EVERYDAY ARGUMENTATION AND KNOWLEGDE CONSTRUCTION IN MATHEMATICAL TASKS

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The aim of this study is to gain insights into relations between knowledge construction and argumentation. This paper presents a case study showing an analysis that combines different tools: the Toulmin´s scheme to reconstruct the argumentation structure, a collection of topical schemes to characterize different types of inferences including everyday inferences and an epistemic action model to describe processes of knowledge construction. Some preliminary results of this case study will illustrate how the combination of these different tools can shed light on relations between argumentation and knowledge construction.

Key words: Argumentation, knowledge construction, everyday argumentation, topical schemes.

INTRODUCTION

The aim of my research is to gain insights into relations between knowledge construction and argumentation. Arguing is an important learning goal for two reasons. Schwarz (2009) distinguishes between learning to argue and argue to learn. Referring to Andriessen et al. he clarifies this distinction:

> „`Learning to argue´ involves the acquisition of general skills such as justifying, challenging, counterchallenging, or conceding. In contrast `Arguing to learn´ often fits a specific goal fulfilled through argumentation, and in an educational framework, the (implicit) goal is to understand or to construct specific knowledge." (Schwarz 2009, 92)

Some researchers like Krummheuer & Brandt (2001) even assume a constitutive function of arguing for learning mathematics in school: Students learn mathematics by participating in argumentation that means practicing argumentation within social interaction as a collective activity. In addition, learning to argue is stated as a goal in many curricula all over the world (e.g. see NCTM 2000). Nevertheless, students have difficulties to learn how to reason in a deductive way. Research shows that everyday argumentation predetermines their way of reasoning in mathematics. For example, like in everyday situations, many students infer general rules from just some examples (Martin & Harel 1989) without feeling the need to prove their inference. In addition, Galbraith (1981) has shown that students often do not realize that one counterexample disproves a mathematical statement because in everyday life an exception does not mean that a rule is not valid. These findings show that students bring experiences of everyday argumentation with them into math classes hindering the learning of mathematical reasoning. The question is, how these experiences could be used as a base upon which mathematical reasoning is built, but we do not exactly know how mathematical argumentation emerges out of everyday argumentation and how this is connected to knowledge construction. This is exactly the point on which I will focus in my study. The leading questions are:

- What elements of everyday argumentation do students use when solving mathematical problems and how does this lead to mathematical argumentation?
- How is knowledge constructed through argumentation? How can the epistemic function of arguing be described?
- How can we describe the relation between argumentation and knowledge construction?
- What components of argumentation processes foster or hinder processes of knowledge construction?

ARGUMENTATION

Schwarz et al. (2003) state that

> "constructing knowledge is a never-ending process of marshalling evidence that the chosen belief is (a) supported by the available evidence and (b) more warranted than plausible rival beliefs" (Schwarz et al. 2003, 222).

Following this statement, I assume that arguing is an epistemic action, i.e. an intentional action to gain knowledge. The construction of new knowledge arises from reasoning or checking the validity of claims. Therefore, arguing has two epistemic functions: Constructing new knowledge and/or convincing others of the validity of one's own hypothesis. The epistemic function of convincing others means: The more people one can convince, the more likely one's own hypothesis will be. In this sense, an argument is a statement that makes a hypothesis more or less likely.

The Toulmin's scheme is an appropriate tool to reconstruct the structure and deepness of argumentation processes (Krummheuer 1995, Knipping 2004) or to characterize abductive and deductive types of arguments (Pedemonte 2002). But how can we grasp the students´ more intuitive methods of concluding? Looking at philosophy and at rhetoric is worthwhile. A collection of topical schemes has the potential to identify the starting point of how mathematical inferences develop. These two tools, the Toulmin's scheme and a collection of topical schemes, will be presented in the following paragraphs.

The Toulmin´s scheme

To analyze processes of argumentation, Toulmin (1958) developed a scheme that classifies elements of an argumentation with regard to their function into data, conclusion, warrant, backing and qualifier. The conclusion is the statement that has to be reasoned. Data are unquestioned facts the conclusion is led back to. Data and conclusion are part of every argumentation. The step of inferring from data to conclusion can be explained by a warrant that sometimes is implicit. To ensure the practicability of a warrant, the warrant can be backed up by additional statements. Qualifiers or exceptions specify the validity of a conclusion.

