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Magnetohydrodynamic Rayleigh Problem with Hall Effect

Haytham Sulieman[(1)], Naji A. Qatanani[(2)]
(1) Palestine Polytechnic University, Hebron, Palestine
(2) Department of Mathematics, An-Najah National University,Nablus, Palestine

Abstract
This paper gives very significant and up-to-date analytical and numerical results to the magnetohydrodynamic flow version of the classical Rayleigh problem including Hall effect. An exact solution of the MHD flow of incompressible, electrically conducting, viscous fluid past a uniformly accelerated and insulated infinite plate has been presented. Numerical values for the effects of the Hall parameter **N** and the Hartmann number **M** on the velocity components **u** and **v** are tabulated and their profiles are shown graphically. The numerical results show that the velocity component u increases with the increases of **N** and decreases with the increase of **M**, whereas, the velocity component v increases with the increase of both **M** and **N**. These numerical results have shown to be in a good agreement with the analytical solution.

Keywords: MHD flow, Hall effect, viscous fluid, accelerated plate.

AMS 2000 mathematics subject classification numbers: 76D05, 76W05, 80A20.

1. Introduction
When the magnetic field diffuses easily through the conducting medium and when the frequency of collision of charge particles is large compared to their frequency of rotation about the magnetic field lines, the current in the medium is controlled by the resistance of the medium and in such a case the generalized Ohm's law is the appropriate law to apply . However, if these conditions are not fulfilled, additional terms will appear in the generalized Ohm's law.
The MHD Stokes or Rayleigh problem was first solved by Rossow [11] without taking into account the induced magnetic field. With the induced magnetic field, it was solved by Nanda & Sundaram [8], Ludford [6], Chang & Yen [2], Rosciszewski [10], and Hashimoto [4]. In these papers, different aspects of the problem were considered. But in an ionized gas where the density is low and / or the magnetic field is very strong, the conductivity becomes a tensor. The conductivity normal to the magnetic field is reduced by the free spiraling of electrons and ions about the magnetic lines of force before they experience collisions, and a current, known as the Hall current is induced in a direction normal to both electric and magnetic fields. Steady state channels flows of ionized gases were studied by Sato [12], Yamanishi [17] and Sherman & Sutton [14]. The effects of Hall current on MHD Rayleigh's problem in ionized gas where studied by Mohanty [7]. Schlichting [13] has studied the unsteady flow due to an impulsive motion of an infinite plate in a fluid of an infinite extent. He showed that this simple problem admitting an exact solution for the Navier-Stokes equation. MHD flow past a uniformly accelerated plate under a transverse magnetic field was studied by Gupta [3], Pop [9] and Soundalgekar [15], neglecting induced magnetic field. Kinyanjui [5] studied the heat and the mass transfer in unsteady free convection flow with radiation absorption passed an impulsively started infinite vertical porous plate subjected to strong magnetic field including the Hall effect. Maleque and Sattar [1] investigated the steady laminar flow on a porous rotating disk with variable fluid properties taking Hall effect into account.
The study of the MHD flow with Hall currents has important engineering applications in problems of MHD generators. Hall accelerators as well as in flight Magnetohydrodynamics. The rotating flow of an electrically conducting fluid in the presence of magnetic field is encountered in cosmical and geophysical fluid dynamics. It is also important in the solar physics involved in the sunspot development, the solar cycle and the structure of rotating magnetic stars.
In this study we have considered the effect of the Hall current on the magnetohydrodynamics flow version of the classical Rayleigh problem. Thus, an exact solution of the MHD flow of incompressible, electrically conducting and insulated infinite plate has been presented. Numerical results for the effects of the Hall parameter **N** and the Hartmann number **M** on the velocity components **u** and **v** are tabulated and their profiles are shown graphically.

2. Formulation of the problem
We consider the flow of an incompressible, electrically conducting, viscous fluid past an infinite and insulated flat plate occupying the plane $y=0$. Let the positive direction of x-axis be chosen along the plate in the direction of the flow and the y-axis is normal to it. A uniform magnetic field H_0 is applied in the direction of the y-axis. The physical configuration and the nature of the flow suggest the following form of velocity vector $\vec{q}$, magnetic induction vector $\vec{H}$, electrostatic field $\vec{E}$ and pressure P, thus:

$$\left.\begin{aligned} \vec{q} &= (u, 0, v) \\ \vec{H} &= (H_x, H_0, H_z) \\ \vec{E} &= (E_x, 0, E_z) \\ P &= \text{constant} \end{aligned}\right\} \tag{2.1}$$

The equations governing the unsteady flow and Maxwell's equations are:
Equation of continuity:

$$\nabla . q = 0 \tag{2.2}$$

Equation of motion:

$$\frac{\partial q}{\partial t} + (q.\nabla) q = -\frac{1}{\rho} \nabla p + \nu \nabla^2 q + \frac{1}{\rho} J \times H \tag{2.3}$$

Equation for current:

$$\nabla \times H = \mu J \tag{2.4}$$

Faraday's Law:

$$\nabla \times E = -\frac{\partial H}{\partial t} \tag{2.5}$$

$$\nabla . H = 0 \tag{2.6}$$

The generalized Ohm's law, neglecting ion-slip effect but taking Hall current is,

$$\frac{J}{\sigma} = (E + \vec{q} \times H) - \frac{(J \times H)}{n_e . e} \tag{2.7}$$

Where $\sigma = \frac{e^2 \tau n}{m}$ (is the electrical conductivity).

