown in Fig. 1.

The effects of the turning angles on driver behavior are shown in Fig. 2, in which the paths of the right-turning vehicles at the North approaches of Suemori-dori 2 Intersection and Kawana Intersection are compared. The lateral trajectory density distribution (D.D.) and the lateral cumulative distribution (C.D.) at three cross sections along the turning paths are also presented. The density and cumulative distributions are drawn in an interval of 0.5 m, starting from the curb of intersection corner. The right-turning vehicle paths at cross-sections 2 and 3 are clearly significantly less distributed at the Kawana Intersection as compared to the Suemori-dori 2 Intersection (at a 95% significance level). This can be attributed to the sharp turning angle from the North approach of the Suemori-dori 2 Intersection, which leads to a wider range of vehicle paths. Furthermore, the West approach of the Suemori-dori 2 Intersection has three exit lanes, whereas the West approach of the Kawana Intersection has only two. More exit lanes give drivers a higher degree of freedom and more choices, which is reflected in wider trajectory distributions.

Another possible factor that might affect driver behavior while turning is the presence of pedestrians. However, Asano et al. (2011) found that the difference in the trajectories of the vehicles that faced and did not face pedestrians or cyclists is not statistically significant at a 95% confidence interval.

According to the previous analysis, it is concluded that there are significant variations in the turning vehicle trajectories dependent on several factors. Furthermore, when excluding the effects of the factors discussed above, significant variations still exist in turning vehicle trajectories, which can be attributed to other external factors that affect driver behavior. The available data is not detailed enough to investigate the effect of vehicle type on the paths of right-turning vehicles.

Fig. 2. Paths of right-turning vehicles at the north approaches of Suemori-dori 2 Intersection and Kawana Intersection.
The variations in vehicle paths significantly affect the positions of potential conflicts with other users. For a better understanding of this relationship, the distribution of the potential conflict points between right-turning vehicles and crossing through traffic entering at the beginning of the next phase is illustrated in Fig. 3. The paths of crossing through traffic are assumed to be in the middle of the approach lane, which means that they do not change their direction inside the intersections. Therefore, cross-section 4 in Fig. 2 is assumed to represent the average path of entering through traffic, which means that all potential conflict points will be located along this cross section. Fig. 3 shows the observed distributions of paths at the Kawana and Suemori-dori Intersections. The horizontal axis represents the position along cross-section 4, where the origin is the point where the 50th percentile of the potential conflict points along the axis is measured. Fig. 3 clearly shows that the potential conflict points are widely distributed at both intersections. The Kawana Intersection has a significantly wider distribution than the Suemori-dori Intersection. The Kawana Intersection has a very long distance from the center of the intersection to the entering hard nose, i.e., almost 35 m. Such a long distance encourages drivers to start turning earlier, which leads to a wider distribution of potential conflict points. These observed wide distributions of potential conflict points might impact the occurrence probability of hazardous conditions.

5. Structure of trajectory model

The objective of this study is to model the distribution of turning vehicle paths as a function of intersection geometry, vehicle type, and speed. The model is intended to be incorporated into a microscopic simulation model for safety evaluation to represent the variation of vehicle positions according to intersection geometries. In general, the modeling method can be divided into the two consecutive steps as follows:

1. Individual vehicle path approximation.
2. Modeling the variations in paths.

The path of a turning vehicle is defined as a set of vehicle positions at each moment while turning. Therefore, modeling vehicle paths means representing vehicle positions sequentially as time proceeds. Instead of sequential representation of vehicle positions, this study approximates the entire turning movement as a combination of straight lines, Euler spiral curves, and circular curves, as shown in Fig. 4. The advantage of this approximation method is that the complicated shape of the paths can be expressed by a few parameters: start and end points of the turning path, circular curve radius, and Euler spiral curve parameter.

Vehicle paths vary as discussed in the previous section. Estimating the distribution of paths is also important for safety evaluation. In this study, the distribution is assumed to be dependent on several representative parameters, which are normally distributed.

