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Dynamic Parameter Identification for Parallel Manipulators

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1. Introduction

The information provided by robot manufacturers regarding the dynamic parameters of robotic systems (the inertial properties of the links and friction parameters at the kinematic joints) is limited and even nonexistent. For instance, friction parameters are generally not provided. Thus, it is necessary to develop efficient procedures for their measurement. The direct measurement of these parameters is not practical since it would imply disassembling the robot. On the other hand, obtaining these parameters from the CAD models has the disadvantage that some parts can not be modeled in full detail and parameters that depend on operational conditions, like friction, can not be determined. For these reasons, parameters identification has turned out to be a widely accepted technique for determining the dynamic parameters. This chapter provides an overview of parameters identification processes applied to parallel manipulators. Practical implementation issues are also considered. In addition, an approach that considers the identification problem as a nonlinear constrained optimization problem is presented. Moreover, an evaluation of the accuracy of the solution of parameters is also addressed.

The importance of inertial and friction parameters lies in their application in most of the recent literature for advanced model-based control algorithms. The accuracy of dynamic parameters plays an important role in the precision, performance, stability and robustness of these control algorithms (Khalil & Dombre, 2002). On the other hand, they are important in dynamic simulation. It is known that the validation of the direct dynamic problem depends considerably on the precision of the dynamic parameters of the mechanical system. An accurately modeled robot permits the substitution of the real mechanical system by a virtual one thus avoiding the expensive experimental tests used to adjust the operational parameters for this system (Hiller et al., 2002). Additionally, another important field in which accurate knowledge of the dynamic parameters is needed is in path planning

algorithms that take into account robot dynamics. The predicted forces depend greatly on the accuracy of the estimated inertial and friction parameters. Hence, inaccurate estimates of the dynamic parameters may lead to an overloaded robot (dynamically or statically), which is the case in approximately 50% of industrial robots (Swevers et al., 2002).

Initially, dynamic parameter identification procedures for estimating the dynamic parameters of open loop mechanical structures were developed in the middles eighties (Khosla & Kanade, 1985; Atkeson et al., 1986; Olsen & Bekey, 1986; Gautier & Khalil, 1988). Since then, they have been widely used and several contributions to serial robot dynamics application control and simulation have been made.

Identification procedures can be classified into two main groups: indirect and direct approaches. On the one hand, indirect procedures act sequentially in several stops. In each step, parameters of a different nature (basically friction and some inertial terms) are identified by means of specifically designed experiments. On the other hand, in the direct approach, all the parameters are identified in the same stage. A detailed comparison between the direct and the indirect approaches, applied to a PUMA industrial robot, can be found in Benimeli et al. (2006). The indirect approach has the disadvantage that errors due to the noise in the measured data are being accumulated throughout the different stages (Khalil & Dombre, 2002). Moreover, it is difficult to maintain the working conditions constant not only throughout these stages, but also within the same one.

For parallel manipulators, the direct approach has been applied (Renaud et al., 1993; Guegan et al., 2003; Farhat et al., 2008). Meanwhile, the indirect approach has been proposed (Grotjahn et al., 2004; Abdellatif et al., 2007). However, apart from error accumulation in each step, the separation of the parameters of a different nature is not straightforward as for open chain manipulators. Due to the fact that the direct approach allows parameters identification in one single experiment, removing the accumulation of error between steps, this chapter will be focus on the direct approach applied to parallel manipulators.

The first part of the chapter deals with conventional direct dynamic parameters identification processes. Thus, the dynamic model, suitable for identification purposes, is developed in its linear form with respect to the dynamic parameters. Due to the fact that the number of parameters is usually greater than the dimension of the equations of motion, an overdetermined system is developed. This overdetermined system is rank deficient, therefore it has to be reduced to another equivalent system that only contains independent columns. These columns correspond to a subset of parameters called the base parameters. Reduction process can be held symbolically (Khalil & Bennis, 1995) or numerically (Gautier, 1991). For experiment design and in order to reduce the sensitivity of the system to the noise signal, procedures have been developed for the trajectories to be executed by the manipulator (Gautier & Khalil, 1992; Swevers et al., 1997). Finally, the dynamic system in its reduced matrix form is solved for the base parameters using the Least Square Method (LSM).

The parameters dynamic identification procedure outlined in the previous paragraph has two main disadvantages: firstly, results could contain non-physically feasible parameters and secondly, it is also limited to linear friction models. Non-physical feasibility can be detected by obtaining a base parameters solution that does not have any physical interpretation when compared with corresponding combinations of the inertial parameters; masses lower than zero or non positive-definite local inertial matrices (Yoshida & Khalil, 2000). This issue not only affects the stability of some of the advanced model-based control algorithms, but is also crucial in the dynamic simulation tasks.

The second part of the chapter will focus on two identification procedures. First, a procedure based on the parameters identification formulated as a nonlinear constrained optimization problem is reviewed. This approach allows not only the implementation of nonlinear friction models to model friction phenomenon at robot joints, but also the consideration of constraint equations in order to ensure the physical feasibility of the identified parameters. The second procedure is established upon the accuracy of the parameter solution, which is called here the *identifiability* of the parameters. Experiments will be held on a class of parallel manipulator. The main conclusions and further research concerning parameters identification for parallel manipulators are presented at the end.

