

HW3

Spring 2010

12-1091

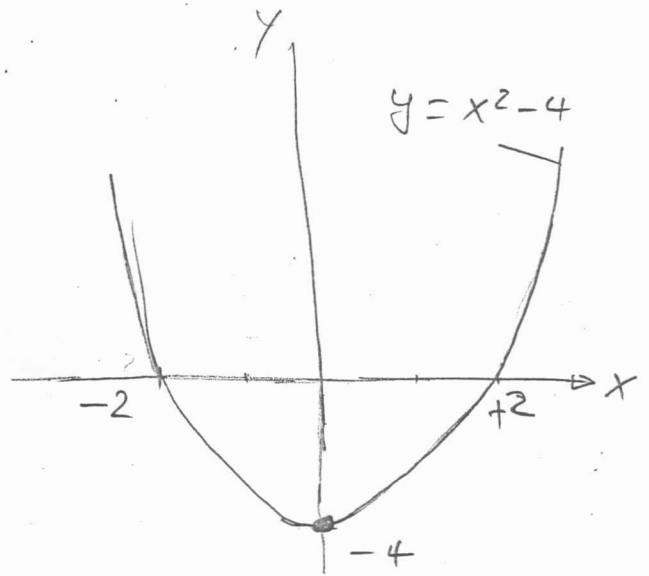
$$y = x^2 - 4$$

$$v = 5 \text{ m/s}$$

$a_t = 0$, since $v = \text{const}$

$$\rightarrow a = a_n$$

$$a_n = \frac{v^2}{r}$$



Since $v = \text{const}$, a is maximum when r is minimum
this is the case when

$$x = 0, \quad y = -4 \text{ m}$$

$\rightarrow$

$$\frac{dy}{dx} = 2x$$

$$\left. \frac{dy}{dx} \right|_{x=0} = 0$$

$$\frac{d^2y}{dx^2} = 2$$

$$\rightarrow r_{\min} = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \bigg|_{\substack{x=0 \\ y=-4}} = \frac{(1+0)^{3/2}}{2} = 0.5 \text{ m}$$

$$a_{\max} = a_{n,\max} = \frac{v^2}{r_{\min}}$$

$$a_{\max} = \frac{5^2}{0.5} = 50 \text{ m/s}^2$$

12-112

Point A

$$a_t = 0 \quad \text{since } v_A = \text{const.}$$

$$a_n = g = 9.81$$

$$v_{xB} = v_{xA} = 50 \text{ m/s}$$

$$\begin{aligned} v_{By}^2 &= v_{y0}^2 + 2a_y(y - y_0) \\ &= 0 + 2(9.81)(0 - 500) \end{aligned}$$

$$\rightarrow v_{By} = 99 \text{ m/s}$$

$$\tan \theta = \frac{v_{By}}{v_{Bx}} = \frac{99}{50} \Rightarrow \theta = 63.21^\circ$$

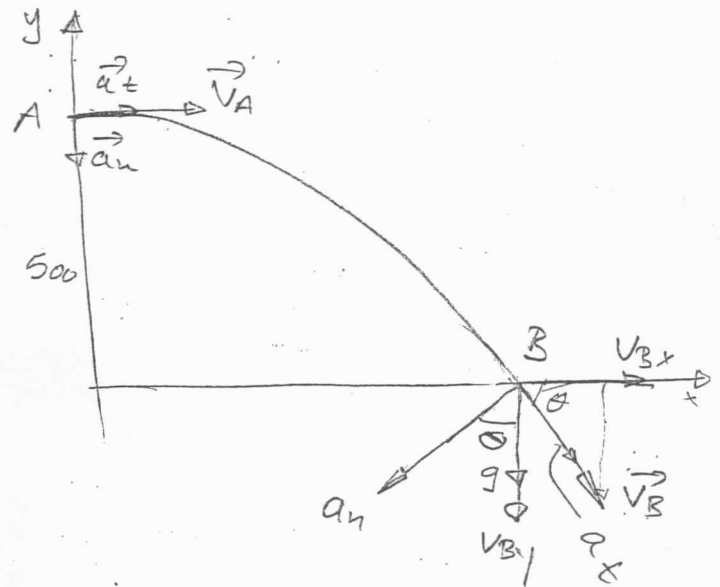
$$v_B = \sqrt{v_x^2 + v_y^2} = 110.91 \text{ m/s}$$

