# HW 12

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$\boxed{22-10}$

given:

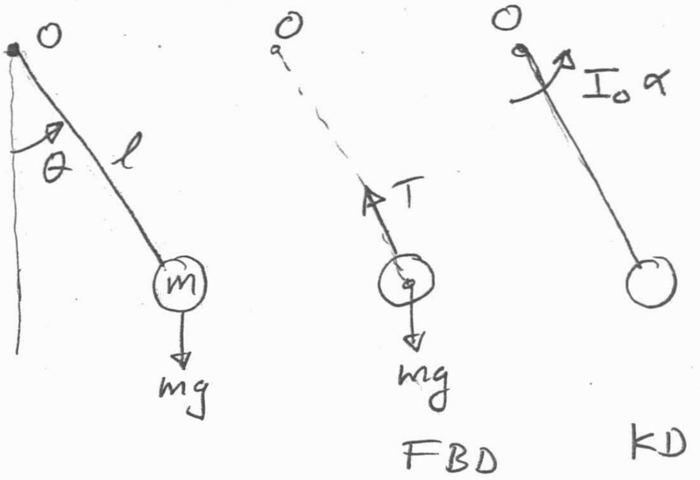
$l = 0.4\,m$

$V_0 = -0.2\,m/s$

$v = \omega r$

$\hookrightarrow \omega_0 = -0.5\ rad/s$

$\theta_0 = 0.3\,rad \triangleq 17.2°$




$\circlearrowleft \sum M_o = I_o \ddot{\theta} \qquad I_o = ml^2$

$-mgl\sin\theta = ml^2\ddot{\theta}$

or $\quad \ddot{\theta} + \frac{g}{l}\sin\theta = 0$

Assume: small oscillations $\rightarrow \sin\theta \approx \theta$

$\ddot{\theta} + \omega_n^2\theta = 0 \qquad \omega_n = \sqrt{g/l} = 4.95\ rad/s$

<u>Solution:</u>

$\theta(t) = A\cos\omega_n t + B\sin\omega_n t$

$\dot{\theta}(t) = -A\omega\sin\omega_n t + B\omega_n\cos\omega_n t$

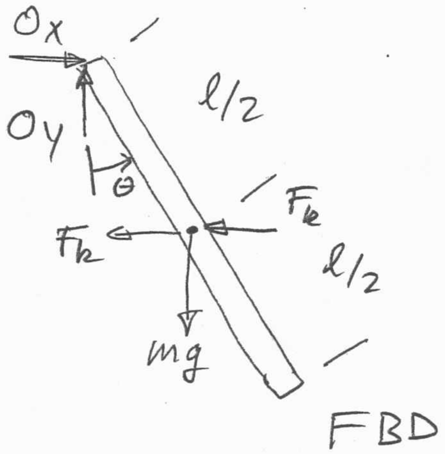
<u>initial conditions:</u>

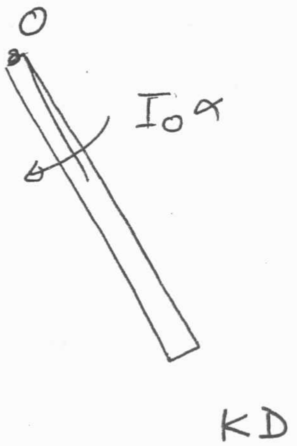
$\theta(0) = 0.3 = A\cos 0 + B\sin 0 \Rightarrow \boxed{A = 0.3}$

$\dot{\theta}(0) = \omega_0 = -0.5 = B\omega_n \Rightarrow B = \frac{-0.5}{\omega_n} = -0.1$

$\hookrightarrow \theta(t) = 0.3\cos 4.95t - 0.1\sin 4.95t$

22-121




$\circlearrowleft \Sigma M_O = I_O \alpha$

$\alpha = \ddot{\theta} \qquad I_O = \frac{ml^2}{3}$

$$-mg\left(\frac{l}{2}\right)\sin\theta - 2F_k\left(\frac{l}{2}\right)\cos\theta = \frac{ml^2}{3}\ddot{\theta}$$

$$F_k = k\,\Delta s = k\left(\frac{l}{2}\right)\sin\theta$$

small oscillations : $\sin\theta \approx \theta \qquad \cos\theta \approx 1$

$$\hookrightarrow m\frac{l^2}{3}\ddot{\theta} + \left[mg\frac{l}{2} + 2k\frac{l}{2}\frac{l}{2}\right]\theta = 0$$

or

$$\ddot{\theta} + \frac{3(mg+kl)}{2ml}\theta = 0$$

$$\omega_n = \sqrt{\frac{3(mg+kl)}{2ml}} = 5.688 \quad \text{rad/s}$$

$$\hookrightarrow f_n = \frac{\omega_n}{2\pi} = 0.905 \quad \text{Hz}$$

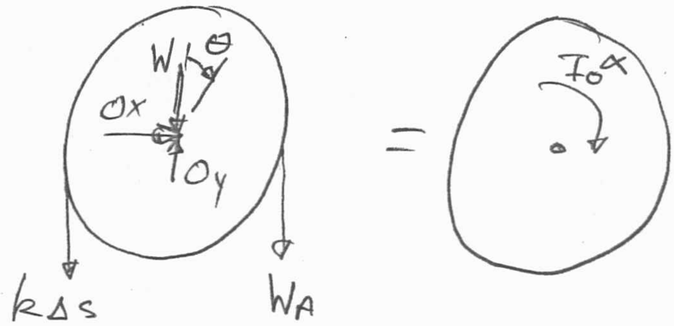
22-20

$r = 0.75\,m$

$m_D = 1529.052\ kg$

$m_A = 305.81\ kg$

$k = 80\,kN/m$




Assume:

block is a particle $\rightarrow I_{AG} = 0$

$I_O = \frac{1}{2} m_D r^2 + m_A r^2 = (\frac{1}{2} m_D + m_A) r^2 = 602.064\ kg\,m^2$