2. Dynamic model

The starting point of the identification process depends on obtaining the dynamic model of the mechanical structure in its linear form with respect to the inertial and friction parameters. For this purpose, the dynamic model can be developed basically by two methods (Kozlowski, 1998); the integral and the differential methods. The integral method is derived from the energy equation and requires measurements of positions, velocities and applied forces on the actuated joints. Measurements of accelerations are not required. This method has been applied on serial manipulators (Gautier & Khalil, 1988; Sheu & Walker, 1989; Khalil et al., 1990; Sheu & Walker, 1991) and extended for parallel manipulators (Bhattacharya et al., 1997). Olsen and others also used the integral method (Olsen & Petersen, 2001; Olsen et al., 2002) where they proposed the use of the *maximum likelihood* method instead of the conventional Least Squares Method (LSM) in the identification process.

On the other hand, the differential method takes the advantage of the equations of motion as a base in the development of the identification process algorithms. As a result, acceleration appears explicitly and needs to be measured. It is known that the equations of motion can be constructed implementing various dynamic principles. Models suitable for the identification process have been developed by means of; the Newton-Euler formulation (Luh et al., 1980; Atkeson et al., 1986; Olsen & Bekey, 1986; Khosla, 1989), the Lagrange formulation (Ha et al., 1989; Sheu & Walker, 1991), Jourdain's principle of Virtual Power (Grotjahn et al., 2004), Gibbs-Appell equations of motion (Benimeli et al., 2003) and recently (Hardeman et al., 2006), a finite element based approach along with Jourdain's principle of Virtual Power was used to develop an automatic generation of the dynamic models for identification.

In order to study the advantages/disadvantages of the integral and differential methods, a comparison was carried out. The experiments were held considering a two degrees of freedom serial manipulator (Prufer et al., 1994). From the results, they concluded that the differential method has advantages over the integral one. Thus, the differential model is used here for parameters identification of parallel manipulators.

2.1 Rigid body model
The equation of motion that describes the dynamic behavior of an open chain manipulator can be obtained by means of Gibbs-Appell equations of motion. For a serial robot (Mata et al., 2002), the rigid body dynamic model can be written as follows,

$$\tau_k = \sum_{i=k}^{n} \left\{ \left(\frac{\partial \, ^i\dot{\vec{\omega}}_i}{\partial \, \ddot{q}_k} \right)^T \cdot \left[^i\mathbf{I}_{G_i} \cdot \, ^i\dot{\vec{\omega}}_i + \, ^i\vec{\omega}_i \times (^i\mathbf{I}_{G_i} \cdot \, ^i\vec{\omega}_i) \right] \right\} + \sum_{i=k}^{n} \left[\left(\frac{\partial \, ^i\ddot{\vec{r}}_{G_i}}{\partial \, \ddot{q}_k} \right)^T \cdot m_i \cdot \, ^i\ddot{\vec{r}}_{G_i} \right] \tag{1}$$

where τ_k and $\ddot{q}_k$ are the generalized forces and accelerations of the joint k, respectively, $^i\dot{\vec{\omega}}_i$ is the angular acceleration, and $^i\ddot{\vec{r}}_{G_i}$ is the acceleration of the center of gravity, m_i is the mass and $^i\mathbf{I}_{G_i}$ is the inertial tensor of the center of gravity, and the superscript/subscript (i) stands for the link number. All of them expressed with respect to the ith local reference frame. The Denavit-Hartenberg modified convention has been considered for modeling the system and i = k..n indicates the sum over all links above joint k, including itself.

For dynamic parameters identification, a linear form with respect to dynamic parameters is necessary. To this end (Atkeson et al., 1986), the linear acceleration of the center of gravity of the ith body is expressed as a function of the linear acceleration of the link coordinate frame i^{th}. Moreover, the link inertial tensor is also expressed about the link coordinate frame by means of parallel axis theorem.

In addition, the following notations are introduced. On the one hand, the cross product $\vec{a} \times \vec{b}$ is expressed using the skew symmetric matrix $\tilde{a}$. Thus, $\vec{a} \times \vec{b} = \tilde{a} \cdot b$ where,

$$\tilde{a} = \begin{pmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{pmatrix} \text{ and } \vec{b} = \begin{bmatrix} b_x & b_y & b_z \end{bmatrix}^T \tag{2}$$

On the other hand, $\mathbf{B} \cdot \vec{a} = \hat{a} \cdot \vec{B}$ where,

$$\hat{a} = \begin{pmatrix} a_x & a_y & a_z & 0 & 0 & 0 \\ 0 & a_x & 0 & a_y & a_z & 0 \\ 0 & 0 & a_x & 0 & a_y & a_z \end{pmatrix} \text{ and } \vec{B} = \begin{bmatrix} B_{xx} & B_{xy} & B_{xz} & B_{yy} & B_{yz} & B_{zz} \end{bmatrix}^T \tag{3}$$