$$a_n = g \cos \theta = 4.42 \text{ m/s}^2$$

$$a_t = g \sin \theta = 8.76 \text{ m/s}^2$$

$$a_n = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{a_n}$$

$$r = 2783 \text{ m}$$



12-125

$$s = r\theta$$

$$s_A = 0.7(2) = 1.4 \text{ m}$$

$$s_B = 4.5(2) = 3 \text{ m}$$

$$\theta_A = \frac{s_A}{r} = \frac{1.4}{5} = 0.28 \text{ rad} = 16.04^\circ$$

$$\theta_B = \frac{3}{5} = 0.6 \text{ rad} = 34.38^\circ$$

Particle A:

$$x = 5 \sin 16.04 = 1.38 \text{ m}$$

$$y = 5(1 - \cos 16.04) = 0.195 \text{ m}$$

$$\vec{r}_A = 1.38 \vec{i} + 0.195 \vec{j}$$

Particle B:

$$x = -5 \sin 34.38^\circ = -2.82 \text{ m}$$

$$y = 5(1 - \cos 34.38^\circ) = 0.873 \text{ m}$$

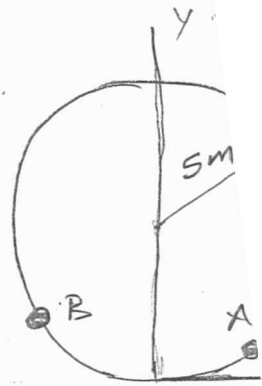
$$\vec{r}_B = -2.82 \vec{i} + 0.873 \vec{j}$$

$$\Delta \vec{r} = \vec{r}_B - \vec{r}_A$$

$$= -4.2 \vec{i} + 0.678 \vec{j}$$

$$\Delta r = \sqrt{(-4.2)^2 + (0.678)^2}$$

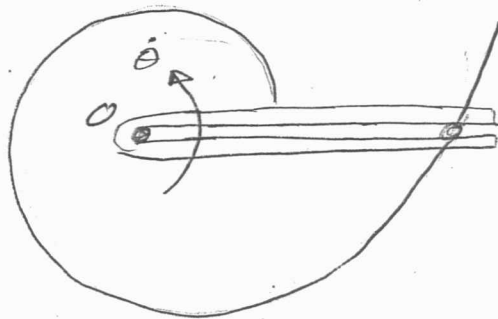
$$\Delta r = 4.26 \text{ m}$$



12-1381

$$\dot{\theta} = 3 \text{ rad/s}$$

$$r(\theta) = 25\left(5 + \frac{\theta}{\pi}\right) \text{ mm}$$



$$\theta = 360^\circ = 2\pi \quad \dot{\theta} = 3 \quad \ddot{\theta} = 0$$

$$r = 25\left(5 + \frac{2\pi}{\pi}\right) = 175 \text{ mm}$$

$$\dot{r} = \frac{25}{\pi} \dot{\theta} = 75/\pi \text{ mm/s} \quad \ddot{r} = 0, \text{ since } \ddot{\theta} = 0$$

$$V_r = \dot{r} = 75/\pi = 23.87 \text{ mm/s}$$

$$V_\theta = r\dot{\theta} = 175(3) = 525 \text{ mm/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - (175)(3)^2 = -1.575 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(75/\pi)(3) = 0.143 \text{ m/s}^2$$

12-1411

When $t = 1 \text{ s}$

$$\theta = 4t = 4 \quad \dot{\theta} = 4 \quad \ddot{\theta} = 0$$

$$r = 2 \sin 2\theta = 1.9787 \text{ m}$$

$$\dot{r} = 4 \cos 2\theta \dot{\theta} = -2.328 \text{ m/s}$$

$$\ddot{r} = -8 \sin 2\theta \dot{\theta}^2 + 8 \cos 2\theta \ddot{\theta} = -126.638 \text{ m/s}^2$$

$$V_r = \dot{r} = -2.33 \text{ m/s}$$

$$V_\theta = r\dot{\theta} = 7.91 \text{ m/s}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -158 \text{ m/s}^2$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = -18.6 \text{ m/s}^2$$