5.1. Individual vehicle path approximation

5.1.1. Curvature calculation

Vehicle paths are assumed to consist of small portions of curves with different curvatures. $P_i(t)$ is assumed as the position of vehicle $i$ at time $t$. Furthermore, $O_i(t)$ is defined as the crossing point between the perpendicular bisector of positions $P_i(t - At)$ and $P_i(t)$ with that of positions $P_i(t)$ and $P_i(t + At)$, as illustrated in Fig. 4a. The curve radius, $R_i(t)$, at position $P_i(t)$ represents the radius of a circle, where its center is at $O_i(t)$ and passes through positions $P_i(t - At), P_i(t)$, and $P_i(t + At)$. The observation time interval, $At$, is set as 0.5 s. Curvature $\kappa_i(t)$ is the reciprocal of $R_i(t)$ (i.e., $\kappa_i(t) = 1/R_i(t)$). Fig. 4b shows an example of the observed curvatures of left and right-turning vehicle paths at the North approach of the Suemori-dori Intersection.

![Fig. 3. Distribution of conflict points between right-turners from the North approaches of Suemori-dori 2 Intersection and Kawana Intersection and crossing through traffic.](image-url)
The horizontal axis of Fig. 4b is normalized so that the position of the stop line corresponds to zero and the position of the end of path measurement (slightly downstream of the exit crosswalk) corresponds to one. Most of the curvature profiles in Fig. 4b start from zero, and increase until the middle of the curve where it starts decreasing to zero again. This characteristic implies that turning-vehicle paths can be represented by modeling the change in the curvatures.

5.1.2. Trajectory elements

For modeling the change in the curvatures, three types of segments are used: straight lines, circular curves, and Euler spiral curves, as shown in Fig. 5. This section explains the features of these curves and their curvature.

The curvature of any straight line is zero, while circular curves have constant curvature. Euler spiral curves are generally installed in road alignment as transition segments between straight and circular curve segments in order to not force drivers to steer suddenly when entering the circular curve. The basic formula of Euler spiral curves is shown below.

\[ R_s L_s = A^2 \]  

where \( R_s \) is the radius of the spiral curve at the end point (the point where it is connected to the circular curve in this study), \( L_s \) is the distance from the spiral starting point, and \( A \) is the parameter of the Euler spiral curve to be estimated. Eq. (1) can be rewritten as follows:

\[ \frac{1}{R_s} = \frac{1}{A^2} L_s \]  

Eq. (2) shows that the curvature \( 1/R_s \) of an Euler spiral curve is a linear function of \( L_s \), with a gradient of \( 1/A^2 \).
5.1.3. Path approximation

According to the curvature profiles in Fig. 4b, it is reasonable to approximate a curvature profile with a polygonal line, which is a combination of straight, Euler spiral, and circular curve segments. This polygonal line starts from a straight segment and then moves to an Euler curve segment with a linear curvature with a gradient of 1/$A_1^2$. This Euler curve is followed by a circular curve segment with a constant curvature of 1/$R_{min}^2$, then an Euler curve segment that has a linear curvature with a gradient of $-1/A_2^2$, and finally a straight segment. These curves are defined as the entering straight segment, the entering Euler spiral curve segment, the circular curve segment, the exit Euler spiral curve segment, and the exit straight segment, respectively. This polygonal line can be uniquely determined by defining five parameters: $A_1$, $A_2$, $R_{min}$, beginning point (BP) of entering the Euler spiral curve, and end point (EP) of exiting the Euler spiral curve, as shown in Fig. 5. By applying the dynamic programming, these parameters for each vehicle trajectory are estimated by minimizing the error between the polygonal line and the measured curvature profile.

To provide a better insight on the path approximation, Table 4 shows the utilized data in the path model estimation for right-turning vehicles. The average and standard deviation of the estimated parameters $A_1$, $A_2$, and $R_{min}$ are shown in Table 4. The sample size in Table 4 is different from that in Fig. 2, since only leading vehicles were considered and the following vehicles were excluded in the modeling.

To reproduce vehicle paths, the lengths of the Euler spiral and circular curve segments are uniquely calculated from the input variables $A_1$, $A_2$, $R_{min}$, and the angle between entering and exiting approaches, $\theta$ (Japan Road Association, 2001). In order to fix the global position of the curves, the global position of the intersection point (IP) is required. IP is defined as the crossing point between the extended lines of the entering and exiting straight segments as shown in Fig. 5.

5.2. Modeling the variations in paths

In this section, the effects of intersection geometry, vehicle speed, and vehicle type on the distribution of approximated paths are modeled. The explanatory variables are vehicle approaching speed, minimum speed ($V_{min}$), and the elements of intersection geometry, such as intersection angle and corner radius. Output variables are the distribution of $A_1$, $A_2$, and $R_{min}$. The entering and exiting Euler spiral curves’ parameters $A_1$ and $A_2$ are estimated by multiple regression models as shown in Table 5.

