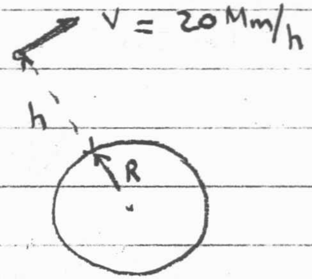
12.107

$$R = \frac{12713}{2} = 6356.5 \text{ km}$$



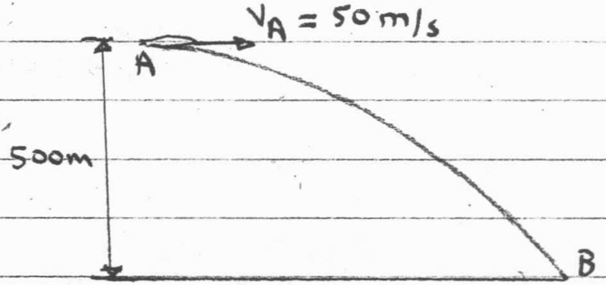
$$a_n = \frac{v^2}{R+h} \Rightarrow 2.5 = \frac{(20\times 10^6/3600)^2}{R+h}$$

$$h = 5990 \text{ km}$$

—x—

12.112)

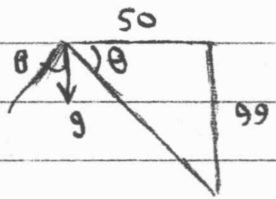
(a) $a_{tA} = 0$ $\quad a_{nA} = -g\hat{j}$



$$a_n = \frac{v^2}{\rho} \Rightarrow \rho = \frac{50^2}{9.81} = 255 \text{m}$$

b) $500 = \frac{1}{2}gt^2 \Rightarrow t = 10.1$ sec, $\quad V_{By} = gt = 9.81 \times 10.1 = 99$ m/s

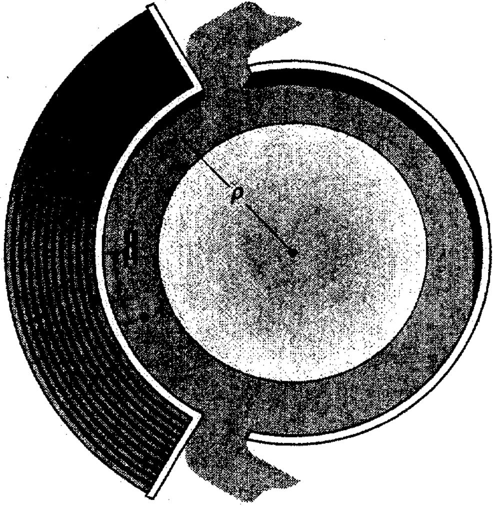
$$\theta = \tan^{-1}\frac{99}{50} = 63.2°, \quad V_B = \sqrt{50^2 + 99^2} = 111 \text{ m/s}$$



$a_{tB} = g\sin\theta = 8.8$ m/s², $\quad a_{nB} = g\cos\theta = 4.42$ m/s²

$$\rho = \frac{v^2}{a_{nB}} = 2783 \text{ m}$$

12-127. The race car has an initial speed $v_A = 15$ m/s at A. If it increases its speed along the circular track at the rate $a_t = (0.4s)$ m/s^2, where s is in meters, determine the time needed for the car to travel 20 m. Take $\rho = 150$ m.



$$a_t = 0.4s = \frac{v\,dv}{ds}$$

$$a\,ds = v\,dv$$

$$\int_0^s 0.4s\,ds = \int_{15}^{v} v\,dv$$

$$\frac{0.4s^2}{2}\Big|_0^s = \frac{v^2}{2}\Big|_{15}^{v}$$

$$\frac{0.4s^2}{2} = \frac{v^2}{2} - \frac{225}{2}$$

$$v^2 = 0.4s^2 + 225$$

$$v = \frac{ds}{dt} = \sqrt{0.4s^2 + 225}$$

$$\int_0^s \frac{ds}{\sqrt{0.4s^2 + 225}} = \int_0^t dt$$

$$\int_0^s \frac{ds}{\sqrt{s^2 + 562.5}} = 0.632\,456t$$

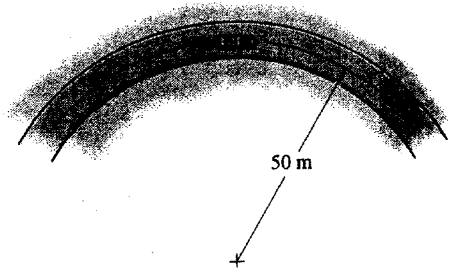
$$\ln\left(s + \sqrt{s^2 + 562.5}\right)\Big|_0^s = 0.632\,456t$$