Here J is the current density, t is the time, ρ, ν, and μ stand for the density, the kinematics viscosity, and the magnetic permeability, e and m are the electric charge and the mass of an electron, n is the electron number density and τ is the mean collision time.

The Lorentz force per unit volume is given by:

$$\vec{J} \times \vec{H} = [-J_z H_0, J_z H_x - J_x H_z, J_x H_0] \tag{2.8}$$

Moreover:

$$\vec{q} \times \vec{H} = [-vH_0, vH_x - uH_z, uH_0] \tag{2.9}$$

where: $\vec{J} = [J_x, 0, J_z]$

with

$$J_x = \frac{\sigma}{1 + \omega^2 \tau^2} [E_x - vH_0 + \omega\tau (E_z + uH_0)] \tag{2.10}$$

$$J_z = \frac{\sigma}{1 + \omega^2 \tau^2} [E_z - uH_0 - \omega\tau (E_x - vH_0)] \tag{2.11}$$

where $\omega = \frac{eH_0}{m}$ (is the electron Larmor frequency).

The initial and boundary conditions are:

$$\left.\begin{aligned} & t \le 0: \quad u = 0, \quad v = 0 \quad \text{for } y \ge 0 \\ & t > 0: \quad u = U_0, \; v = 0 \quad \text{for } y = 0 \\ & u \to 0: \; v = 0 \quad \text{as } y \to \infty \\ & H_x \to 0 \;\; H_y = H_0, \quad H_z \to 0 \quad \text{as } y \to \infty \end{aligned}\right\} \tag{2.12}$$

At infinity the magnetic induction is uniform with components $(0, H, 0)$, and hence the current density in (2.4) vanishes. And since the free stream is at rest, it follows from generalized Ohm's low that $E = 0$ as

$y \to \infty$. Assuming small magnetic Reynolds number for the flow, the induced magnetic field is neglected in comparison to the applied constant field H_0.
On introducing the non-dimensional quantities:

$$y^* = \frac{U_0 . y}{\nu}, \; u^* = \frac{u}{U_0}, \; v^* = \frac{v}{U_0}, \; t^* = \frac{U_0^2 \, t}{\nu} \tag{2.13}$$

then, we can write $u\,(y, t) = u^*\,(y^*, t^*)$

$$\frac{\partial u}{\partial t}(y, t) = \frac{\partial u^*}{\partial t^*}(y^*, t^*) = \frac{U_0^2}{\nu}\frac{\partial u^*}{\partial t^*} \quad \text{and} \quad \frac{\partial^2 u}{\partial y^2} = \frac{U_0^2}{\nu^2}\frac{\partial^2 u^*}{\partial y^{*2}}$$

in this case we obtain :

$$\frac{1}{\rho}(J \times H) = \frac{-1}{\rho}(J_z \, H_0)$$

or

$$\frac{1}{\rho}(J \times H) = -\frac{1}{1+\omega^2\tau^2}\left[\frac{\sigma H_0^2}{\rho}(u + \omega\tau v)\right] \tag{2.14}$$

Likewise, $v\,(y, t) = v^*(y^*, t^*)$ yields

$$\frac{\partial v}{\partial t} = \frac{U_0^2}{\nu}\frac{\partial v^*}{\partial t^*} \quad \text{and} \quad \frac{\partial^2 v}{\partial y^2} = \frac{U_0^2}{\nu}\frac{\partial^2 v^*}{\partial y^{*2}}$$

Then we obtain: $\frac{1}{\rho}(J \times H) = \frac{1}{\rho}(J_x \, H_0)$

or

$$\frac{1}{\rho}(J \times H) = \frac{1}{1+\omega^2\tau^2}\left[\frac{\sigma H_0^2}{\rho}(\omega\tau u - v)\right] \tag{2.15}$$

Consequently, the equation of motion (2.3) in component term becomes
(dropping the stars):

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} - \frac{\sigma H_0^2 \, \nu}{\rho U_0^2 + \rho U_0^2 \, \omega^2\tau^2}(u + \omega\tau v) \tag{2.16}$$

$$\frac{\partial v}{\partial t} = \frac{\partial^2 v}{\partial y^2} + \frac{\sigma H_0^2 \, \nu}{\rho U_0^2 + \rho U_0^2 \, \omega^2\tau^2}(\omega\tau v - v) \tag{2.17}$$

Now let $M^2 = \frac{\sigma H_0^2 \, \nu}{\rho U_0^2}$ is the Hartmann number and $N = \omega\tau$ is the Hall parameter **u** and **v** are the velocity components in the **x** and **y** direction respectively. The initial and boundary conditions become:

$$\left.\begin{array}{l} u(0, y) = v(0, y) = 0 \\ u(t, 0) = 1, v(t, 0) = 0 \\ u(t, y) \text{ and } v(t, y) \to 0 \text{ as } y \to \infty \end{array}\right\} \tag{2.18}$$