In processes of argumentation with more than one step, conclusions that are already accepted by (most of) the other participants of the process can turn into new data. If backings, warrants or data are questioned, these elements have to be reasoned in a separate process of argumentation before they can be used in the primal argumentation. In complex processes of argumentation, Krummheuer & Brandt (2001) name these separate processes of argumentation as lines of argumentation.

A collection of topical schemes

Topical thinking means that someone has access to some basic ideas for generating arguments. These basic ideas for concluding methods are called topical schemes. In this sense, topical schemes are facilities to create steps of an argumentation or to ensure persuasively the power of argumentation steps. Ottmers (2007) presents a collection of topical schemes that is divided into two prime classes: everyday logical schemes and convention-based schemes. The everyday logical schemes contain five types: causal-based, comparison-based, contrast-based, classification-based and example-based conclusions. The first four types conclude from general to special statements (schemes of inferential nature), the last type concludes from special cases to general statements (schemes of inductive nature). Everyday logical schemes are redolent of formal logical rules. Causal-based conclusions use causal relations to ensure plausibility. Causal relations are those between cause and effect, between reason and consequence of human activities or between means and end. Comparison-based conclusions relate different parameters. They refer to equality, diversity, or more or less probable cases. Contrast-based conclusions refer to relevance between contrasts. These contrasts can be absolute, relative, or alternative. Classification-based conclusions use relations between parts and the whole issue, between species and genus, or between definition and the defined issue. Example-based conclusions use a lot of examples or examples as a type for something general. A general example that is appropriate for reasoning in the direction of generalization is called an inductive example. An example that just shows that a rule is valid in this case is called an illustrative example.

Convention-based conclusions do not use any kind of logical structures but concluding methods that are established within a group. Therefore it is not possible to present a complete collection of convention-based concluding schemes. Ottmers presents two topical schemes as examples for this prime class: authority-based conclusions and metaphor-based conclusions. Authority-based concluding means referring to another person (a special group/special institution/etc.) that is accepted as an authority in the relevant field. Metaphor-based conclusions are similar to example-based or comparison-based conclusions. They relate the conclusion to other similar cases for plausibility. There are two important differences between these concluding methods. Metaphor-based conclusions refer to one example. In contrast, example- or comparison-based conclusions regard an amount of similar cases or examples. Furthermore, the similar cases in this concluding method stem from another field and explain the conclusion in a more metaphorical way [1].

Knowledge construction

Bikner-Ahsbahs (2005) developed an epistemic action model to analyze the epistemic process in interest-dense situations in comparison to other learning situations [2]. This model consists of three epistemic actions that shape epistemic processes. These epistemic actions are gathering mathematical meanings, connecting mathematical meanings and structure-seeing. Gathering means assembling similar mathematical entities; connecting means linking a limited amount of these and other entities; structure-seeing means constructing or reconstructing a mathematical structure that refers to an unlimited number of mathematical entities.

METHODOLOGY AND METHODS

This study is embedded into the research project *"Effective knowledge construction in interest-dense situations"* that investigates processes of in-depth knowledge construction and its background conditions by linking two theories of constructing mathematical knowledge. This project is a joint study between two research teams from Israel and one team from Germany and it is supported by the German-Israeli Foundation for Scientific Research and Development (Grant 946-357.4/2006). Within this project, three different tasks have been developed that offer an opportunity to construct mathematical knowledge. In each country, three pairs of students (grade 10 in Germany, grade 11 in Israel) are video- and audiotaped when solving these tasks in a varying order. There is no teacher, however, in some instances an interviewer takes over the role of a teacher. The audio- and videodata are transcribed and translated into English to exchange them between the research teams. Including some transcripts of the project's pilot phase, there are 11 German transcripts and 12 translated transcripts at my disposal.

The transcripts will be analysed interpretatively in turn-by-turn-analyses and in several steps according to the leading questions above. The kind of student's involvement and the epistemic process is reconstructed through interpretation on three levels: the locutionary, the illocutionary and the perlocutionary level. The locutionary level consists of what is said. On this level, the construction of mathematical knowledge is reconstructable. The illocutionary level consists of what is said through an utterance (the underlying subtext) and the perlocutionary level contains the people's intentions and the impact of utterances (Davis 1980). Reconstructing the illocutionary and the perlocutionary level shows how mathematical knowledge is constructed socially (Arzarello et al. 2009).