$$\ln\left(s + \sqrt{s^2 + 562.5}\right) - 3.166\,196 = 0.632\,456t$$

At $s = 20$ m,

$t = 1.21$ s **Ans**

12.132 The truck travels at a speed of 4 m/s along a circular road that has a radius of 50 m. For a short distance from $s = 0$, its speed is then increased by $\dot{v} = (0.05s)$ m/s^2, where s is in meters. Determine its speed and the magnitude of its acceleration when it has moved $s = 10$ m.



Velocity **:** The speed v in terms of position s can be obtained by applying $v dv = a ds$.

$$v dv = a ds$$

$$\int_{4\,\text{m/s}}^{v} v dv = \int_0^s 0.05 s ds$$

$$v = \left(\sqrt{0.05s^2 + 16}\right) \text{ m/s}$$

At $s = 10$ m, $\quad v = \sqrt{0.05(10^2) + 16} = 4.583 \text{ m/s} = 4.58 \text{ m/s}$ **Ans**

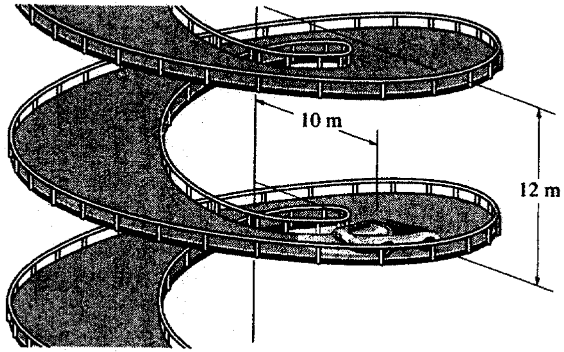
Acceleration **:** The tangential acceleration of the truck at $s = 10$ m is $a_t = 0.05(10) = 0.500 \text{ m/s}^2$. To determine the normal acceleration, apply Eq. 12 – 20.

$$a_n = \frac{v^2}{\rho} = \frac{4.583^2}{50} = 0.420 \text{ m/s}^2$$

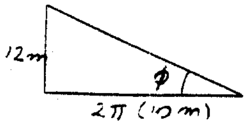
The magnitude of the acceleration is

$$a = \sqrt{a_t^2 + a_n^2} = \sqrt{0.500^2 + 0.420^2} = 0.653 \text{ m/s}^2$$ **Ans**

12.155 The automobile is traveling from a parking deck down along a cylindrical spiral ramp at a constant speed of $v = 1.5$ m/s. If the ramp descends a distance of 12 m for every full revolution, $\theta = 2\pi$ rad, determine the magnitude of the car's acceleration as it moves along the ramp, $r = 10$ m. *Hint:* For part of the solution, note that the tangent to the ramp at any point is at an angle of $\phi = \tan^{-1}(12/[2\pi(10)]) = 10.81°$ from the horizontal. Use this to determine the velocity components v_θ and v_z, which in turn are used to determine $\dot{\theta}$ and $\dot{z}$.



$$\phi = \tan^{-1}\left(\frac{12}{2\pi(10)}\right) = 10.81°$$



$v = 1.5$ m/s

$v_r = 0$

$v_\theta = 1.5\cos 10.81° = 1.473$ m/s

$v_z = -1.5\sin 10.81° = -0.2814$ m/s

Since

$r = 10 \qquad \dot{r} = 0 \qquad \ddot{r} = 0$

$v_\theta = r\dot{\theta} = 1.473 \qquad \dot{\theta} = \frac{1.473}{10} = 0.1473$

Since $\ddot{\theta} = 0$

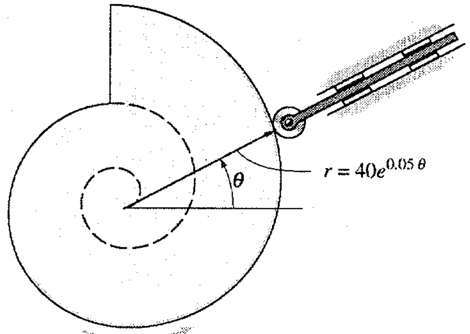
$a_r = \ddot{r} - r\dot{\theta}^2 = 0 - 10(0.1473)^2 = -0.217$