$\Delta s = r\theta$

From static equilibrium position

$\circlearrowleft \Sigma M_{(O)} = I_O \alpha$

$I_O \ddot{\theta} + k r^2 \theta = 0$

$\ddot{\theta} + \omega_n^2 \theta = 0$

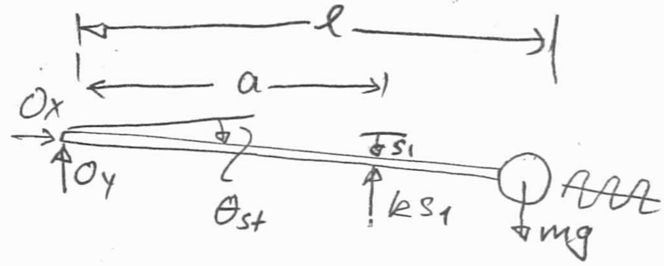
$$\omega_n = \sqrt{\frac{k r^2}{I_O}} = \sqrt{\frac{80000(0.75^2}{602.064}}$$

$\omega_n = 8.645\ rad/s$

$\tau_n = \frac{2\pi}{\omega_n} = 0.727\ s$

22-34

$l = 0.75 m$ $\qquad a = 0.5 m$




$s_1 = \theta_{st} \cdot a$

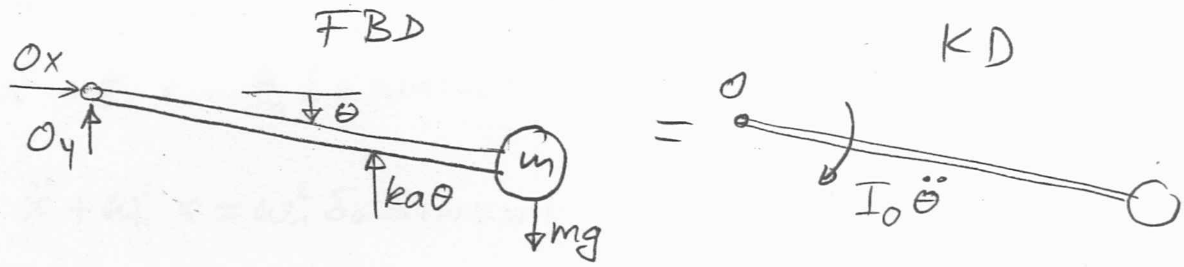
Static equilibrium $\theta_{st}$

$\circlearrowright \Sigma M_O = 0$

$k s_1 (a) - mgl = 0$

$k \theta_{st} a^2 - mgl = 0 \Rightarrow \boxed{\theta_{st} = \frac{mgl}{ka}}$ $\qquad \theta_{st} = 0.196 \text{ rad} \equiv 11.24°$

Dynamic equilibrium:




Assume:

small oscillation

vibration about the static equilibrium position

$\circlearrowright \Sigma M_O = I_O \ddot{\theta}$ $\qquad I_O = ml^2 = 20(0.75)^2 = 11.25 \text{ kgm}^2$

$-ka^2\theta = ml^2 \ddot{\theta}$

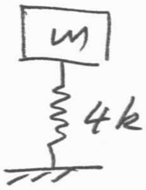
or $\ddot{\theta} + \frac{ka^2}{ml^2}\theta = 0$

$\ddot{\theta} + \omega_n^2 \theta = 0$

$\omega_n = \sqrt{\frac{ka^2}{ml^2}} = \sqrt{\frac{150(0.5)^2}{2(0.75)^2}} = 5.7735 \text{ rad/s}$

$f_n = \frac{\omega_n}{2\pi} = 0.92 \text{ Hz}$

22-49




given:

$m = 18\,kg \qquad k = 130\,N/m$

$y(t) = \delta_0 \sin \omega_0 t \qquad \delta_0 = 0.17\,m$

$f_0 = 7\,Hz \rightarrow \omega_0 = 2\pi(7) = 43.982\ rad/s$

Determine the vertical displacement of the mass

Equation of motion:

$m\ddot{x} + kx = ky$

$\ddot{x} + \frac{k}{m}x = \frac{k}{m}\delta_0 \sin \omega_0 t$

or

$\ddot{x} + \omega_n^2 x = \omega_n^2 \delta_0 \sin \omega_0 t \qquad \omega_n = \sqrt{\frac{4k}{m}} = 5.3748\ rad/s$

$x(t) = X_0 \sin \omega_0 t$

where $$X_0 = \frac{\delta_0}{1 - \left(\frac{\omega_0}{\omega_n}\right)^2} = \frac{0.17}{1 - \left(\frac{43.982}{5.3748}\right)^2}$$

$X_0 = 2.57726\ mm$