An-Najah National University
Faculty of Engineering
Mechanical Engineering Department
Control Systems I (67471)
Second Exam

Instructor: Dr. Nidal Farhat
Credit Hours: 3
Date: Thursday, April 09, 2015
Exam Duration: 50 min

Student Name: .....................
Registration Number: ..............
Total Exam Mark: 100
Exam Weight: 20

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Student Grade

Exam Notes:

1. Solve all the problems.
2. Closed books and notes.
3. Read each problem carefully before attempting to solve it.
4. Write all work on this exam paper.

Instructors:

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<th>Dr. Bashir Nouri</th>
<th>Dr. Nidal Farhat</th>
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Question 1: (55 points)
For the following transfer function, and using diagonal canonical form, determine:
1) The signal flow graph model

\[ T(s) = \frac{Y(s)}{R(s)} = \frac{4s + 12}{s^2 + 8s + 12} \]

\[ s_{1,2} = -4 \pm \sqrt{8} \]

\[ \Phi_{21}(s) = \frac{X_2(s)}{X_1(s)} = \frac{4s + 12}{(s+2)^2} \]

2) The state differential equation in matrix form.

\[ \begin{align*}
\dot{x}_1 &= -2x_1 + r(t) \\
\dot{x}_2 &= -6x_2 + r(t)
\end{align*} \]

\[ \mathbf{x} = \begin{bmatrix} -2 & 0 \\ 0 & -6 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r(t) \]

3) \( \Phi_{21}(s) \) from the signal flow graph

\[ \Phi_{21}(s) = \frac{X_2(s)}{X_1(s)} = \frac{4s + 12}{(s+2)^2} \]

\[ \text{No path from } X_1(0) \text{ to } X_2(s) \Rightarrow P = 0 \]

\[ \Rightarrow \Phi_{21}(s) = 0 \text{ (Diagonal Canonical form)} \]

\[ \Rightarrow \Phi_{21}(t) = 0 \]
Question 2: (45 points)

For a system represented by the following block diagram model, determine:

1) Determine the sensitivity of the system to the change of the parameter $K$

$$ S_K = \frac{\partial T}{\partial K} $$

$$ T = \frac{K+11s}{s(s+1)+K+11s} = \frac{K+11s}{s+12s+s+K} $$

$$ \frac{\partial T}{\partial K} = \frac{(S^2+12s+K) - (K+11s)}{(S^2+12s+K)^2} = \frac{S^2+5}{(S^2+12s+K)^2} \cdot \frac{K}{K+11s} $$

$$ S_K = \frac{S^2+5}{(K+11s)^2} \cdot \frac{K}{K+11s} $$

2) The steady state error of the system to a unit step input.

$$ E_a(s) = R(s) - Y(s) = \frac{1}{1 + G(s)} R(s) $$

$$ E_a(s) = \frac{1}{1 + \frac{K+11s}{s(s+1)}} R(s) = \frac{S(s+1)}{S(s+1)+K+11s} \cdot \frac{1}{s} = \frac{s+1}{s^2+12s+k} $$

$$ \lim_{s \to 0} s E_a(s) = \lim_{s \to 0} s \left( \frac{s+1}{s^2+12s+k} \right) = 0 $$

3) The effect of unit step disturbance on the output response of the system.

$$ Y(s) = \frac{1}{s(s+1)+(K+11s)} T_d(s) $$

$$ T_d(s) = \frac{1}{s^2+12s+k} $$

$$ \lim_{s \to 0} s Y(s) = \lim_{s \to 0} s \left( \frac{1}{s^2+12s+k} \right) = \frac{1}{k} $$

$$ Y(\infty) = K $$

Depending on the value of $K$:

- $0 \leq K \leq 36$ = overdamped
- $36 < K$ = underdamped