By doing the above mentioned, upon substituting (2)-(3) in (1) and using some vector identities, the dynamic model linear with respect to dynamic parameters can be written as (Mata et al., 2005),

$$\tau_k = \begin{cases} ^k\vec{z}_k^T \cdot \sum_{i=k}^{n} {}^k\mathbf{R}_i \left[{}^i\hat{\eta}_i \cdot {}^i\vec{I}_i + \left({}^i\tilde{r}_{O_k O_i} \, {}^i\tilde{\eta}_i - {}^i\ddot{\vec{r}}_{O_i} \right) \cdot m_i \, {}^i\vec{r}_{O_i G_i} + {}^i\tilde{r}_{O_k O_i} \, {}^i\ddot{\vec{r}}_{O_i} \cdot m_i \right] & k:R \\ \\ ^k\vec{z}_k^T \cdot \sum_{i=k}^{n} {}^k\mathbf{R}_i \cdot \left(m_i \cdot {}^i\ddot{\vec{r}}_{O_i} + {}^i\tilde{\eta}_i \cdot m_i \, {}^i\vec{r}_{O_i,G_i} \right) & k:P \end{cases} \tag{4}$$

where $^i\hat{\eta}_i = \left({}^i\dot{\vec{\omega}}_i + {}^i\tilde{\omega}_i \, {}^i\hat{\omega}_i \right)$ y $^i\tilde{\eta}_i = \left({}^i\tilde{\omega}_i \, {}^i\tilde{\omega}_i + {}^i\dot{\tilde{\omega}}_i \right)$.

In equation (4), $^i\vec{r}_{O_i G_i}$ locates the center of mass of the ith link with respect to its own reference system. $^i\ddot{\vec{r}}_{O_i}$ is the acceleration of the origin of the ith local reference system. $^k\mathbf{R}_i$ is the rotation matrix between the ith and k^{th} reference systems. $^i\tilde{r}_{O_k O_i}$ is the position vector from the ith reference system to the kth reference system with respect to the ith reference

system. ${}^{k}\vec{z}_{k} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}^{T}$. P and R denote the type of the corresponding joint, revolute or prismatic, respectively. Equation (4) can be written in the following matrix form,

$$\mathbf{K}\left(\bar{q},\dot{\bar{q}},\ddot{\bar{q}}\right) \cdot \vec{\Phi}_{rb} = \bar{\tau} \tag{5}$$

K can be denoted as the observation matrix of a single configuration; this matrix depends on the generalized kinematic variables. The vector of dynamic parameters $\vec{\Phi}_{rb}$ regroups the elements of the inertia tensor $\begin{bmatrix} I_{xx} & I_{xy} & I_{xz} & I_{yy} & I_{yz} & I_{zz} \end{bmatrix}^{T}$, calculated with respect to the local reference frame, the mass m and the first mass moments with respect to the center of gravity $\begin{bmatrix} mx & my & mz \end{bmatrix}^{T}$, for all the bodies contained in the system.

Equation (5) can be applied for the dynamic parameter identification of open chain manipulators. Under other circumstances, such as parallel manipulators, its application is not straightforward. For parallel manipulators, the dynamic model can be obtained by making a cut at one or more joints so that the manipulator can be dealt with as an open-chain mechanical systems with a tree structure. By doing so, equation (5) can be applied for the several open chain mechanical systems obtained after the cut. However, the constraint equations representing the union at the cutted joints should be fulfilled. These equations have the following form,

$$f_{i}(q_{1},q_{2},...,q_{n}) = 0 \quad i = 1,2,...,m \tag{6}$$

where $(q_{1},q_{2},...,q_{n})$ are the generalized coordinates and *m* is the number of independent constraint equations. The degree of freedom (n_{DOF}) of the system is obviously (*n-m*). Taking the first and the second time derivatives of the previous equations, the following the acceleration constraint equations are obtained,

$$\mathbf{A}(\vec{q}) \cdot \ddot{\vec{q}} - \vec{b}(\vec{q},\dot{\vec{q}}) = \vec{0} \tag{7}$$

where **A** is the Jacobian matrix of the constraint equations with respect to the generalized coordinates, and $\vec{b}$ is a vector that contains all the terms that remain after removing all the acceleration dependent terms from the acceleration constraint equations. Regrouping the terms of the matrix A, according to the coordinated partition, in independent/dependent generalized accelerations,

$$\begin{bmatrix} \mathbf{A}_{i} & \mathbf{A}_{e} \end{bmatrix} \begin{bmatrix} \ddot{\vec{q}}_{i} \\ \ddot{\vec{q}}_{e} \end{bmatrix} = \vec{b} \tag{8}$$

where $\mathbf{A}_{i}$ and $\mathbf{A}_{e}$ are obtained when the above mentioned coordinated partition is applied to the Jacobian matrix of the constraint equations.

Similarly, regrouping equation (5) but this time according to the independent and dependent generalized coordinates,

$$\mathbf{K}_{i} \cdot \vec{\Phi}_{rb} = \bar{\tau}_{i} \quad \mathbf{K}_{e} \cdot \vec{\Phi}_{rb} = \bar{\tau}_{e} \tag{9}$$

The subindices i and e refer to independent and dependent generalized coordinates respectively.

