On the Estimation of Population Size of West Bank and Gaza Strip

By

Dr. Mohammed Ass'ad and Adnan A.Solqan
An-Najah National University
Nablus - Palestine

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Dr. Mohammed Ass'ad and Adnan A.Solqan
An-Najah N. University,
P.O. Box 7, Nablus, West Bank, Palestine.
e-mail : stsolqan@najah.edu.

Abstract

A method for estimating the population size of the Palestinians in West Bank (WB) and Gaza Strip (GS) is proposed. Given the fact that the only comprehensive census in these two areas was made by the Israeli Occupation Authorities in 1967, our method is based on a *mathematical model* and on some reliable figures such as the numbers of students in schools taken from these two areas. Sensitivity is studied, in particular elasticity between the estimated size of the considered population and its growth rate, has been discussed. Finally, Several comparisons were made between our estimates and those made by four main Israeli sources. Three levels estimates were calculated, namely the minimum, medium and maximum of population size in WB and GS. These three levels are estimated up to the year 2010 . Thus our proposed model can serve as a flexible substitute for an absent census or/and as a concrete basis for any future census.

***Key words**: Mathematical Model , Palestinian population, Annual Growth Rate, CBS estimates, Stochastic Model, Sensitivity* .

1.0 Introduction :

The statistical study of human population is called Demography. Vital statistics are the "statistics of life" such as the numbers of birth, marriages, deaths, emigration and immigration. The primary results of demographic studies are statistical tables classifying a community by variables such as age, sex, material condition, health, schools and geographical distribution. Secondary statistics such as birth rate, death rate and increasing rate of population size are also calculated .

Demographic studies are of great importance to political, economical and scientist planners at both national and local levels. They are of commercial value to marketing organizations, as well .

From the information provided by the population census and vital registration, many statistics; called "Demographic rates " can be calculated: Birth rates, Death rates and others .

We have also discussed G*eometric Progression* to give an estimate to the population size in GS, where we have noticed that no great difference between this estimation and that given as the minimum level of this population size.

As far as West Bank (WB) and Gaza Strip (GS) are concerned, it is widely known, now (i.e up to the date of our study) that there is no Arab specialized Statistical Center there. Consequently it is very difficult, so far , to find pure Arab or an independently conducted statistics or information, in the course of conducting a demographic study .

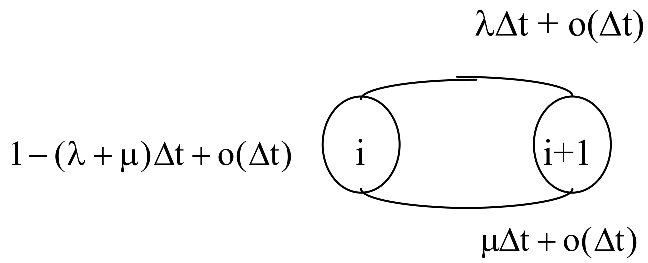
With some exception of those separate individuals who managed to collect some partial data in the fields of demography, education, agriculture, industry, labor, vocational training and health in WB and GS, we could not find an integrated statistics there .

Since the only statistics available here are those issued by several Israeli institutions (most of which are governmental), it should be noted from the outset that the information explored in this study is based to some extent on figures taken from Israeli resources. Finally, we conclude our attempt by analyzing expectation and predication for the future .

2.0 Mathematical Model :

Consider a population process $\{W_t ,t \in [0,\infty]\}$ where W_t is the size of the population at time t. Let λ be the birth rate, and μ death and emigration rate.

