Obtain the transfer function $G(s) = Y(s)/U(s)$ and determine the response of the system to a unit step input.

E3.22 Consider the system in state variable form

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + \mathbf{D}u$$

with

$$\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 3 & 4 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \mathbf{C} = [1 \quad 0], \text{ and } \mathbf{D} = [0].$$

(a) Compute the transfer function $G(s) = Y(s)/U(s)$. (b) Determine the poles and zeros of the system. (c) If possible, represent the system as a first-order system

$$\dot{x} = ax + bu$$
$$y = cx + du$$

where a, b, c, and d are scalars such that the transfer function is the same as obtained in (a).

E3.23 Consider a system modeled via the third-order differential equation

$$\dddot{x}(t) + 3\ddot{x}(t) + 3\dot{x}(t) + x(t) = \dddot{u}(t) + 2\ddot{u}(t) + 4\dot{u}(t) + (t).$$

Develop a state variable representation and obtain a block diagram of the system assuming the output is $x(t)$ and the input is $u(t)$.

PROBLEMS

P3.1 An *RLC* circuit is shown in Figure P3.1. (a) Identify a suitable set of state variables. (b) Obtain the set of first-order differential equations in terms of the state variables. (c) Write the state differential equation.

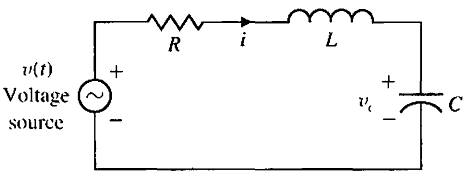

FIGURE P3.1 *RLC* circuit.

P3.2 A **balanced** bridge network is shown in Figure P3.2. (a) Show that the **A** and **B** matrices for this circuit are

$$\mathbf{A} = \begin{bmatrix} -2/((R_1 + R_2)C) & 0 \\ 0 & -2R_1R_2/((R_1 + R_2)L) \end{bmatrix},$$

$$\mathbf{B} = 1/(R_1 + R_2)\begin{bmatrix} 1/C & 1/C \\ R_2/L & -R_2/L \end{bmatrix}.$$

(b) Sketch the block diagram. The state variables are $(x_1, x_2) = (v_c, i_L)$.

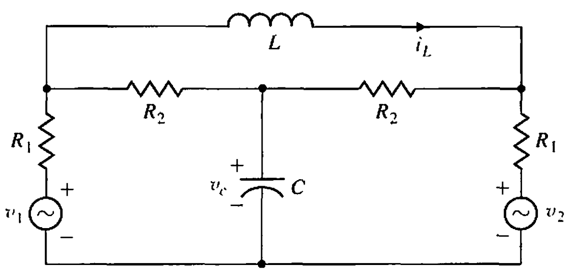

FIGURE P3.2 Balanced bridge network.

P3.3 An *RLC* network is shown in Figure P3.3. Define the state variables as $x_1 = i_L$ and $x_2 = v_c$. Obtain the state differential equation.

Partial answer:

$$\mathbf{A} = \begin{bmatrix} 0 & 1/L \\ -1/C & -1/(RC) \end{bmatrix}.$$

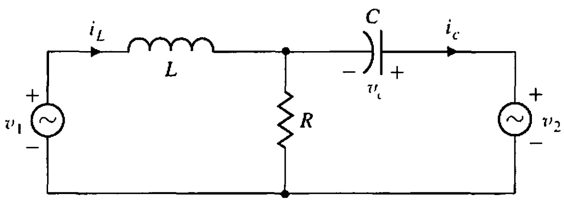

FIGURE P3.3 *RLC* circuit.

P3.4 The transfer function of a system is

$$T(s) = \frac{Y(s)}{R(s)} = \frac{s^2 + 2s + 10}{s^3 + 4s^2 + 6s + 10}.$$

Sketch the block diagram and obtain a state variable model.

P3.5 A closed-loop control system is shown in Figure P3.5. (a) Determine the closed-loop transfer function $T(s) = Y(s)/R(s)$. (b) Sketch a block diagram model for the system and determine a state variable model.