The analyses of the argumentation processes and of the epistemic processes will be done in separate steps. First, the epistemic processes will be reconstructed by identifying the epistemic actions gathering, connecting and structure-seeing and some social actions like asking for an explanation, valuing, initiating and contrasting. A diagram will illustrate the processes of knowledge construction as well as the social processes in a compressed way. Secondly, the structure of the argumentation is reconstructed with the aid of Toulmin's scheme, and the kind of warrants the students use is characterized by means of Ottmer's collection of topical schemes. The results will be shown in a diagram as well. In a last step, these diagrams will be compared and contrasted (Prediger, Bikner-Ahsbahs & Arzarello 2008). The comparison of the results of the different analyses will shed light on relations between argumentation and knowledge construction. However, the problem to compare a diagram showing a process (the epistemic actions) with a diagram showing products (argumentation) is not answered yet. Hence, some elements of the diagrams will be developed and modified in the study.

PRELIMINARY RESULTS

Two high-achieving students (grade 10 of a German Gymnasium), Tim and Matthias, solve a task asking them to interpret the continued fraction $1+\cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{1+\cfrac{2}{\cdots}}}}$. The task is structured into two parts. In the first part, the students are asked to find the first 7 elements of the sequence, reflect on how they computed them, extend the computation to 20 elements and write them as simple as well as decimal fractions. In the second part, the students are asked to make a conjecture on the sequence from the first part and justify their conjecture.

In the case of Tim and Matthias, the transcript can be divided into main three parts according to the task. The first main part of the transcript corresponds with the first part of the task (line 31 – 718). In this part, Tim and Matthias compute the elements of the sequence very accurately always checking that they made no miscalculation. Their accurate and precautious way of solving the task becomes apparent in the following parts as well. In the second main part, Tim and Matthias make conjectures on the sequence (line 719 – 1465); in the third main part (line 1466 – 2630) they try to justify their conjectures whereupon the interviewer directs the students´ focus on some aspects. This paper presents the analysis of the second main part of the transcript that can illustrate some interesting relations between argumentation and knowledge construction.

When Tim and Matthias are asked to make conjectures on the sequence, they focus on the decimal places. They look at the elements of the sequence from f(7) on and observe that the amount of same decimal places increases.

729 Matthias: […] ,wait here were only three zeros (points at the following on the sheet) ,then three nines ,then three zeros again ,then four nines ,then four zeros ,then four nines again ,so always after […]

731 /Matthias: always after- ,always after three. ,it becomes ,it becomes one more.

Matthias gathers information of their previous work (three zeros, three nines ...). This information is unquestioned and can be used as data later on. When gathering the information, Matthias uses words like “only”, “then” and “again”. This is a hint that he already combines these data in order to make a conjecture. Matthias infers example-based that after every third element of the sequence the amount of nines respectively zeros behind the decimal point increases by one. Tim starts counting the amount of nines respectively zeros behind the decimal point from the first element of the sequence on. He realizes that the results are not in line with Matthias´ conjecture. But instead of abandoning this conjecture now, they first check if they have made any mistake. They find that they did no miscalculation, and therefore they change the conjecture a bit:

744 Tim: yes at least ,one notices that always (.) ,always three nines ,three zeros ,the amount of nines and zeros is identical ,and then it always switches after three or four.

The whole second main part of the transcript (line 719 – 1465, approximately 30 minutes) consists of making conjectures on the sequence and sharpening the formulation of these conjectures. All the conjectures are associated. They do not contradict, but deal with different aspects of the sequence. For lack of space it is not possible to show all diagrams of the second main part. Some selected parts are presented to illustrate important observations. Figure 1 shows how Tim and Matthias develop a new aspect concerning their conjecture.