People in the population behave independently of one another . Let (Δt) be length of a short time interval, then $\lambda\Delta t + o(\Delta t)$ will be the probability that a member will give a birth to another member, and $\mu\Delta t+o(\Delta t)$ will be the probability that a member will die or emigrate .
$1 - (\lambda + \mu)\Delta t + o(\Delta t)$ be the probability that a member has not born, die or emigrate. Let α be rate of immigration joining the population, then
$\alpha\Delta t + o(\Delta t)$ will be the probability that that an immigrant will join the population . Any other possible changes must have probability $o(\Delta t)$.
If we consider the state of population process be characterized by the size of population, then the *transition diagram* of the process could be given by :



Let W_{to} be the initial size population and equal to i members .
Consider the process in the time interval $(t , t + \Delta t)$, then
$W_{t+\Delta t} = W_t + X_1 + X_2 + \dots + X_{to} + Z$,
where :

$$Z = \begin{cases} 1 & \text{if there is an immigrant joining the population in } (t, t+\Delta t) \\ & \text{with probablity } \alpha\Delta t + o(\Delta t) \\ 0 & \text{otherwise with probability } 1 - \alpha\Delta t + o(\Delta t) \end{cases}$$

$$X_1 = \begin{cases} 1 & \text{if the } (1\,th) \text{ member gives a birth in } (t, t+\Delta t) \text{ with probability } 1\Delta t + o\Delta t \\ -1 & \text{if the } (1\,th) \text{ member dies or emigrates in } (t, t+\Delta t) \text{ with probability } \mu\Delta t + o(\Delta t) \\ 0 & \text{otherwise, i.e with probability } 1 - (\lambda + \mu)\Delta t + o\Delta t \end{cases}$$

Thus it can be shown that

$$\Pr[W_{t+\Delta t} = j \mid W_t = i] = \Pr[X_1 + X_2 + \ldots + X_i + Z = j - i]$$

$$= \begin{cases} (i\lambda + \alpha)\Delta t + o(\Delta t) & j = i + 1 \\ 1 - (i\lambda + i\mu + \alpha)\Delta t + o(\Delta t) & j = i \\ i\mu\Delta t + o(\Delta t) & j = i - 1 \\ o(\Delta t) & \text{otherwise} \end{cases}$$

(see, Coleman, 1974).

Define $P_{ij}(t)$: to be the probability that the system would be in state j when it was in state i before t time units, and $P(t) = (P_{ij}(t))$ to be the *transition matrix.*

Then from the above we get the *transition matrix* P(t) :

P(t) =

	0	1	2	3	4	5	...
0	1-(o+o+α)1+o	(oλ+α)1+o	0	0	0	0	...
1	1μ.1+o	1-(λ+μ+α)1+o	(λ+α)1+o	0	0	0	...
2	0	2μ.1+o	1-(2λ+2μ+α)	2λ+α	0	0	...
3	0	0	3μ	1-(3λ+3μ+α)	3λ+α	0	...
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

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	0	1	2	3	4	5	...
0	1-α	α	0	0	0	0	...
1	μ	1-(λ+μ+α)	λ+α	0	0	0	...
2	0	2μ	1-(2λ+2μ+α)	2λ+α	0	0	...
3	0	0	3μ	1-(3λ+3μ+α)	3λ+α	0	...
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

The *Kolmogorov* differential equations relate P(t) to Q (Cinlar, 1975), where Q is the rate matrix which is equal to:

Q =

	0	1	2	3	4	5	...
0	$-\alpha$	α	0	0	0	0	...
1	μ	$-\mu-\lambda-\alpha$	$\lambda+\alpha$	0	0	0	...
2	0	2μ	$-2\lambda-2\mu-\alpha$	$2\lambda+\alpha$	0	0	...
3	0	0	3μ	$-3\mu-3\lambda-\alpha$	$3\lambda+\alpha$	0	...
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.