P3.6 Determine the state variable matrix equation for the circuit shown in Figure P3.6. Let $x_1 = v_1$, $x_2 = v_2$, and $x_3 = i$.

P3.7 An automatic depth-control system for a robot submarine is shown in Figure P3.7. The depth is measured

$K_1 = 0.5$. (a) Determine the closed-loop transfer function

$$T(s) = \frac{\omega(s)}{R(s)}.$$

(b) Determine a state variable representation. (c) Determine the characteristic equation obtained from the **A** matrix.

P3.10 Many control systems must operate in two dimensions, for example, the x- and the y-axes. A two-axis control system is shown in Figure P3.10, where a set of state variables is identified. The gain of each axis is K_1 and K_2, respectively. (a) Obtain the state differential equation. (b) Find the characteristic equation from the **A** matrix. (c) Determine the state transition matrix for $K_1 = 1$ and $K_2 = 2$.

P3.11 A system is described by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & -2 \\ 2 & -3 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

and $x_1(0) = x_2(0) = 10$. Determine $x_1(t)$ and $x_2(t)$.

P3.12 A system is described by its transfer function

$$\frac{Y(s)}{R(s)} = T(s) = \frac{8(s+5)}{s^3 + 12s^2 + 44s + 48}.$$

(a) Determine a state variable model.
(b) Determine $\Phi(t)$, the state transition matrix.

P3.13 Consider again the *RLC* circuit of Problem P3.1 when $R = 2.5$, $L = 1/4$, and $C = 1/6$. (a) Determine whether the system is stable by finding the characteristic equation with the aid of the **A** matrix. (b) Determine the transition matrix of the network. (c) When the initial inductor current is 0.1 amp, $v_c(0) = 0$, and $v(t) = 0$, determine the response of the system. (d) Repeat part (c) when the initial conditions are zero and $v(t) = E$, for $t > 0$, where E is a constant.

P3.14 Determine a state variable representation for a system with the transfer function

$$\frac{Y(s)}{R(s)} = T(s) = \frac{s + 50}{s^4 + 12s^3 + 10s^2 + 34s + 50}.$$

P3.15 Obtain a block diagram and a state variable representation of this system.

$$\frac{Y(s)}{R(s)} = T(s) = \frac{14(s+4)}{s^3 + 10s^2 + 31s + 16}.$$

P3.16 The dynamics of a controlled submarine are significantly different from those of an aircraft, missile, or surface ship. This difference results primarily from the moment in the vertical plane due to the buoyancy effect. Therefore, it is interesting to consider the control

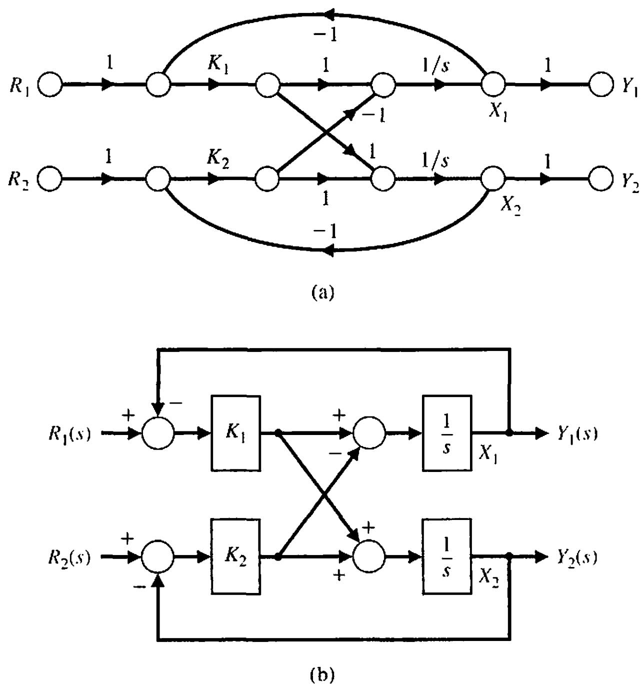

FIGURE P3.10 Two-axis system. (a) Signal-flow graph. (b) Block diagram model.