1HW

SS 2009/2010

12-9

$s = t^3 - 3t^2 + 2$

find v, $v_{av,sp}$ a at $t = 4$ s

$t = 0$; $\underline{s_0 = 2}$ $\quad s(t=4) = 4^3 - 3\cdot4^2 + 2 = 18$ m

$\Delta s = 18 - 2 = 16$ m

$v = \frac{\Delta s}{\Delta t} = \frac{16}{4} = 4$ m/s

$v = \frac{ds}{dt} = 3t^2 - 6t = 0 \Rightarrow t = 0$ or $t = 2$

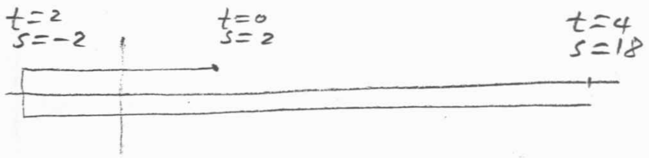
$0 \le t < 2$ v is negative, particle moves to the left

$t > 2$ v is positive particle moves to the right

$t = 2 \rightarrow s = 2^3 - 3(2)^2 + 2 =$

$S_T = 2 + |-2| + 2 + 18 = 24$

$v_{av,sp} = \frac{S_T}{\Delta t} = \frac{24}{4} = 6$ m/s



12-10

$a = -2v$

$a\,ds = v\,dv$; $\quad ds = \frac{v}{-2v}dv = -\frac{1}{2}dv$

$\int_0^s ds = -\frac{1}{2}\int_{20}^{v} dv \Rightarrow \boxed{v = 20 - 2s}$

to stop $v = 0 = 20 - 2s \Rightarrow s = 10$ m

د. محمد ابو هلال

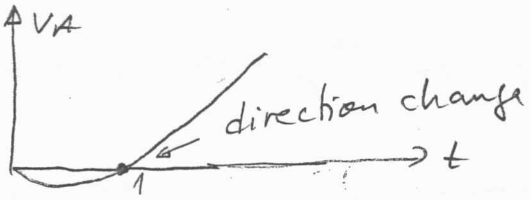
12-17

Car A: $t=0,\ v_0=s_0=0$

$a_A = 6t-3 = \frac{dv_A}{dt}$

$\hookrightarrow v_A = 3t^2-3t$

$v_A=0 \rightarrow t=0,\ t=1$

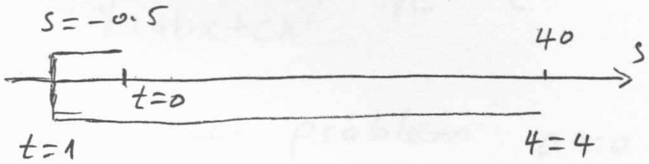


$ds_A = v_A\,dt = (3t^2-3t)\,dt$

$\hookrightarrow s_A = t^3 - \frac{3}{2}t^2$

$s_A(t=1) = -0.5\ m$

$s_A(t=4) = 40\ m$



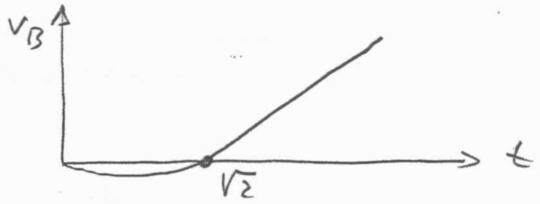
$s_{TA} = 2|-0.5| + 40 = 41\ m$

Car B: $t=0,\ v_0=s_0=0$

$a_B = 12t^2-8 = \frac{dv_B}{dt}$

$\hookrightarrow v_B = 4t^3-8t$

$v_B=0:\ t=0,\ t=\sqrt{2}$

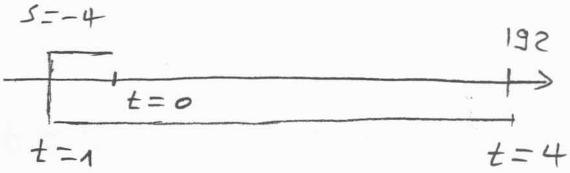


$ds_B = v_B\,dt$

$\hookrightarrow s_B = t^4-4t^2$

$s_B(t=\sqrt{2}) = -4\ m$

$s_B(t=4) = 192\ m$



$s_{TB} = 2|-4| + 192 = 200\ m$

At $t=4s$ the distance between cars A and B is

$d_{AB} = 192-40$

$d_{AB} = 152\ m$

12–23

$a = a(s) = 6 + 0.02s$ $\quad\quad t=0: \; s=0, v_0$

$ads = vdv \quad \longrightarrow \quad \int_0^s (6+0.02s)ds = \int_0^v vdv$

$6s + 0.01s^2 = \frac{v^2}{2} \quad \Rightarrow \quad v = \sqrt{12s + 0.02s^2}$

$v = \frac{ds}{dt}$; $\quad \sqrt{12s+0.02s^2} = \frac{ds}{dt}$

$\hookrightarrow \int_0^t dt = \int_0^s \frac{ds}{\sqrt{12s+0.02s^2}}$

Appendix A:

$\int \frac{dx}{\sqrt{a+bx+cx^2}} = \frac{1}{\sqrt{c}} \ln\left[\sqrt{a+bx+cx^2} + x\sqrt{c} + \frac{b}{2\sqrt{c}}\right]$ for $\underline{\underline{c>0}}$

in our problem $\quad a=0 \quad b=12 \quad c=0.02>0$

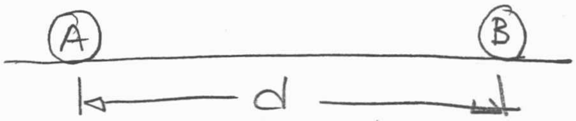
$t = \frac{1}{\sqrt{0.02}}\left[\ln\left(\sqrt{12s+0.02s^2} + s\sqrt{0.02} + \frac{6}{\sqrt{0.02}}\right) - \ln\frac{6}{\sqrt{0.02}}\right]$

at $\quad s=100 \quad$ we get

$t = 5.623$ sec

12 - 32

Car A:



to stop: $a_A = \text{const.}$

$v = v_0 + a_c t$

$0 = v_A - a_A t \quad \Rightarrow t = \frac{v_A}{a_A}$ $-$: deceleration

$s_A = s_0 + v_0 t + \frac{1}{2} a_c t^2$ $s_0 = 0,\ v_0 = v_A \quad a_c = -a_A$

$s_A = 0 + v_A \frac{v_A}{a_A} - \frac{1}{2} a_A \frac{v_A^2}{a_A^2}$

$\hookrightarrow s_A = \frac{1}{2} \frac{v_A^2}{a_A}$

Car B:

$s_B = s_0 + v_0 t + \frac{1}{2} a_c t^2$ $s_0 = 0,\ a_c = 0 \quad v_0 = v_B$

$s_B = v_B t = v_B \frac{v_A}{a_A}$ the same time

$s_B = \frac{v_A v_B}{a_A}$

$\left| \frac{v_A}{2 a_A} (2 v_B - v_A) \right|$

$d = |s_B - s_A| = \left| \frac{v_A v_B}{a_A} - \frac{1}{2} \frac{v_A^2}{a_A} \right|$

$d = \frac{v_A}{2 a_A} |2 v_B - v_A|$