The *forward equations* are given by:

$P'_{ij}(t) = \frac{d}{dt} P_{ij}(t)$, then the following are true; where $W_{to} = i$

$P_{io}^{\copyright} = -\alpha P_{io} + \mu P_{il}$

$P'_{ij} = [(j-1)\lambda+\alpha]P_{i,j-1} - (j(\lambda+\mu)+\alpha)P_{ij} + (j+1)\mu P_{i,\,j+1}$, $j = 1,2,......$ ...(1)

(Coleman, 1974)

$P_{ij} = 0 \,, \ \forall j < 0$

multiply (1) by S^j to get the *z-transformation* of these functions, and define π to be

$$\pi_t(S) = \sum_{j=-\infty}^{\infty} p_{ij}(t)s^j,$$ *the probability generation function.*

To get the moment generating function (m.g.f) of π_t (s) we find, and obtain :

$$\frac{\partial \pi_t(S)}{\partial t} + (\lambda S - \mu)(1-S)\frac{\partial \pi_t(S)}{\partial S} = \alpha(S-1)\pi_t(S) \qquad ...(2)$$

Solve this partial differential equation; to get :

$$\frac{\lambda S - \mu}{1-S} e^{-(\lambda-\mu)t} = \text{constant}, \ \lambda \neq \mu \qquad ...(3)$$

and

$$\frac{\alpha}{\lambda}\left[\frac{\lambda dS}{\lambda S - \mu}\right] = -\frac{d\pi_t(S)}{\pi_t(S)}$$

therefore :

$$(\lambda S - \mu)^{\alpha/\lambda} \pi_t(S) = \text{constant} \qquad ...(4)$$

From (3), (4) and the fact that (2) is a first order partial differential equation; it can have only one arbitrary constant, we conclude that :

$$(\lambda S - \mu)^{\alpha/\lambda} \pi_t(S) = f\left\{\frac{\lambda S - \mu}{1-S} e^{-(\lambda-\mu)t}\right\} \qquad ...(5)$$

Using the initial condition $W_{to} = i$, i.e. $\pi_o(S) = S^i$, we obtain

$$(\lambda S - \mu)^{\alpha/\lambda}(S^i) = f\left\{\frac{\lambda S - \mu}{1-S}\right\} \qquad ...(6)$$

Differentiating equation (2) w.r.t .S, we get:

$$\frac{\partial^2 \pi_t(S)}{\partial S \partial t} + (\lambda S - \mu)(1-S)\frac{\partial^2 \pi_t(S)}{\partial S^2}\{(\lambda S - \mu)(-1) + \lambda(1-S) - \alpha(S-1)\}\frac{\partial \pi_t(S)}{\partial S} = \alpha \pi_t(S)$$

But $\pi_t(1) = 1$, $\left.\frac{\partial \pi_t(S)}{\partial S}\right|_{S=1} = M_t = E(W_t)$

and $\left.\frac{\partial^2 \pi_t(S)}{\partial S \partial t}\right|_{S=1} = \left.\frac{\partial}{\partial t}\left(\frac{\partial \pi_t}{\partial S}\right)\right|_{S=1} M_t^{'}$

let $S \to 1$ in (7), we get a first order linear differential equation :

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$$\frac{dM_t}{dt} + (\mu - \lambda)M_t = \alpha \qquad ...(8)$$

where

$$M_t = E(W_t) = \left.\frac{\partial \pi_t}{\partial S}\right|_{S=1}$$ is the *expected value* of the population process .

Using the fact that $M_0 = E(W_{t_0}) = i$ in (8) we get

$$M_t = E(W_t) = \begin{cases} \frac{\alpha}{\mu-\lambda} + (i - \frac{\alpha}{\mu-\lambda})^{e^{-(\mu-\lambda)t}} & \text{if } \lambda \neq \mu \\ \alpha t + i & \text{if } \lambda = \mu \end{cases} \qquad ....(9)$$

Let $\lambda - \mu = \beta$, where β is the *annual rate* of growth in the population size, then (9), becomes

$$E(W_t) = \begin{cases} (i + \frac{\alpha}{\beta})e^{\beta t} - \frac{\alpha}{\beta} & \text{if } \lambda \neq \mu \\ \alpha t + i & \text{if } \lambda = \mu \end{cases} \qquad ....(10)$$

Equation (10) can be approximated if we consider the approximate case,

$$\alpha \to 0, \text{ then } \lim E(W_t) = \begin{cases} ie^{\beta t} & \text{if } \lambda \neq \mu \\ i & \text{if } \lambda = \mu, \text{ equilibrium state} \end{cases} \qquad .....(11)$$

$E(W_t) = ie^{\beta t}$ exhibiting *exponential growth* or *decay* according as $\lambda > \mu$ or $\lambda < \mu$, where i is the size of population at time t = 0.