In a similar way to that introduced in (Udwadia & Kalaba, 1998), and starting from equation (4) and equation (5), it can be proved that the dynamic equation for parallel manipulators in its linear form with respect to the dynamic parameters can be written in the form,

$$\left[\mathbf{K}_i - \mathbf{X}^T \cdot \mathbf{K}_e\right] \cdot \vec{\Phi}_{rb} = \vec{\tau}_i - \mathbf{X}^T \cdot \vec{\tau}_e \tag{10}$$

where $\mathbf{X} = \mathbf{A}_e^{-1} \cdot \mathbf{A}_i$.

If the dependent generalized forces correspond to passive joints then, they can be dropped form the equation. Hence, equation (6) is reduced to,

$$\left[\mathbf{K}_i - \mathbf{X}^T \cdot \mathbf{K}_e\right] \cdot \vec{\Phi}_{rb} = \vec{\tau}_i \tag{11}$$

The observation matrix for a given trajectory can be found by appending this equation over all the configurations (n_{pts}) of the trajectory. This gives,

$$\begin{bmatrix} \left[\mathbf{K}_i - \mathbf{X}^T \cdot \mathbf{K}_e\right]_1 \\ \left[\mathbf{K}_i - \mathbf{X}^T \cdot \mathbf{K}_e\right]_2 \\ \vdots \\ \left[\mathbf{K}_i - \mathbf{X}^T \cdot \mathbf{K}_e\right]_{n_{pts}} \end{bmatrix} \cdot \vec{\Phi}_{rb} = \begin{bmatrix} \left[\vec{\tau}_i\right]_1 \\ \left[\vec{\tau}_i\right]_2 \\ \vdots \\ \left[\vec{\tau}_i\right]_{n_{pts}} \end{bmatrix} \tag{12}$$

The left-hand side of this equation is the observation matrix for a given trajectory ($\mathbf{W}_{rb}$) and the right-hand side is the corresponding applied forces ($\vec{\tau}$), or in a compact form,

$$\mathbf{W}_{rb}\left(\vec{q}, \dot{\vec{q}}, \ddot{\vec{q}}\right) \cdot \vec{\Phi}_{rb} = \vec{\tau} \tag{13}$$

2.2 Friction models

Several friction models have been proposed in the literature (Olsson et al., 1998). The classical friction model used for identification is a linear model which includes Coulomb and viscous friction. Equation (14) represents a general asymmetrical linear model (Armstrong, 1988) for both Coulomb and viscous friction,

$$F_f = \begin{cases} F_c^+ + F_v^+ \dot{q} & \dot{q} > 0 \\ -F_c^- - F_v^- \dot{q} & \dot{q} < 0 \end{cases} \tag{14}$$

where F_c and F_v stand for the Coulomb and viscous friction coefficients, respectively. (+ve) and (-ve) superscripts correspond to the velocity sign. Considering different coefficients for the different velocity sign, this model is asymmetrical. Applying the friction model to all joints and for all the configurations (n_{pts}),

$$\mathbf{W}_f\left(\dot{\vec{q}}\right) \cdot \vec{\Phi}_f = \vec{\tau}_f \tag{15}$$

The previous equation is applicable in the case of linear friction model. In the other case, if the friction at joints seems to have nonlinear tendency, nonlinear friction models can be used. In the identification process different nonlinear friction models have been used. For example the following models (Grotjahn et al., 2001),

$$F_2(v) = F_C + F_{v_1} v + F_{v_2} \, atan(F_{v_3} v) \tag{16}$$

where, F_C, F_{v_1}, F_{v_2} and F_{v_3} are friction model parameters. On the other hand, another model proposed (Farhat, 2006) for friction modeling has the form,

$$F_f = \begin{cases} F_c^+ + F_v \, |v|^\delta & v > 0 \\ -F_c^- - F_v \, |v|^\delta & v < 0 \end{cases} \tag{17}$$

where, F_c , F_v are Coulomb and viscous parameters and δ a geometry dependent variable that takes into account the *Stribeck effect*.

2.3 Actuator dynamics

In some cases, a considerable part of the actuator torque is consumed by accelerating or decelerating its rotor inertia (J_{ri}) and its driven system (for instance, a ball screw drive , J_s). Then, the rotor and the driving system inertia have to be considered. The corresponding equation for the actuator of the ith joint **can** be written as follows,

$$\tau_{ri} = (J_{ri} + J_s) \cdot \ddot{q}_i \tag{18}$$

Equations (18) is linear. The actuator dynamic for all the joints and for all the configurations (n_{pts}) can be expressed in matrix form as,

$$\mathbf{W}_r \left(\ddot{\vec{q}} \right) \cdot \vec{\Phi}_r = \vec{\tau}_r \tag{19}$$

2.4 Complete robot model

If only linear friction models are considered in the identification process, equations (13), (15), and (18) can be grouped. Thus the complete dynamic model of the manipulator can be express as follows,

$$\begin{bmatrix} \mathbf{W}_{rb} & \mathbf{W}_f & \mathbf{W}_r \end{bmatrix} \begin{bmatrix} \vec{\Phi}_{rb}^{\,T} & \vec{\Phi}_f^{\,T} & \vec{\Phi}_r^{\,T} \end{bmatrix}^T = \mathbf{W} \cdot \vec{\Phi} = \vec{\tau} \tag{20}$$