Now, multiply both sides of equations (2.16) and (2.17) by e^{-st} and integrate from 0 to ∞ with respect to t we get:

$$\frac{d^2 \hat{u}}{d y^2} - \left(\frac{M^2}{1 + N^2} + s\right)\hat{u} = \frac{NM^2}{1 + N^2}\hat{v} \tag{2.19}$$

$$\frac{d^2 \hat{v}}{d y^2} - \left(\frac{M^2}{1 + N^2} + s\right)\hat{v} = -\frac{NM^2}{1 + N^2}\hat{u} \tag{2.20}$$

where:

$$\hat{u}(s,y) = L\{u(t,y)\} = \int_0^\infty u(t,y)\, e^{-st}\, dt$$

$$\hat{v}(s,y) = L\{v(t,y)\} = \int_0^\infty v(t,y)\, e^{-st}\, dt$$

By introducing the complex function $\hat{q} = \hat{u} + i\hat{v}$, then equations (2.19) and (2.20) can be combined into the single equation:

$$\frac{d^2 \hat{q}}{d y^2} - \{ \frac{M^2}{1+N^2}(1 - iN) + s \} \hat{q} = 0 \tag{2.21}$$

3. Analytical Solution

By introducing the complex function $\hat{q} = \hat{u} + i\hat{v}$, then equations (2.16) and (2.17) yield

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} - (\frac{M^2}{1+N^2})(1 - iN)\, q \tag{3.1}$$

The initial and boundary conditions take the form:

$$\left. \begin{array}{l} q(0,y) = 0, \quad q(t,0) = 1 \\ q(t,y) \to 0 \quad \text{as } y \to \infty \end{array} \right\} \tag{3.2}$$

Using the abbreviation $\alpha = -\frac{M^2}{1+N^2}(1 - iN)$, equation (3.1) can be written as:

$$\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial y^2} + \alpha q \tag{3.3}$$

let:

$$\Phi(t,y) = e^{-\alpha t} q(t,y) \tag{3.4}$$

and multiplying equation (3.3) by ($e^{-\alpha t}$) we get:

$$\frac{\partial \Phi}{\partial t} = \frac{\partial^2 \Phi}{\partial y^2} \tag{3.5}$$

From equations (3.2) and (3.4) we conclude that:

$$\left. \begin{array}{l} \Phi(0,y) = 0, \quad \Phi(t,0) = e^{-\alpha t} \\ \Phi(t,y) \to 0 \quad \text{as } y \to \infty \end{array} \right\} \tag{3.6}$$

To solve (3.5) subject to the initial and the boundary conditions (3.6) we apply the Laplace transform method.

$$L\{ \frac{\partial \Phi}{\partial t} \} = L\{ \frac{\partial^2 \Phi}{\partial y^2} \}$$

$$s\hat{\Phi}(s,y) = \frac{d^2 \hat{\Phi}}{dy^2} \tag{3.7}$$

where:

$$\hat{\Phi}(s,y) = L\{\Phi(t,y)\}$$

$$\hat{\Phi}(s,0) = \frac{1}{s+\alpha}$$

$$\lim_{y\to\infty} \hat{\Phi}(s, y) = 0 \tag{3.8}$$

The auxiliary equation for equation (3.7) can be written as:

$$\beta^2 - s = 0 \tag{3.9}$$

hence,

$$\hat{\Phi}(s, y) = C_1 e^{-\sqrt{s}\, y} + C_2 e^{\sqrt{s}\, y} \tag{3.10}$$

We claim $C_2 = 0$

Proof of claim:

divide both sides of equation (3.10) by $e^{\sqrt{s}\, y}$

$$e^{-\sqrt{s}\, y}\, \hat{\Phi}(s, y) = C_1 e^{-2\sqrt{s}\, y} + C_2$$

Now ,taking the limit of both sides of the above equation as $y \to \infty$:

$$0.0 = C_2 + C_1 . 0 \quad \text{i.e. } C_2 = 0$$

Furthermore:

$$\hat{\Phi}(s, y) = C_1 e^{-\sqrt{s}\, y} \tag{3.11}$$

Setting $y = 0$ in equation (3.11) and from equation (3.8), we obtain:

$$\hat{\Phi}(s, 0) = C_1 . 1 = \frac{1}{s + \alpha} \qquad \text{i.e. } C_1 = \frac{1}{s + \alpha}$$

This gives

$$\hat{\Phi}(s, y) = \frac{1}{s + \alpha} e^{-\sqrt{s}\, y} \tag{3.12}$$

Taking the inverse transform

We have

$$\Phi(t, y) = L^{-1}\{ \hat{\Phi}(s, y) \} = L^{-1}\left\{ \frac{s}{s + \alpha} . \frac{e^{-\sqrt{s}\, y}}{s} \right\} \tag{3.13}$$

Now, we use the following fact (convolution theorem) about Laplace transformation:

$$L\{ \int_0^t f(t-\tau)\, g(\tau)\, d(\tau) \} = L\{ f(t) \} . L\{ g(t) \} = \hat{f}(s)\, \hat{g}(s)$$

where:

$$f(t) = L^{-1}\{ 1 - \frac{\alpha}{s + \alpha} \} = \delta(t) - \alpha e^{-\alpha t}$$

$$g(t) = L^{-1}\{ \frac{e^{-\sqrt{s} y}}{s} \} = \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} \right) = \frac{2}{\sqrt{\pi}} \int_{\frac{y}{2\sqrt{t}}}^{\infty} e^{-u^2}\, du$$

thus, $\Phi(t, y) = L^{-1}\{ \frac{s}{s+\alpha} . \frac{e^{-\sqrt{s} y}}{s} \} = \int_0^t [\, \delta(t-\tau) - \alpha e^{-\alpha(t-\tau)} \,] \times [\, \frac{2}{\sqrt{\pi}} \int_{\frac{y}{2\sqrt{\tau}}}^{\infty} e^{-u^2} du \,]\, d\tau$

$$\Phi(t, y) = \frac{2}{\sqrt{\pi}} \int_{\frac{y}{2\sqrt{t}}}^{\infty} e^{-u^2}\, du - \frac{2\alpha e^{-\alpha t}}{\sqrt{\pi}} \int_0^t \left(e^{\alpha\tau} \int_{\frac{y}{2\sqrt{\tau}}}^{\infty} e^{-u^2}\, du \right) d\tau \tag{3.14}$$