It is interesting to note that $\lim_{\substack{t \to \infty \\ \lambda \geq \mu}} M_t \to \infty$ where ,

$$\lim_{\substack{t \to \infty \\ \lambda < \mu}} M_t \to \frac{\alpha}{\mu - \lambda} = -\frac{\alpha}{\beta} > 0$$

These results suggest that in the second case, where $\lambda < \mu$, the population size stabilizes in the long run in some form of *statistical equilibrium* .

3.0 Sensitivity :

In order to get an idea about the effect of change of β and t on the size of population we can determine the first partial derivative of $E(W_t)$ with respect to β and to t, using equation (11).

$\frac{\partial E(W_t)}{\partial t} = i\beta e^{\beta t}$, which yields that $E(W_t)$ varies exponentially with respect to the time .

$\frac{\partial E(W_t)}{\partial \beta} = i\beta e^{\beta t} = ite^{\beta t}$, again , $E(W_t)$ varies exponentially with respect to the annual growth rate β.

We know that the *elasticity* " ∈ " is a measure of population of relative change between two quantities, which is defined mathematically as

$$\in_{f,x} = \frac{\frac{\partial f}{f}}{\frac{\partial x}{x}} = \frac{\partial \ln f}{\partial \ln x} ,$$ (Bartman, 1989)

Using this definition of elasticity and Equation (11), we get

$$\in_{E(W_t),t} = \beta t \qquad ....(12)$$

$$\in_{E(W_t),\beta} = \beta t \qquad ....(13)$$

From (12), we conclude that if the value of β increased by r%, then the size of population will increase by rβt%. Same conclusion can be reached from (13).

Equations (12) and (13) imply that the elasticity over β and over t is linearly dependent on β and on t respectively.

4.0 Available figures :

It is very important to notice that no census has been taken since 1967 in West Bank and Gaza.Strip (up to the date of this study) The data on the size of population in this region is based on estimates and *Statistical Models* done by four sources :

1. The Central Bureau of Statistics (CBS) .
2. Interior Ministry (MOI).
3. Statistics office in Military Government (MG).
4. The West Bank Data Project (WBDP).

CBS estimates represent permanent population at the end of the calendar year where births are added and deaths are estimated by using parallel figures from within Israel and neighbouring countries.

MOI figures are based on population registration (identity cards) data. MOI figures represent residents who are temporarily or permanently living abroad (temporary population).

CBS figures for December 31,1987 shows that the number of West Bank population was 858,000 while MOI figures for November 1987 was 1,252,000 (Benvenisti,1988). The difference attributed to :

1. CBS figures represent "present" population while MOI figures represent permanent and "temporary population ".
2. The under-reporting of deaths and permanent emigration.

There were 586,000 resident in West Bank and 380,000 in Gaza, at the end of 1967 according to CBS figures. And at the end of 1987, the figures were 858,000 and 564,000 for the West Bank and Gaza strip, respectively. Thus the increase was about 270,000 or about 46% in 20 years in the West Bank, which shows an approximate *average annual growth rate* 2.3%. And the increase in GS was about 183,000 or about 4.8% in 20 years, which shows an approximate *average annual groth rate* 2.4%.

WBDP figures shows that the size of permanent population in the West Bank, at the end of year 1987 was 1,067,000 where the present population was 977,000 . It seems reasonable to assume that the actual figures for permanent and present residents of West Bank are higher than CBS estimates and lower than MOI figures.

Finally, the figures published by the staff officer for statistics in the Military Government (MG) in 1980, for the West Bank's population was 750,000 . (Benvenisti,1988).