In equation (20), $\mathbf{W}$ is the observation matrix of the system and $\vec{\Phi}$ is the vector grouping all the dynamic parameters. In the case that nonlinear friction models are considered, the complete dynamic model of the manipulator can be expressed in the form,

$$\begin{bmatrix} \mathbf{W}_{rb} & \mathbf{W}_r \end{bmatrix} \begin{bmatrix} \vec{\Phi}_{rb}^{\,T} & \vec{\Phi}_r^{\,T} \end{bmatrix}^T + \left(\vec{F}_{f_i} - \mathbf{X}^T \cdot \vec{F}_{f_e} \right) = \mathbf{W} \cdot \vec{\Phi} + \left(\vec{F}_{f_i} - \mathbf{X}^T \cdot \vec{F}_{f_e} \right) = \vec{\tau} \tag{21}$$

where $\vec{F}_{f_i}$ and $\vec{F}_{f_e}$ stand for the friction in the independent and dependent joints, respectively. These vectors include the friction parameters ($\vec{\Phi}_f$). $\vec{\Phi}$, in this case, contains the body and the actuator dynamic parameters.

2.5 Base parameters

An important characteristic of the system expressed by equation (20) is that some inertial parameters do not affect the dynamics of the manipulator and others have a relative effect with respect to the other parameters on the external generalized forces. Mathematically, this can be expressed by the presence of zero columns in the observation matrix (**W)** and the dependence that exist between others. Hence, this system never could be considered as a determined one. The minimal number of parameters that are needed to determine uniquely the dynamics of the system has to be calculated. This set of parameters is known as the base parameters and can be considered as a combination with the others that have an effect. The number of base parameters is equal to the rank of the observation matrix.

The base parameter combination can be calculated principally in two ways; analytically (Khalil & Bennis, 1995) or numerically using the Singular Values Decomposition (SVD) or QR factorization (Gautier, 1991). The analytical analysis will not be considered here since for a parallel manipulator its application is not direct. Analytical calculation of the base parameters have already been presented for close chains mechanical systems (Khalil & Bennis, 1995), however, the method is applicable only to some particular topologies. The use of SVD is characterized by its precision, while the QR factorization by its low computational cost. The first case is of interest to us since the precision of the resulting inertial parameters identified is more important.

As an example, consider the 3-DOF RPS (revolute, prismatic and spherical joints) parallel manipulator depicted in Fig. (1), this manipulator (a virtual model and an actual one) are used here for the experimental evaluation of the dynamic parameter identification process.

As can be seen in the figure, this parallel manipulator consists of a fixed base and a moving platform interconnected by three RPS limbs. The axes of rotation of the revolute joints are assumed to share the same plane of the base. Spherical joints are modeled as three successive revolute joints with the corresponding axes of rotation passes through the center of the spherical joint. The linear motion of each prismatic joint of the actual parallel robot is achieved through a ball screw linear actuator driven by a DC motor.

This parallel robot consists of 7 bodies: 3 limbs in which two of then contain two bodies and one with 3 bodies including the platform. Then, considering linear friction models and for a trajectory of n_{pts} configurations, the manipulator dynamic model in its linear form is appended in the following matrix form,

$$\mathbf{W}_{(n_{DOF} \cdot n_{pts}) \times (70 + n_{fric} + n_J)} \cdot \vec{\Phi}_{(70 + n_{fric} + n_J) \times 1} = \bar{\tau}_{(n_{DOF} \cdot n_{pts}) \times 1} \qquad (22)$$

As described before and because of the dependence between the inertial parameters the SVD has been used to reduce this model to its base form (reduced form) expressed by the following equation,

$$\mathbf{W}_{red}\left(\vec{q}, \dot{\vec{q}}, \ddot{\vec{q}}\right) \cdot \vec{\Phi}_{base} = \bar{\tau} \qquad (23)$$

where $\mathbf{W}_{red}$ is the reduced matrix and $\vec{\Phi}_{base}$ is the base parameters vector. This vector is composed of the 25 parameters listed in Table (1). It has been found that friction parameters, as well as the screw and rotor inertial parameters, are linearly independent.

Fig. 1. The 3-DOF RPS parallel robot; ADAMS model on the left and Actual 3-RPS parallel robot on the right (built at the Polytechnic University of Valencia).

No	Base Parameters	No	Base Parameters
1	mx(1)	14	mx(4)
2	my(1)*, **	15	my(4)*
3	Izz(1)+Iyy(2)*, **	16	Iyy(5)+ Izz(4)*
4	mx(2)	17	m(5)-2.531my(3)+m(3)+m(2)*,**,†
5	mz(2)* , **	18	mx(5)
6	Ixx(3)-0.3952my(3)* , **	19	mz(5)*
7	Ixy(3)+0.2282my(3)	20	mx(6)
8	Ixz(3)		
9	Iyy(3)+0.3952my(3)-0.2082(m(3)+m(2))　　* , **	21	my(6)*
10	Iyz(3)	22	Iyy(7)+ Izz(6)*
11	Izz(3)-0.2082 (m(3)+m(2))* , **	23	m(7)+2.531my(3)* , **,†
12	mx(3)+0.5774my(3)　　　　　　　-0.4563(m(3)+m(2))　　* , **,†	24	mx(7)
13	mz(3)	25	mz(7)*

Table 1. Rigid body base parameters of a 3-RPS robot.