For left-turning vehicles, the estimated path parameters $A_1$, $A_2$, and $R_{min}$ are dependent on intersection geometry, as shown in Tables 5 and 6. However, for right-turning vehicles, the effect of intersection geometry was found to be insignificant on the entering and exiting Euler spiral curves’ parameters $A_1$ and $A_2$ except the distance from IP to the entering and exiting median hard nose ($D_{HIN,IN}$ and $D_{HIN,OUT}$), which has significant impact.

The radius of the circular curve, $R_{min}$, is estimated by assuming normal distribution as shown in following equation.

$$R_{min} \sim N(\mu, \sigma)$$

$$\mu = f(x_1, x_2 \sim x_n) = a_{1,1}x_1 + a_{1,2}x_2 + \ldots + a_{1,n}x_n + a_{1,n+1}$$

$$\sigma = f(x_1, x_2 \sim x_n) = a_{2,1}x_1 + a_{2,2}x_2 + \ldots + a_{2,n}x_n + a_{2,n+1}$$

where $\mu$ is the mean of the normal distribution, $\sigma$ is the standard deviation of the normal distribution, $x_1, \ldots, x_n$ are independent variables, and $a_{1,1}, \ldots, a_{1,n}$ and $a_{2,1}, a_{2,n}$ are the model coefficients estimated by the maximum likelihood method. The estimation results for left-turning and right-turning vehicles are shown in Table 6.

---

**Fig. 5.** Trajectory approximation of right-turning and left-turning vehicles using Circular and Euler spiral curves.
Vehicle minimum speed, $V_{min}$ along the whole maneuver for leading vehicles that did not face any pedestrian or cyclist is used as one of the explanatory variables in modeling the parameters for the entering and exiting Euler spiral curves and in modeling the radius of the circular curve, $R_{min}$ as well. $V_{min}$ is modeled by assuming normal distribution as shown in following equation.

$$V_{min} \sim N(\mu, \sigma)$$

$$\mu = \beta_{1,1}X_1 + \beta_{1,2}X_2 \sim \beta_{1,n}X_n + \beta_{1,n+1}$$

$$\sigma = \beta_{2,1}X_1 + \beta_{2,2}X_2 \sim \beta_{2,n}X_n + \beta_{2,n+1}$$

where $\mu$ is the mean of the normal distribution, $\sigma$ is the standard deviation of the normal distribution, $x_1, \ldots, x_n$ are independent variables, and $\beta_{1,1}, \ldots, \beta_{1,n}, \beta_{2,1}$ and $\beta_{2,n}$ are the model coefficients estimated by the maximum likelihood method. The
estimation results of $V_{\text{min}}$ for left-turning and right-turning vehicles are shown in Table 6. $V_{\text{min}}$ of turning vehicles along the turning path is dependent on approaching speed, $V_{\text{in}}$, as shown in Table 6, which is quite rational. Turning vehicles with higher running speeds approaching the intersection will reach higher minimum speeds compared to vehicles that are approaching the intersection with low running speeds.

In reality, vehicle path is not independent from speed. This correlation between vehicle path and speed is empirically proven in this study. Vehicle path is affected by the entering speed of the vehicle, $V_{\text{in}}$, which significantly affects $V_{\text{min}}$ along the turning path. This phenomenon is reflected in the developed empirical models as shown in Tables 5 and 6.

Since the currently available data of right-turning heavy vehicles is not sufficient to reflect the effect of vehicle type, all developed models for right-turning vehicle paths do not consider vehicle type.

### 6. Comparison between estimated and observed paths

Although the estimated parameters are well-fitted, each variable explains only a part of the whole path. For validation, generated distributions are compared with the actual observed distributions.

Fig. 6 compares the estimated and observed path distributions of left-turning passenger cars at the West approach of the Nishi-osu Intersection. The IP is set for path generation by assuming that the entering and exiting straight segments are located at the midpoint of each lane. The observed lane usage ratio of exit lanes is used in the developed path model to generate turning vehicle paths. Monte-Carlo simulation with 1000 trials is conducted to generate these path distributions.

The red-colored numbers of each graph in Fig. 6 correspond to the cross sections defined along vehicle path. The estimated paths clearly fit well, especially the paths before reaching the exit crosswalks. The shape of the estimated distribution at cross-section 3 tends to be stepwise since the exiting straight segments are assumed to be located at the center of traffic lanes. However, in reality, the positions of the exiting straight segments may also vary laterally inside each lane.