So it is clear upon viewing and examining the Israeli statistical estimates of the Israeli four centres, that there are great deal of differences in figures and results estimated by them for the population number of the WB and GS. Hence we have chosen an official exact figure concerning the numbers of students who sat for the General Secondary School Examination, in the WB (GSSE, Tawjihi), 1982 - 1992 as a reference . These statistics can been assumed to reflect in one way or another (even partially) the annual growth rate of population of WB and GS.

In the following table we exhibit, in its first part, the resident population figures of the CBS for the WB and GS, n1 and n3 respectively, and the figures of GSSE students, n2, in the second part of the same table.

We demonstrate the calculated *percentage annual growth* (PAG) according to those figures i.e. N_1 , N_2 and N_3 . λ^1_t% , λ^2_t% and λ^s_t% denote the PAG for the WB , GS and GSSE figures, respectively .

Table(1).
Population Estimates and Source of its Growth
for WB and GS (Different years).

Population at end of period for the WB and GS & figures of GSSE				Percentage annual growth .		
Year	n1	n2	n3	λ^1_t%	λ^s_t%	λ^2_t%
1967	585900		380800			
1968	583100		356800	-0.5		-6.3
1969	597900		363900	2.5		2.0
1970	607800		370000	1.7		1.7
1971	622600		378800	2.4		2.4
1972	633500		387000	1.8		2.2
1973	652400		401500	3.0		3.7
1974	669700		414000	2.7		3.1
1975	675200		425500	0.8		2.8
1976	683300		437400	1.2		2.8
1977	695700		450800	1.8		3.1
1978	708000		463000	1.8		2.7
1979	718600		444700	1.5		2.5
1980	724300		456500	0.8		2.7
1981	731800		468900	1.0		2.7
1982	749300	14007	477300	2.4		1.8
1983	771800	13873	444500	3.0	-0.96	3.6
1984	793400	13608	509900	2.8	-1.9	3.1
1985	815500	12658	527000	2.8	-7.0	3.4
1986	837700	12843	545000	2.7	1.86	3.4
1987	860000	13457	564100	2.7	4.37	2.7
1988		13125			-2.48	
1989		14197			8.17	
1990		14678			3.37	
1991		16023			3.2	
1992		16885			5.38	

Sources : (1) CBS, The Statistical Abstract of Israel, CBS , 1992.
(2) Lajnat Al Emtehanat Al'amah Fe Addefah Al Gharbeyah, (The Committee of General Exams in West Bank), Tulkarem , West Bank, 1992.

We shall consider that the average annual growth rate for the size of population in WB to be equal to 3.5% and for GS to be 3.8%. This can be interpreted according to some studies done by individual statisticians and demographians in WB Universities, (Ass'ad, 1992). According to the Israeli CBS, the population growth rate estimate β in WB is equal to 2.34% and in Gaza β=2.41%

If we looked at this table, we'll see that the *percentage annual growth rate* of the table of the students of GSSE during the years of Intifada (1987 - 1992) has largely increased, compared with the years before Intifada. This can be interpreted due to Labor market fluctuations and due to working attitudes among youths. For it is founded through the last few years before Intifada that a relatively large number of students of ages between 14 and 19 has preferred to go to work inside the Green Line motivated by economical reasons and lack of job opportunities after accomplishing university. But the situation has become different after the first tow years of Intifada . Labor inside the Green Line in general has become almost impossible. Consequently, students of the same ages have chosen to continue their study. In fact students of this age were confused during the first tow years of Intifada , where no obvious plans for their educational, economical or social future. In certain studies like (Idris,1987) it was found that the rates of drop out among male students in the three existing learning stages (Elementary, Preparatory & Secondary) in the districts of Tulkarm, Ramallah and Hebron were; 0.092, 0.024, 0.059, respectively .