3. Experiment design

An accurate and efficient dynamic parameter identification process requires special design experiments. The trajectories to be executed by the robot not only have to be able to reduce the sensitivity of the identification solution to the noise signal, but also the data processing (position and forces) needs to be kept as simple and accurate as possible.

The condition number of a matrix can be considered as an upper limit for input-output error transmissibility (Horn & Johnson, 1985). Therefore one of the criteria for designing a "well excited" trajectory is to minimize the condition number of the matrix $\mathbf{W}_{red}$. This can be treated as an optimization process where the objective function can be written as,

$$f\left(\vec{q},\dot{\vec{q}},\ddot{\vec{q}}\right) = \text{Cond}\left(\mathbf{W}_{red}\right) \tag{24}$$

As can be observed from equation (24), the variables of the optimization process are the generalized coordinates and there time derivatives. Several approaches have been proposed (Armstrong, 1989; Gautier & Khalil, 1992; Presse & Gautier, 1993) in which the trajectory is parameterized. However, the finite Fourier series trajectory parameterization proposed by (Swevers et al., 1996) is the most widely implemented.

$$
\begin{aligned}
q_i(t) &= q_{i0} + \sum_{j=1}^{n_H}\left[\frac{a_{ij}}{2\pi f \cdot j}\sin\left(2\pi f \cdot j \cdot t\right) - \frac{b_{ij}}{2\pi f \cdot j}\cos\left(2\pi f \cdot j \cdot t\right)\right] \\
\dot{q}_i(t) &= \sum_{j=1}^{n_H}\left[a_{ij}\cos\left(2\pi f \cdot j \cdot t\right) + b_{ij}\sin\left(2\pi f \cdot j \cdot t\right)\right] \\
\ddot{q}_i(t) &= \sum_{j=1}^{n_H}\left[-2\pi f \cdot j \cdot a_{ij}\cdot\sin\left(2\pi f \cdot j \cdot t\right) + 2\pi f \cdot j \cdot b_{ij}\cdot\cos\left(2\pi f \cdot j \cdot t\right)\right]
\end{aligned}
\tag{25}
$$

where t is the time, q_{i0}, a_{ij} and b_{ij} are the coefficients of the Fourier series that will be the new variables of the optimization process, n_H is the harmonic number and f is the fundamental frequency.

Generally, there exist many limitations on the movement of the actual manipulator. These can be converted to constraints in the optimization process. Depending on their complexity with respect to the variables of the optimization process they could be linear or nonlinear. For example, limitations on the displacements and the velocity of the prismatic joints (actuators) – if exist – are linear, meanwhile limitations to avoid the singularity regions or physical constraints such as the aperture angle of the spherical joints are nonlinear.

It will be important to mention that the condition number of the observation matrix is not the only criteria for finding exciting trajectories. In a statistical frameworks the covariance matrix of the maximum likelihood has been used (Swevers et al., 1997). The criteria for optimization is called *d-optimality criterion* and has the following form,

$$f\left(\vec{q},\dot{\vec{q}},\ddot{\vec{q}}\right) = -\log\left(\det\left(\mathbf{W}^t\Sigma^{-1}\mathbf{W}\right)\right) \tag{26}$$

where Σ is the diagonal covariance matrix of the measured actuator forces. Recently, this criteria was used for the case in which the parameterization of the trajectories was based on combining Fourier series and polynomial functions (Park, 2006).

As mentioned previously above, the exciting trajectories must allow simple and accurate data processing. This can be achieved if measurement position is fitted to the finite Fourier series.

For the considered parallel manipulator presented in the previous section, the exciting trajectories were obtained according to the criteria of minimization of the condition number of the reduced observation matrix. The trajectory was parameterized by means of a finite Fourier series with 11 harmonics functions. Limitations on the movement of the actual manipulator that were introduced as constraints in the optimization process can be outlined as follows,

1. Limitations on the stroke of the linear actuators, upper and lower bounds.
2. Limitations on the velocities of the linear actuators, upper and lower bounds.
3. Limitation on the aperture angle of the spherical joints. Observe that this is a sufficient condition to avoid singularity regions of this manipulator.

The optimization process was carried out in FORTRAN environments using a nonlinear Sequential Quadratic Programming (SQP) optimization routine provided by the NAG commercial library.