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 10(0) + 2(0)(0.1473) = 0$

$a_z = \ddot{z} = 0$

$a = \sqrt{(-0.217)^2 + (0)^2 + (0)^2} = 0.217 \text{ m/s}^2$ **Ans**

12-159. The partial surface of the cam is that of a logarithmic spiral $r = (40e^{0.05\theta})$ mm, where θ is in radians. If the cam is rotating at a constant angular rate of $\dot{\theta} = 4$ rad/s, determine the magnitudes of the velocity and acceleration of the follower rod at the instant $\theta = 30°$.



$r = 40e^{0.05\theta} \quad r = 40e^{0.05\left(\frac{\pi}{6}\right)} = 41.0610$

$\dot{r} = 2e^{0.05\theta}\dot{\theta} \quad \dot{r} = 2e^{0.05\left(\frac{\pi}{6}\right)}(4) = 8.21$ mm/s **Ans**

$\ddot{r} = 0.1e^{0.05\theta}(\dot{\theta})^2 + 2e^{0.05\theta}\ddot{\theta}$

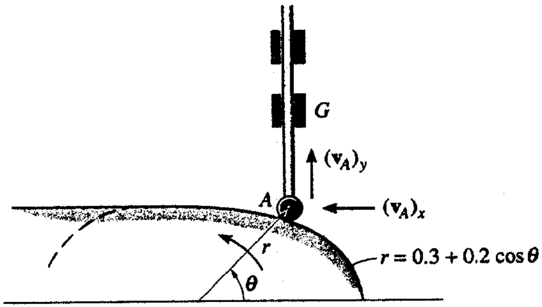
$\ddot{r} = 0.1e^{0.05\left(\frac{\pi}{6}\right)}(4)^2 + 0 = 1.64$ mm/s^2 **Ans**

$\theta = \frac{\pi}{6}$

$\dot{\theta} = 4$

$\ddot{\theta} = 0$

12.169 The mechanism of a machine is constructed so that the roller at A follows the surface of the cam described by the equation $r = (0.3 + 0.2\cos\theta)$ m. If $\dot{\theta} = 0.5$ rad/s and $\ddot{\theta} = 0$, determine the magnitudes of the roller's velocity and acceleration when $\theta = 30°$. Neglect the size of the roller. Also compute the velocity components $(\mathbf{v}_A)_x$ and $(\mathbf{v}_A)_y$ of the roller at this instant. The rod to which the roller is attached remains vertical and can slide up or down along the guides while the guides translate horizontally to the left.



$\dot{\theta} = 0.5$

$\ddot{\theta} = 0$

$r = (0.3 + 0.2\cos\theta)$

$\dot{r} = -0.2\sin\theta\,\dot{\theta}$

$\ddot{r} = -0.2(\cos\theta\,\dot{\theta}^2 + \sin\theta\,\ddot{\theta})$

At $\theta = 30°$

$r = 0.473$

$\dot{r} = -0.05$

$\ddot{r} = -0.0433$

$v_r = \dot{r} = -0.05$

$v_\theta = r\dot{\theta} = 0.473(0.5) = 0.237$

$v = \sqrt{(-0.05)^2 + (0.237)^2} = 0.242$ m/s **Ans**

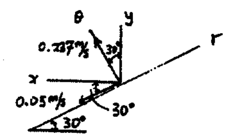
$a_r = \ddot{r} - r\dot{\theta}^2 = -0.0433 - 0.473(0.5)^2$

$a_r = -0.162$ m/s^2

$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 + 2(-0.05)(0.5)$

$a_\theta = -0.05$ m/s^2

$a = \sqrt{(-0.162)^2 + (-0.5)^2} = 0.169$ m/s^2 **Ans**



$(\overset{+}{\leftarrow})$ $v_x = 0.05\cos 30° + 0.237\sin 30°$

$v_x = 0.162$ m/s **Ans**

$(+\uparrow)$ $v_y = -0.05\sin 30° + 0.237\cos 30°$

$v_y = 0.180$ m/s **Ans**