Recall,

$$q(t, y) = e^{\alpha t}\, \Phi(t, y)$$

Then we get:

$$q(t, y) = e^{\alpha t} \operatorname{erfc}\left(\frac{y}{2\sqrt{t}} \right) - \alpha \int_0^t e^{\alpha\tau} \operatorname{erfc}\left(\frac{y}{2\sqrt{\tau}} \right) d\tau \tag{3.15}$$

where

$$\operatorname{erfc}(x) = 1 - \operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-u^2}\, du$$

Now, writing $q(t, y)$ as $u + iv$

We have,

$$q(t, y) = e^{at} \cos bt \,\operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right) - \int_0^t e^{a\tau} \operatorname{erfc}\left(\frac{y}{2\sqrt{\tau}}\right)$$

$$[a\cos(b\tau) - b\sin(b\tau)]\, d\tau + i[\, e^{at} \sin(bt) \operatorname{erfc}\left(\frac{y}{2\sqrt{t}}\right)$$

$$- \int_0^t e^{a\tau} \operatorname{erfc}\left(\frac{y}{2\sqrt{\tau}}\right) [a\sin(b\tau) + b\cos(b\tau)]]\, d\tau \qquad (3.16)$$

where: $\alpha = a + ib$

with $a = -\dfrac{M^2}{1 + N^2}, \quad b = \dfrac{M^2 N}{1 + N^2}.$

4. Numerical Solution for The Second Order BVP

In order to get a clear understanding of the flow fluid we have carried out numerical calculations of equation (2.21). The boundary-value problem can be stated as:

$$\frac{d^2 \hat{q}}{d y^2} - \omega \hat{q} = 0 \qquad (4.1)$$

$$\hat{q}(0, s) = \frac{1}{s}, \quad \hat{q}(\infty, s) = 0 \qquad (4.2)$$

where:

$$\omega = \left(\frac{M^2}{1 + N^2} + s\right) - i\,\frac{N M^2}{1 + N^2}.$$

To ensure that the Laplace Transforms are well-defined, it should be assumed that $s > 0$. This implies $\operatorname{Re}(\omega) = \dfrac{M^2}{1 + N^2} + s > 0$. Hence there exists η in the complex number such that $\eta^2 = \omega$ with $\operatorname{Re}(\eta) < 0$

Furthermore,

$$\hat{q}(y, s) = \frac{e^{\eta y}}{s} \qquad (4.3)$$

Satisfies the boundary value problem (4.1) -(4.2) .

For **y = 0** we have:

$$\hat{q}(0, s) = \frac{1}{s} = \int_0^\infty 1 \,.\, e^{-st}\, dt = \int_0^\infty (1 + 0i)\, e^{-st}\, dt.$$

Thus,

$$u(0, t) \equiv 1 \ \text{ and } \ v(0, t) \equiv 0 \ \text{ for all } t$$

Recall that the inverse Laplace Transform is:

$$q(y, t) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \hat{q}(y, s)\, e^{st}\, ds$$

Where $\gamma > 0$ is chosen so that all the singularities of $\hat{q}(y, s)$ are to the left of γ. The above integral is over the vertical line z = γ in the complex plane. Since $\hat{q}(y, s) = \dfrac{e^{\eta y}}{s}$, we can choose γ to be any positive number. In the calculations below we choose $\gamma = 0.25$. We will define q strictly as a function of t using Mathematics' NIntegrate command. We will approximate the integral above by integrating from $0.25 - 500\,i$ to $0.25 + 500\,i$.

We also define $\omega = \left(\frac{M^2}{1 + N^2} + s\right) - i\,\frac{N M^2}{1 + N^2}$,

where $M^2 = \frac{\sigma H_0^2 \nu}{\rho U_0^2}$ is the Hartmann number, and $N = \omega\tau$ is the Hall parameter.

Next, we will define the range of t values as required in cases (**1.a-4.a)**
t = {0.4 , 0.8 , 1.2 , 1.6 , 2 , 3 , 4 , 5 , 6 , 7 , 8 , 9 }.

5. Numerical Results

The effect of the Hall parameter **N** and the Hartmann number **M** in the velocity components **u** and **v** is illustrated in the following cases:

Case 1.a

In this case, M= 1, N = 1/2, and y = 0. This case has already been eliminated since y=0. Hence this is the perfect opportunity to check the numerical method that will be employed in the other cases.