There were 14,007 students in the final secondary stage in the WB at the end of 1982, and 13,125 at the end of 1988; which means that there was decrease of 882 students or -6.3 percent in 6 years. Table (1) shows considerable changes in the annual growth rate. In the interval 1983 - 1985 the annual growth rate stood at -3.3 percent, in the interval 1986 - 1987 at 3.1 percent, and in 1988 it decreased to -2.48 percent. In the interval 1989 - 1992 the rate jumped to 6.53 percent. In the following tables we shall give an estimation of the WB and GS population size where we shall adopt the annual growth rate β to be the difference between the average birth rate and the average death and emigration rates; that is :

$$\beta = \lambda - \mu .$$

5.0 Calculations :

The symbols which are to be used in tables (2) , (3) and (4) are :N_1, the size of population determined by using the limiting formula (11) derived before, $N_1 = ie^{\beta t}$, i is the initial population size, then considering N_1 as a function of t, N_1 can be approximated using *Taylor* series by expanding it about t = 0 (to give another estimate population size). This Taylor series is then cut after the second term to get $N_3 = i[1+\beta t]$, and finally, it will be cut to the third term to get a better approximation to the population size,

$$N2 = i\left[1 + \beta t + \frac{(\beta t)^2}{2}\right]$$

$N_4 = i(1 + \beta)^t$ represents the *Geometric Growth* .

Table (2) gives an estimation of the size of the WB population at different years; 1992, 2002 and 2010, assuming the initial size of population at the end of year 1967 to be 585,900 (Benvenisti,1984).

Table (3) shows the estimated size of population in GS assuming the size of population in 1967 to be 380,800 .

Tables (4) and (5) give the total estimated size of population in WB and GS (assuming 1967 and 1987 the base periods, respectively) .

Table(2)
Estimates of Palestinian Resident Population
WB (t_o = 1967)

	$\beta = 0.035$			β=0.035
Year	N_1	N_2	N_3	N_4
1992	1,405,501	1,322,852	1,098,563	1,384,625
2002	1,994,500	1,743,235	1,303,628	1,953,150
2010	2,638,984	2,131,218	1,467,680	2,571,926

Table(3)
Estimates of Palestinian Resident Population
GS (t_o = 1967)

	β = 0.038			β=0.038
Year	N_1	N_2	N_3	N_4
1992	938,638	914,396	742,560	967,455
2002	1,439,821	1,224,063	887,264	1,404,767
2010	1,951,345	1,511,387	1,003,027	1,893,142

Table(4)
Estimates of Palestinian Resident Population
WB and GS (t_o = 1967)

Year	N_1	N_2	N_3	N_4
1992	2,390,139	2,237,248	1,841,123	2,352,080
2002	3,434,321	2,967,298	2,190,892	3,357,917
2010	4,590,329	3,642,605	2,470,707	4,465,068

Table(5)
Estimates of Palestinian Resident Population
WB and GS (t_o = 1987)

Year	N_1	N_2	N_3	N_4
1992	1,706,608	1,705,130	1,681,774	1,701,150
2002	2,451,274	2,427,498	2,197,137	2,427,798
2010	3,275,410	3,103,525	2,609,423	3,227,392

In Tables (2), (3), (4) and (5) we can consider N_1, N_2 and N_3 to be the maximum, medium and minimum size of population, respectively . Those can be easily compared with N_4 which is another estimate for the same population size, calculated by considering the geometric growth.

Table (6) below gives an estimation of the size of Arab population in Jerusalem area where we have assumed that the *annual growth rate* β = 0.035, i.e. assuming β is the same as in the case of WB population growth.

The initial population size of Jerusalem in 1967 was 73,348 (Combined from CBS and Benvenisti, 1988).

Table (6)
Estimates of Palestinian Residence Population
Jerusalem (t_o = 1967)

	β = 0.035			β=0.035
Year	N_1	N_2	N_3	N_4
1992	175,953	165,606	137,528	173,339
2002	249,687	218,233	163,200	244,512
2010	330,370	266,804	183,737	321,197

6.0 Conclusion :

A considerable period of time has passed since this paper was achieved in its first form .

Despite this fact, we feel now that we are more confident of the figures we have reached and used for the estimation of the population size of the WB and GS . This is because there is a great coincidence of our figures and some other published figures from other resources such as those recently published figures by the Palestinian Authority .(see for example : Central Statistics Department , 1996) .

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