It is important to mention that in the optimization process, one can take use of methods for scaling the reduced observation matrix. The scaled matrix can be obtained as follow,

$$\left(\mathbf{W}_{\text{red}} \right)_{\text{eq}} = \mathbf{R} \cdot \mathbf{W}_{\text{red}} \cdot \mathbf{C} \tag{27}$$

where $\mathbf{R}$ is the $r \times r$ diagonal matrix and $\mathbf{C}$ is the $b \times b$ diagonal matrix, both including row and column scaling factors on their diagonal, respectively. r and b are the dimensions of $\mathbf{W}_{\text{red}}$. To keep on the benefits of the condition number reduction of $\mathbf{W}_{\text{red}}$, scaling must be introduced into the identification process. Another approach that can be used for scaling $\mathbf{W}_{\text{red}}$ is the normalization of the matrix by means of the division of its columns by their norms. The use of scaling factors or the normalization rescale has to be taken into account in the identified model parameters afterwards.

For the purpose of experimental parameter identification, several trajectories were found. For example, one of the trajectories that have been obtained had the condition number of 595. It is depicted in Fig. (2).

4. Physical feasibility

Once an optimized trajectory has been found and it is applied to the manipulator, data is measured so that the dynamic system represented by equation (23) is ready to be solved. At this point an important issue in the dynamic parameters identification arises: the physical feasibility of the identified parameters. Because of the incompleteness of the dynamic model and the existence of noise in the measurements, the solution of equation (23) by LSM, or the identified base parameters, is susceptible to have no physical interpretation when compared with corresponding combinations of the inertial parameters; masses lower than zero or non positive-definite local inertial matrices (Yoshida & Khalil, 2000). This issue not only affects the stability of some of the advanced model-based control algorithms, but it is also a crucial one in the case of dynamic simulations.

To insure the physical feasibility of the identified parameters a nonlinear *constrained* optimization process, instead of the LSM, could be implemented to solve equation (1). The corresponding physical feasibility constraint equations have the following form,

$$
\begin{aligned}
& m_i > 0 \\
& I_{xx}^g > 0 \\
& I_{xx}^g I_{yy}^g - \left(I_{xy}^{xy} \right)^2 > 0 \\
& \det\left({}^i\mathbf{I}_{G_i} \right) = I_{xx}^g I_{yy}^g I_{zz}^g + 2\, I_{xy}^g I_{xz}^g I_{yz}^g - I_{xx}^g \left(I_{yz}^g \right)^2 - I_{yy}^g \left(I_{xz}^g \right)^2 - I_{zz}^g \left(I_{xy}^g \right)^2 > 0
\end{aligned}
\tag{28}
$$

where I_{xx}^g, I_{xy}^g, I_{xz}^g, I_{yy}^g, I_{yz}^g and I_{zz}^g are the components of body inertia matrix ${}^i\mathbf{I}_{G_i}$ calculated with respect to its center of gravity.