Next, we obtain the value η such that $\eta^2 = \omega$ with $\mathrm{Re}(\eta) < 0$.

t	u(t)	v(t)
0.4	0.9992	0
0.8	1.0005	0
1.2	1.0007	0
1.6	1.0003	0
2	0.9997	0
3	1	0
4	1.0002	0
5	0.9997	0
6	1.0005	0
7	0.9995	0
8	1.0004	0
9	0.9998	0

Table (1): The velocity components u and v for different values of t

As the above table indicates $u(t) = \mathrm{Re}(q(t)) \approx 1$ and $v(t) = \mathrm{Im}(q(t)) \approx 0$ for all t. this is consistent with the exact results that we obtained for the case y = 0.

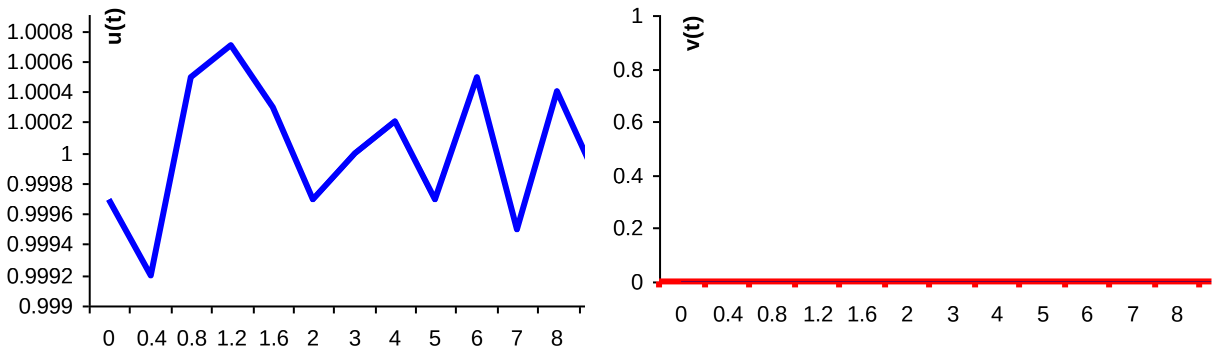


Fig 1: The velocity components for different values of t when M=1; N =1/2; y = 0

We observe that the velocity component v(t) equals zero and the velocity component u(t) approaches to one at y =0 , this means that the fluid is filling the whole space between the two plates .

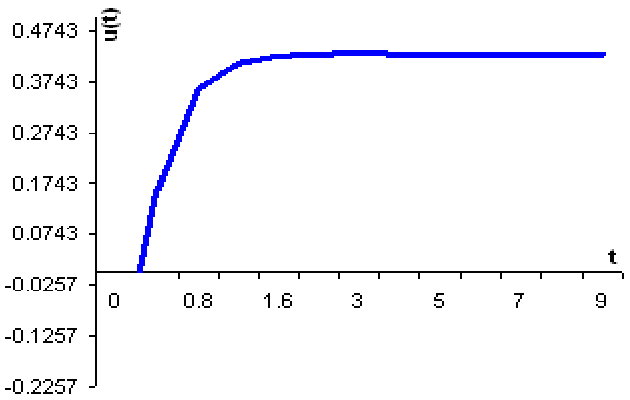
Case 2.a

Define the variables: $M = 1; \quad N = 1/2; \quad y = 1;$

Next, we obtain the value η such that $\eta^2 = \omega$ with $\mathrm{Re}(\eta) < 0$.

t	u(t)	v(t)
0.4	0.1525	0.0467
0.8	0.3621	0.0576
1.2	0.4099	0.0628
1.6	0.4251	0.0656
2	0.4302	0.0671
3	0.4303	0.0688
4	0.4291	0.0693
5	0.4285	0.0695
6	0.4275	0.0696
7	0.4283	0.0696
8	0.4272	0.0696
9	0.4281	0.0696

Table (2): The velocity components u and v for different values of t



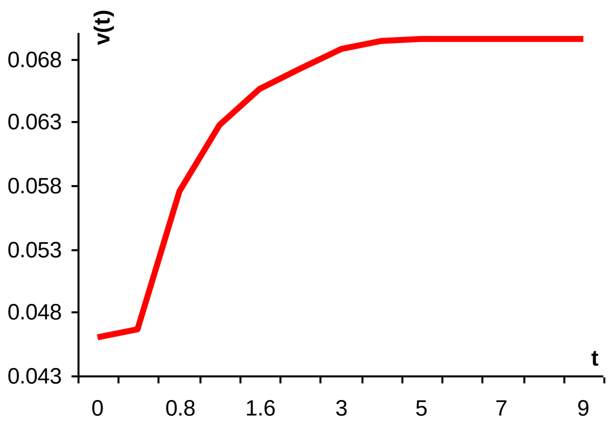


Fig 2: The velocity components for different values of t when M=1; N =1/2; y = 1

Case 3.a

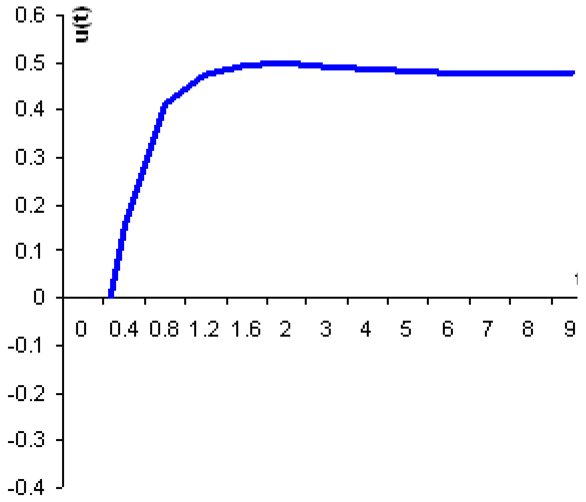
Define the variables:

$M = 1; \quad N = 1 \; ; \; y = 1;$

Next, we obtain the value η such that $\eta^2 = \omega$ with $\mathrm{Re}(\eta) < 0$.

t	u(t)	v(t)
0.4	0.1517	0.0723
0.8	0.4088	0.0879
1.2	0.4726	0.0964
1.6	0.4927	0.1016
2	0.498	0.1049
3	0.4922	0.1092
4	0.4849	0.1108
5	0.4804	0.1115
6	0.4775	0.1118
7	0.4774	0.1119
8	0.4762	0.1119
9	0.4771	0.1119

Table (3): The velocity components u and v for different values of t



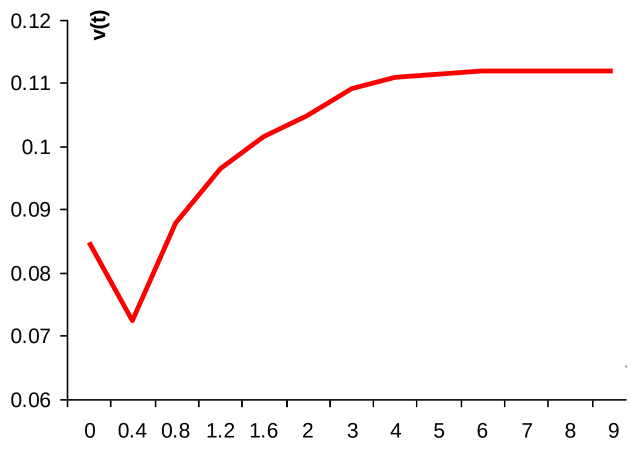


Fig 3: The velocity components for different values of t when M=1; N =1; y = 1

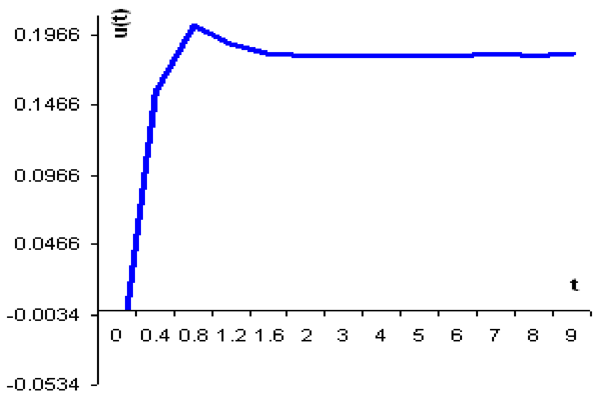
Case 4.a

Define the variables: $M=2$; $N=1;$ $y=1;$

Next, we obtain the value η such that $\eta^2 = \omega$ with $Re(\eta) < 0$.

t	u(t)	v(t)
0.4	0.1561	0.1008
0.8	0.2027	0.1185
1.2	0.1896	0.1202
1.6	0.1834	0.1201
2	0.1823	0.1199
3	0.1819	0.1199
4	0.1821	0.1199
5	0.1823	0.1199
6	0.1818	0.1199
7	0.1826	0.1199
8	0.1816	0.1199
9	0.1825	0.1199

Table (4): The velocity components u and v for different values of t



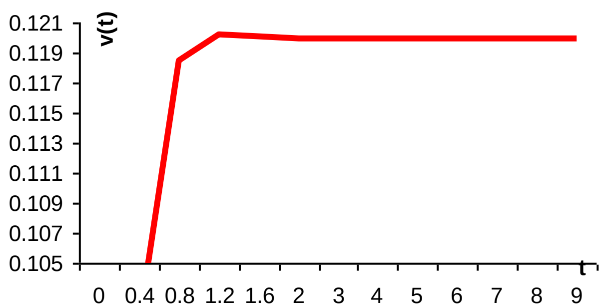


Fig 4: The velocity components for different values of t when M=2; N =1; y = 1

For the next table we'll redefine q as a function of y.

$$q(y,t) = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \hat{q}(y,s)\, e^{st}\, ds \quad \text{and} \quad \omega := \left(\frac{M^2}{1+N^2} + s\right) - i\frac{N M^2}{1+N^2}$$

Case 1.b

Define the variables: $M=1;$ $N=1/2;$ $t=1/2;$

Next, we define ω and obtain the value η such that $\eta^2 = \omega$ with $\mathrm{Re}(\eta) < 0$.

t	u(t)	v(t)
0	1.9993	0
0.4	0.6713	0.0398
0.8	0.3791	0.0515
1.2	0.1129	0.0458
1.6	-0.1213	0.0301
2	-0.2506	0.0113
3	0.1163	-0.0079
4	-0.1269	0.0017
5	0.0937	0.0007
6	0.1066	-0.0001
7	0.0883	0
8	0.0165	0.0003
9	-0.0636	-0.0001