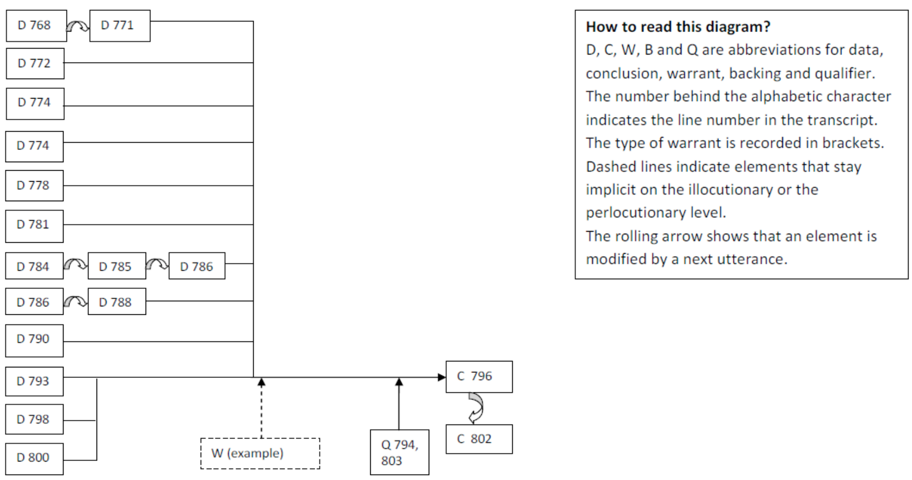

Figure 1: diagram of line 768 - 803

Tim and Matthias start gathering a lot of data. Some data is modified. They combine the data immediately what is shown by words like “then”, “here again”, etc. They realize that f(8) till f(10) have two nines or zeros behind the decimal point, f(11) till f(13) have three nines or zeros and f(14) till f(17) have five nines or zeros behind the decimal point. F(8) till f(10) and f(11) till f(14) are rows of three elements with the same amount of nines or zeros behind the decimal point. F(14) till f(17) is a row of four elements with the same amount of nines or zeros behind the decimal point [3]. From these data, they infer example-based.

796 Tim: I would say ,ohm- ,as soon as it is a row of three four five whatever ,ohm- ,lets just say a row of five ,o- or no ,lets say a row of four ,and twenty-five is inside of it ,so five squared ,from then on ,are ,five nines ,five times

They sharpen their formulations and even create a new term in order to specify what they mean. The term "space of places" describes a space (some following elements of the sequence) where the amount of nines or zeros behind the decimal point is the same. For instance, f(4) till f(7) is a space of places as there is one nine respectively one zero behind the decimal point. F(8) till f(10) is the following space of places. With the aid of this term, they develop and clarify their conjectures.

During the whole second main part of the transcript, Tim and Matthias write down their conjectures in a very accurate and careful manner. They always discuss their formulation and do not note it down until both agree on it. Here is what they wrote down as their conjectures:

> "The amount of zeros or nines behind the decimal point is the same in a particular space of places. If you go from one to another space of places, the amount of nines or zeros increases by another nine or zero. If there are nines or zeros behind the decimal point, depends on the x-value. If the x-value is even, there are nines behind the decimal point and a one in front of it. If the x-value is odd, there are zeros behind the decimal point and a two in front of it. The length of the space of places changes to c as soon as the space of places contains c^2."

All of these conjectures are inferred example-based. Before a new aspect of a conjecture arises, Tim and Matthias gather a lot of data. In the following, they sharpen and modify this conjecture. A characteristic argumentation line during this part is shown in figure 2.

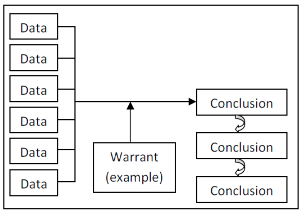

Figure 2: characteristic argumentation line in part 2 of the transcript

Obviously, several data in the form of examples are necessary for example-based inferences. However, such argumentation lines with a high concentration of data do not seem to be characteristic only for example-based inferences. In another case study (see Cramer 2010), the first argumentation lines contained a lot of data as well. The conclusions in this case were inferred causal-based. This could be a hint, that in general a high concentration of data is characteristic for argumentation lines in the beginning. This result corresponds with the finding of interest-dense situations, that gathering phases until a certain saturation level are necessary to provide a basis for further increases of knowledge.

The fact that conjectures arise example-based is obvious as well. It is a heuristic strategy stemming from everyday life and the way how discovery learning works. The question is how to encourage students to justify their conjecture in a mathematical way. Tim and Matthias insist several times that their conclusions are conjectures. They believe that they are valid, but they are not sure about. The way they become more and more sure in the second part of the transcript stems from everyday life as well. When they observe that nines and zeros behind the decimal point alternate as well as one and two in front of the decimal point, they take the alternation as an argument for the validity of their conjecture.

