AN-NAJAH NATIONAL UNIVERSITY
ENGINEERING FACULTY
MECHANICAL & MECHATRONICS ENGINEERING DEPARTMENT

Control Systems (1) (67471)
First Exam

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Student Name: ...

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Total Exam Mark: 100
Exam Weight: 20

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Student Grade

Exam Notes:

1. Solve all the problems.
2. Closed books and notes.
3. Read each problem carefully before attempting to solve it.
4. Write all work on this exam paper.

Good Luck
Q1: (ILO 1) 20 pts

The output \( y \), and the input \( x \) of a device are related by \( y = x + 0.79x^3 \). Obtain a linearized model about the operating point \( x_0 = 2 \).

\[
\frac{dy}{dx} = \left. \frac{dy}{dx} \right|_{y=x_0} \ dx
\]

\[
\frac{dy}{dx} = 1 + 3(0.79)x^2 = 1 + 2.37x^2
\]

\[
\left. \frac{dy}{dx} \right|_{x=2} = 1 + 2.37(2)^2 = 10.48
\]

\[
\Rightarrow \Delta y = 10.48 \Delta x
\]

\[
y_0 = 2 + 0.79(2)^2
\]

\[
= 8.32
\]
The transfer function $G(s)$ of a system is given by: \[
\frac{Y(s)}{R(s)} = \frac{10(s+2)}{s^2 + 8s + 15}
\]

a) Find the zeros and poles of the system $G(s)$ and sketch them on the $s$-plane.

Zeros: $s = -2 \quad (s+2 = 0)$

Poles: $s^2 + 8s + 15 = (s+3)(s+5) = 0 \quad s = -3, -5$

b) Determine the natural frequency and damping ratio of the system.

\[
\omega_n = \sqrt{8} \quad 2\zeta \omega_n = 8 \Rightarrow \zeta = \frac{\frac{\omega_n}{\sqrt{8}}}{2\zeta} = 1.03
\]

c) Determine $y(t)$ when $r(t)$ is a unit step input.

\[
Y(s) = \frac{10(s+2)}{s^2 + 8s + 15} \cdot \frac{1}{s} = \frac{10(s+2)}{(s+3)(s+5)s} = \frac{k_1}{s+3} + \frac{k_2}{s+5} + \frac{k_3}{s}
\]

\[
k_1 = \frac{10(s+2)}{(s+3)s} \bigg|_{s = -3} = \frac{10(-1+2)}{(-3+3)(-3)} = -\frac{10}{-6} = \frac{5}{3} \approx 1.667
\]

\[
k_2 = \frac{10(s+2)}{(s+5)s} \bigg|_{s = -5} = \frac{10(-5+2)}{(-5+5)(-5)} = -\frac{10}{10} = -1
\]

\[
k_3 = \frac{10(s+2)}{(s+3)(s+5)} \bigg|_{s = 0} = \frac{10(2)}{3(5)} = \frac{20}{15} = \frac{4}{3}
\]

\[
y(t) = \frac{5}{3} e^{-3t} + 3e^{-5t} + \frac{4}{3}
\]

\[
y_{ss} = \lim_{t \to \infty} y(t) = \frac{4}{3}
\]

d) Determine the steady state response to a unit step input.
Q3: (ILOs 2)  

A system is shown in Fig Q#3-a

\[ \frac{1}{s+5} + \frac{1}{s+10} \]

Fig: Q#3-a

a) Determine \( G(s) \) and \( H(s) \) of the block diagram shown in Fig Q#3-b that are equivalent to the block diagram of Fig Q#3-a.

\[ G(s) = \frac{1}{(s+5)(s+10)} \]

\[ H(s) = s+10 \]

\[ G(s) = \frac{1}{s^2+16s+55} \]

b) Using the obtained results from part (a), determine \( T(s) = Y(s)/R(s) \) for Fig Q#3-b.

\[ T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{1}{(s+5)(s+10) + (1)(s+10)} \]

\[ = \frac{1}{s^2+16s+55 + s+10} = \frac{1}{s^2+17s+65} \]
Q4: (ILO 2)

Figure Q4 presents a signal flow graph model of an open loop dc motor control with velocity as the output. Determine the transfer function of the given dc motor.

![Signal Flow Graph]

3 loops -

\[ l_1 = -\frac{5}{5} \quad \frac{1}{5} \quad l_2 = -\frac{2}{5} \quad \frac{1}{5} \quad l_3 = -\frac{3}{5} \]

2 poles -

\[ p_1 = (1) \left( \frac{1}{5} \right) (5) (1) \left( \frac{1}{5} \right) 6 \left( \frac{1}{5} \right) (1) = \frac{30}{5^3} \]

\[ p_2 = (1) (5) (1) \left( \frac{1}{5} \right) (6) \left( \frac{1}{5} \right) (1) = \frac{30}{5^3} \]

\[ D = 1 - (l_1 + l_2 + l_3) + (l_1 l_2 + l_1 l_3 + l_2 l_3) = \left( l_1 l_2 l_3 \right) \]

\[ = 1 - \left( -\frac{5}{5} - \frac{2}{5} - \frac{3}{5} \right) + \left( \frac{5}{5} \right) \left( \frac{2}{5} \right) + \left( \frac{5}{5} \right) \left( \frac{3}{5} \right) + \left( \frac{2}{5} \right) \left( \frac{3}{5} \right) \]

\[ = 1 + \frac{10}{5} + \frac{210}{5^2} + \frac{30}{5^3} \]

D_1 = 1 \quad D_2 = 1

\[ T = \left( \frac{30}{5^3} \right) (1) + \left( \frac{30}{5^3} \right) (1) \times \frac{5^3}{5^3} = \frac{30 + 30s}{s^3 + 10s^2 + 31s + 30} \]