Table (5): The velocity components u and v for different values of y

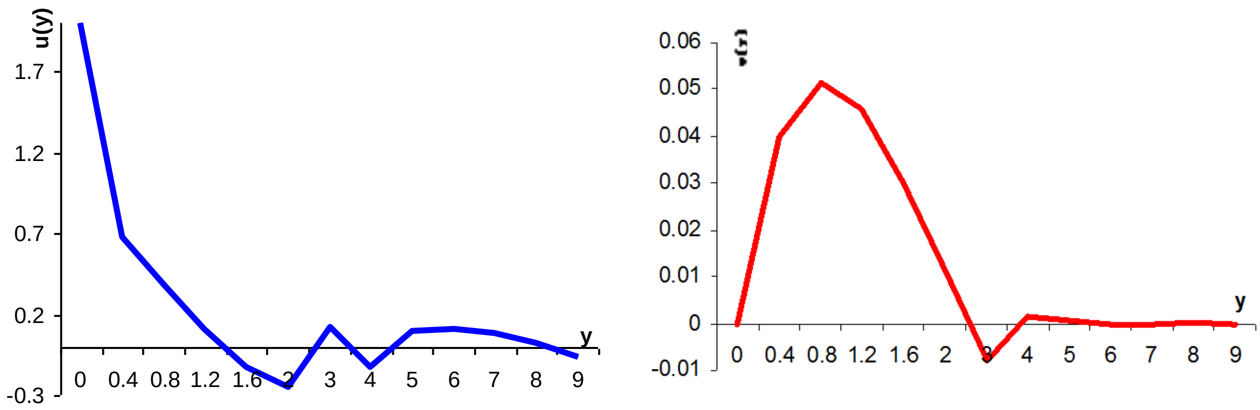


Fig 5: The velocity components for different values of t when M=1;N =1/2; t= 1/2

Case 2.b

Define the variables: $M=1;\quad N=1/2;\quad t=1;$

Next, we define ω and obtain the value η such that $\eta^2 = \omega$ with $\mathrm{Re}(\eta) < 0$.

t	u(t)	v(t)
0	2.0014	0
0.4	0.7076	0.0428
0.8	0.4862	0.059
1.2	0.3116	0.0593
1.6	0.1685	0.0504
2	0.0528	0.0367
3	-0.108	0.0009
4	-0.0053	-0.0091
5	0.0534	0.0034
6	-0.0555	-0.0008
7	0.46	0.0007
8	-0.0224	-0.001
9	-0.0215	0.0007

Table (6): The velocity components u and v for different values of y

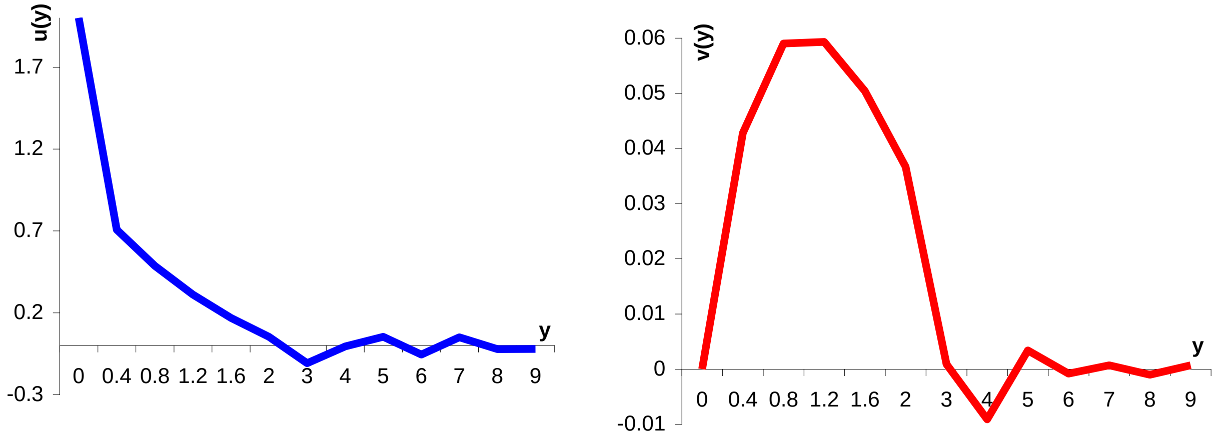


Fig 6: The velocity components for different values of t when M=1; N =1/2; t= 1

Case 3.b

Define the variables: $M = 1;\quad N = 1;\quad t = 1/2;$

Next, we define ω and obtain the value η such that $\eta^2 = \omega$ with $\mathrm{Re}(\eta) < 0$.

t	u(t)	v(t)
0	1.9993	0
0.4	0.708	0.0558
0.8	0.4111	0.0766
1.2	0.1076	0.0729
1.6	-0.1789	0.053
2	-0.3431	0.0259
3	0.16	-0.0114
4	-0.1691	-0.0003
5	0.1272	0.0009
6	0.1435	0
7	-0.1195	0
8	0.0224	0.0003
9	-0.0859	-0.0001