1000	Tim:	Yes ,that always switches ,thats why it is logical

The implicit argument is based on previous experiences. The students are asked to look for patterns and regularities. Something alternating is a pattern, therefore it is more likely that their conjecture is true. Ottmers does not mention experience-based conclusions. It could be a type of example-based conclusions in the sense that previous experiences are kind of examples. But the related examples are not mentioned explicitly. Therefore it is not possible to check the relevance of these experiences, and for this reason I would describe it as a convention-based scheme. Another example of this concluding scheme appears later on. Tim and Matthias realizes that the space of places from f(4) till f(7) has a length of four. This is not in line with their conjecture that the length of the space of places changes to c as soon as the space of places contains c^2. Tim would like to solve the problem in this way:

1345	Tim:	Yes exc- ,exception ,or then write exceptions prove the rule next to it

This is an everyday proverb that even contradicts to mathematical argumentation. This finding shows that Tim and Matthias are influenced by everyday argumentation on the one hand. On the other hand, they insist that their rule has the status of a conjecture. This could be a hint that they are aware of the fact that mathematical argumentation works different. A little episode of the third part of the transcript illustrates this awareness. At the end of the second part they mentioned that the sequence tends to two. They are now working on a justification. Matthias remembered that $0,\overline{9}1=$ and justified this fact authority-based referring to his teacher.

1446	Matthias:	So one found out that ohm- ,on(e)- one say ,our teacher told us that ohm- ,one point nine period equals two.

Such a convention-based conclusion seems to be insufficient or unsatisfactory, therefore Matthias justifies it classification-based.

1449	Tim:	[...] ,because one plus nine ninth is precisly two ,but nine ,one ninth ,is zero point one one one one one

Tim and Matthias use convention-based schemes seldomly. Whenever they use these schemes yet, they give up these argumentations lines quickly. This is a hint that they regard these concluding methods as insufficient or unsatisfactory as convention-based schemes do not lead to deeper understanding why a conclusion is valid.

CONCLUSIONS

The combination of the Toulmin´s scheme and a collection of topical schemes turned out to be an appropriate tool to describe argumentation and to identify elements of everyday argumentation. In this case, everyday argumentation was the starting point to develop mathematical conjectures. Everyday logical schemes can turn into strategies to come to conjectures and to create ideas of how to justify them. The analysis of the episode presented in this paper focuses on this aspect. To find out more about typical argumentation structures and relations between knowledge construction and argumentation, analysis of different episodes have to be compared. A first hint concerning relations between knowledge construction and argumentation is that argumentation lines with a high concentration of data prevail in the beginning of epistemic processes. This corresponds to phases of gathering and combining in terms of the epistemic action model. In structure-seeing phases, argumentation lines occur where warrants and backings are formulated and sharpened. In the future, several case studies will be compared to deliver deeper insights into relations between argumentation and knowledge construction.

NOTES

1. Ottmers calls these metaphor-based conclusions analogy-based ones. His example for such an analogy-based conclusion makes clear that these analogies are metaphorical. As analogy is understood in a different way in mathematics, I decided to call these conclusions metaphor-based ones.

2. In this study, Bikner-Ahsbahs´ epistemic action model is used to describe processes of knowledge construction. The emergence of interest-dense situations will not be analysed. Therefore, I do not enlarge upon this term here. For further information about the theory of interest-dense situations see Bikner-Ahsbahs 2004; 2005.

3. To follow the excerpts of the transcript, here are the elements of the sequence presented as decimal fractions:

	Decimal fraction		Decimal fraction		Decimal fraction		Decimal fraction		Decimal fraction
f(0)	1	f(4)	1,909090909	f(8)	1,994152047	f(12)	1,999633834	f(16)	1,999977112
f(1)	3	f(5)	2,047619048	f(9)	2,002932551	f(13)	2,000183117	f(17)	2,000011444
f(2)	1,666666667	f(6)	1,976744186	f(10)	1,998535871	f(14)	1,99990845	f(18)	1,999994278
f(3)	2,2	f(7)	2,011764706	f(11)	2,000732601	f(15)	2,000045777	f(19)	2,000002861

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