Fig. 7 compares the estimated and observed path distributions of right-turning passenger cars from the North approach of the Nishi-osu Intersection. The results are very similar to those of left-turning vehicles. The estimated path distributions at cross-sections 1 and 2 are not significantly different (95% confidence level) from the observed distributions, while at cross-section 3, they are significantly different. This difference is attributed to the assumed exit positions, which are located in the mid-point of each exit lane.

### 7. Sensitivity analysis

The effect of intersection geometry on left-turning vehicle paths is demonstrated by the developed models. Fig. 8a shows estimated path distributions at hypothetical intersections with a corner radius of 15 m. Left-turning vehicle paths are esti-
mated at three different intersection angles: 60°, 90°, and 120°. The origin point of the X axis is assumed to be the entering point of the intersection. Furthermore, all hypothetical intersections are assumed to have one exit lane. The lateral distance between the center of the vehicle and the curb is set as 1.5 m. The results show that the approach with a smaller angle tends to have larger path variations. This can be attributed to the ability of left-turning drivers to maintain their higher speeds at larger angle approaches, while at smaller angles, drivers need to reduce their speed, which reflects directly on the turning

![Figure 6](image1.png)

**Fig. 6.** Comparison between observed and estimated paths of left-turning passenger cars at the West approach of Nishi-osu Intersection.

![Figure 7](image2.png)

**Fig. 7.** Comparison between observed and estimated paths of right-turning vehicles at the North approach of Nishi-osu Intersection.
radius and the whole maneuver. It is rational to conclude that the probability that drivers might change their maneuver (speed and position) becomes smaller if intersection angle is large and vehicle speed is high.

Fig. 8b shows the estimated path distributions at two hypothetical intersections with corner radii of 10 m and 20 m, while intersection angle is the same (90°) for both intersections. The intersection radius clearly and significantly affects the distribution of vehicle paths. Larger corner radii will result in longer turning maneuvers as shown in Fig. 8b.

To illustrate the effect of the angle between entering and exiting approaches, \( \theta_r \), on the variation of right-turning vehicle paths, Fig. 9 is presented. It is assumed that all right-turning vehicles will exit in one lane, which is the median lane. The results conclude that as \( \theta_r \) increases, vehicle paths become less distributed. These results are rational since the possibility that drivers can maintain their speed increases as \( \theta_r \) increases, which means that the effect will lessen. This is in accordance with the results of left-turning vehicles.

Fig. 8. Sensitivity of left-turning vehicle paths to intersection geometric parameters.
8. Conclusions

In this study, the trajectory distribution of turning vehicles is analyzed and modeled as a function of intersection geometry, vehicle type, and speed. According to the empirical data analysis, the path distribution of left-turning vehicles is determined to be mainly dependent on the intersection angle, corner radius, number of exit lanes, vehicle type, and approaching speed. The path distribution of right-turning vehicles is determined to be dependent on the angle between entering and exiting approaches, the positions of the entering and the exiting hard noses of the median (see Fig. 1), number of exit lanes, and vehicle approaching speed. Furthermore, this study proved that the variations in vehicle paths significantly affect the distribution of potential conflict points, which implies that the path distribution may strongly impact the occurrence probability of severe conflicts.

A unique model that can reflect the variations in turning vehicle paths is proposed. A comparison between the estimated and the observed paths shows that the developed model reasonably represents the effect of intersection geometry not only on average vehicle trajectories but also on their distribution. This quantitative representation of detailed microscopic behavior can be used to predict the changes in vehicle maneuver as a result of improvements in the intersection layout. Furthermore, the proposed model combined with other behavioral models can be utilized to stochastically evaluate intersection safety.

However, the proposed trajectory model still has several limitations. The exit positions are assumed as input to the trajectory model. Since exit positions are widely distributed, as shown in Fig. 2, incorporating a lane choice model in the proposed method is necessary to provide a complete procedure to reproduce the spatial maneuver of turning vehicles. Furthermore, the number of turning lanes and the turning lane width are not considered in the developed method since the available data does not include various sites with different numbers of turning lanes and widths.

The developed turning vehicle path model is planned to be used in a microscopic simulation tool for the safety evaluation of signalized intersections. However, a trajectory model will not be sufficient; thus, a probabilistic speed profile model, which considers the yielding behavior as a result of intersection geometry and interaction with other users, is necessary to provide a successful presentation of the left-turning vehicle behavior in a microscopic simulation environment.

Acknowledgements

The authors are very grateful to the Takata Foundation and the Japan Society for the Promotion of Science (JSPS) for supporting this research.

References