Table (7): The velocity components u and v for different values of y

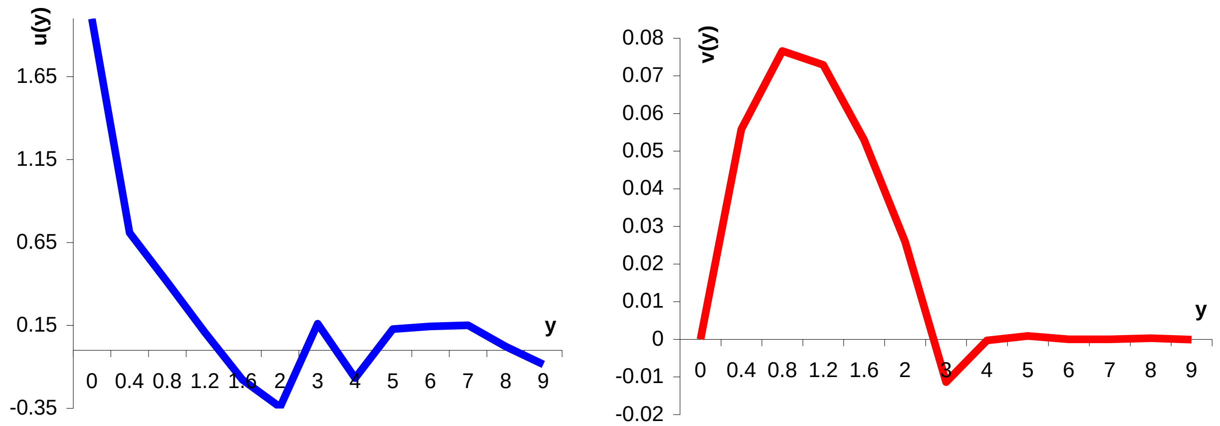


Fig 7: The velocity components for different values of t when M=1;N =1; t= 1/2

Case 4.b

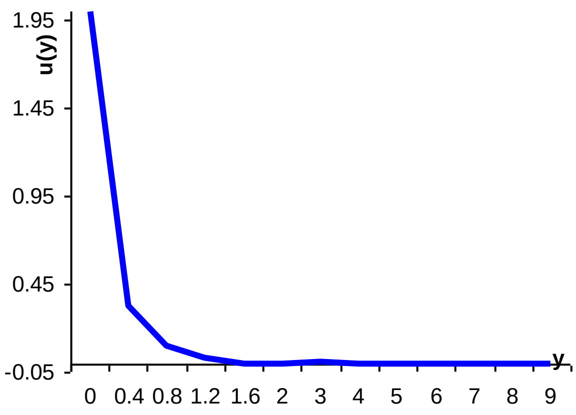
Define the variables:

$M = 3;\quad N = 1/2;\quad t = 1/2;$

Next, we define ω and obtain the value η such that $\eta^2 = \omega$ with $\mathrm{Re}(\eta) < 0$.

t	u(t)	v(t)
0	1.9993	0
0.4	0.3273	0.0829
0.8	0.0995	0.054
1.2	0.0301	0.0257
1.6	-0.0064	0.0103
2	-0.0007	0.0031
3	0.0013	-0.0008
4	-0.0005	0.0002
5	0	0
6	0.0004	0
7	0	0
8	0.0002	0
9	-0.0001	0

Table (8): The velocity components u and v for different values of y



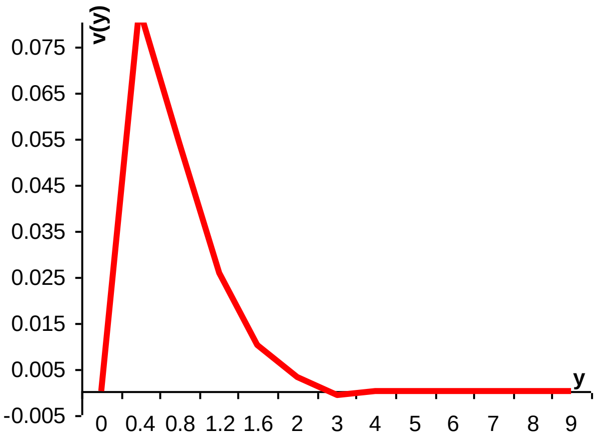


Fig 8: The velocity components for different values of t when M=3;N =1/2; t= 1/2

6. Conclusions

The set of equations describing the MHD flow of an incompressible electrically conducting fluid in the presence of Hall effect are a compensation of the Navier-Stokes equations of fluid dynamics and Maxwell's equations of electromagnetism. These differential equations have been solved analytically and numerically. It must be revealed that only in few special cases an exact solution can be obtained. One of these cases occurs when the compressibility effects of the medium are considered to be negligible. That is, the fluid is taken as in compressible and the other fluid properties such as viscosity, thermal conductivity and electrical conductivity are regarded as constants, for the numerical solution of the partial differential equations the velocity components u and v which are dependent on **M** and **N** have been calculated for different values of times **t** and heights **y.** The numerical results show the following observations:

(i) The velocity component **u** increases with the increase of **N** at equal heights of **y** and attains a steady state earlier with the increase of **N**.
(ii) The velocity component **u** decreases with the increase of **M**.
(iii) Attaining the steady state is delayed as **N** decreases.
(iv) The velocity component **v** increases with the increase of **M**.
(v)The velocity component **v** increases with the increase of parameter **N.**
(vi) When **y** increases at different values of **t**, **u** decreases, while **v** increases for fixed values of **M** and **N**. Moreover the velocity component **u** gets unstable at different values of **N**, and **y** along with the increase of **t**. However, the velocity component **v** often increases as **t** increases.
(vii) Finally, it can be stated that these new numerical results are in close agreement with the analytical solution that has been obtained by the method of Laplace transform.

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