t1	t2

t3	t4

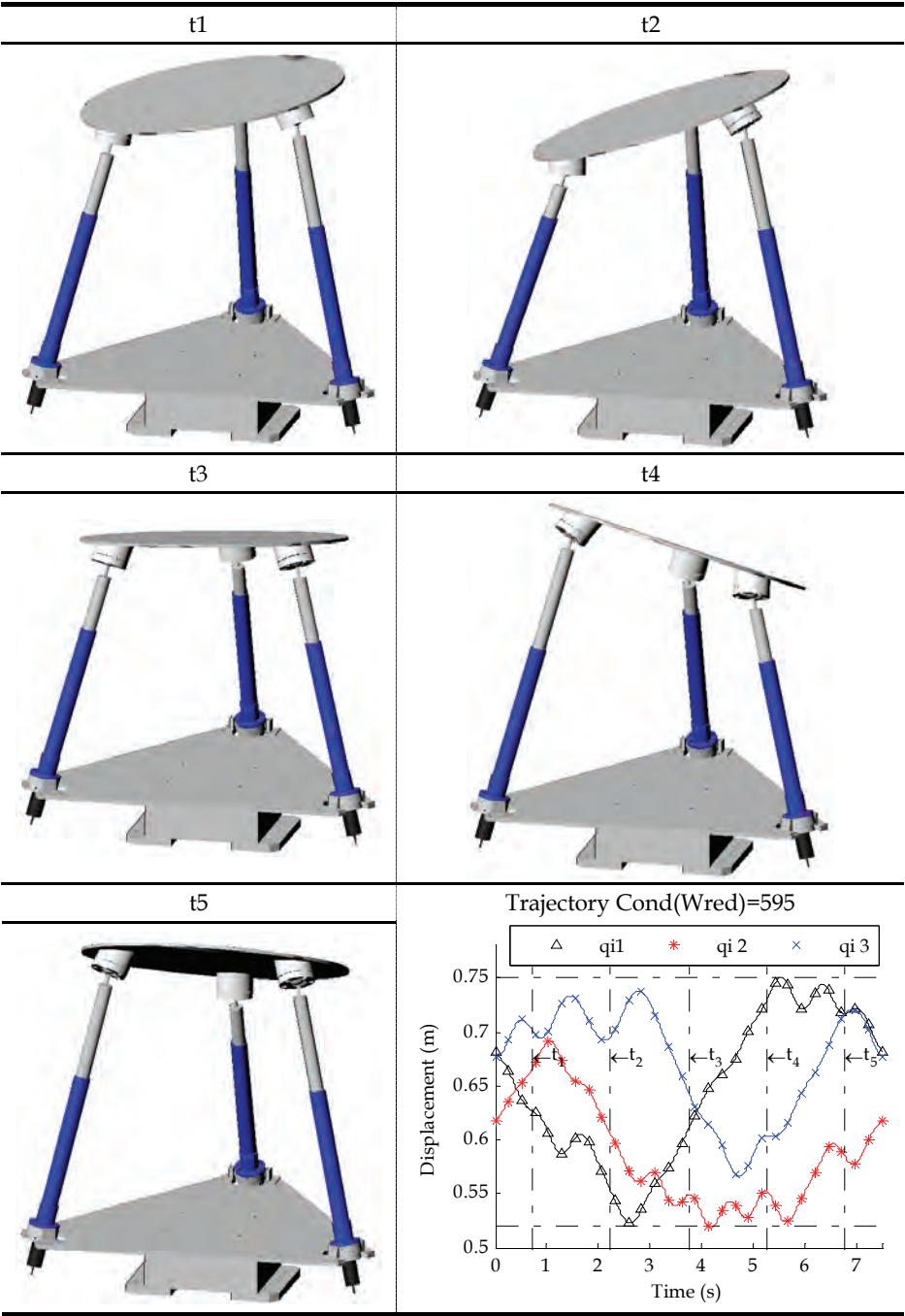| t5	Trajectory Cond(Wred)=595

Fig. 2. An optimized trajectory with a Condition Number of 595.

Note that in equation (28), local inertia matrixes are calculated with respect to the center of gravity, meanwhile the inertia matrix terms of the identified base parameters are calculated with respect to the origin of the local reference system. Hence, to verify the physical feasibility of a set of numerical values of the base parameters, they must be compared with the corresponding combinations of the inertial parameters. That is to say, if it is possible to obtain a set of inertial parameters that verify the equality between the linear combinations and the numerical values of the base parameters then this set of base parameters – numerical values – is judged to be physically feasible, and vice versa. A scheme was proposed (Yoshida & Khalil, 2000) to judge whether the values of the base parameters correspond to a set of physically feasible inertial parameters, or not. This scheme evaluates the physical feasibility after the parameter identification process has been carried out. Other authors subject formulate the identification process as a nonlinear constrained optimization problem (Mata et al., 2005; Farhat et al., 2008). This approach allows on the one hand, ensuring the physical feasibility of the identified parameters. On the other hand, it permits the implementation of nonlinear friction models, like the one expressed by equations (16) and (17), to model friction phenomenon at robot joints. To this end, the objective function of the identification problem has the form,

$$f\left(\bar{q},\dot{\bar{q}},\ddot{\bar{q}},\bar{\tau}\right)=\bar{\tau}-\underbrace{\left(\mathbf{W}_{\text{red}}^{I}\right)_{\text{eq}}\cdot\left(\Phi_{\text{base}}^{I}\right)}_{\text{Linear part}}+\underbrace{\left(\left(\bar{F}_{f_i}-\mathbf{X}^{T}\cdot\bar{F}_{f_e}\right)\right)}_{\text{Nonlinear friction vector}},\qquad(29)$$

constrained by equation (28) applied to all links constituting the robot.

5. Practical issues

5.1 Measurements

Recall that the inputs of the identification process are the external forces, positions and there time derivatives, all measured for an optimized trajectory. Generally, the external forces or the torque exerted by motors is not directly available. An acceptable approach is to assume a linear relation between the current and the torque,

$$\tau_m = K_m i_m \qquad(30)$$

where τ_m is the motor torque, i_m is the motor current and K_m is the torque constant which can be found, among other methods, by means of a previously performed experiments on the dismounted motors or by in situ experiments (Corke, 1996).

On the other hand, for kinematic variables, most industrial robots are provided with a precise position sensor. However, the direct measurement of joints velocities and joints accelerations is not normally available. Thus, they are obtained by taking the time derivative of the measured positions. These derivatives could be obtained numerically using central difference algorithms (Khalil & Dombre, 2002). For the velocity,

$$\dot{q}_i\left(t\right)=\frac{\left(q_i\left(t+1\right)-q_i\left(t-1\right)\right)}{2\Delta t}\qquad(31)$$

Other approaches (Swevers et al., 1996) suggest, using the exciting trajectory found by the optimization process. In this manner the new trajectory coefficients are found by refitting

the measured positions to the Fourier series. Therefore, the first and second time derivatives could be obtained analytically. Other authors (Gautier et al., 1995) propose filtering the measured positions by a low pass filter before taking the derivative numerically.

The authors of this chapter made a comparison between three different method for finding the time derivatives (Diaz-Rodriguez et al., 2007). The comparison was carried out over a 3-RPS parallel manipulators which was equipped with accelerometers located at the generalized independent coordinated. The following methods were used,

1. Filtering the measured positions by a low pass filter before taking the derivative numerically (Gautier et al., 1995)
2. Refitting the measured positions to the Fourier series used in the trajectory optimization process (Swevers et al., 1996)
3. Approach based on local fitting (Page et al., 2006). A local regression of the measured trajectory was executed using a third order polynomial. When performing the local regression the whole set of samples were considered, but with different statistical weights. After applying the local regression, the velocities and accelerations were derived from each polynomial.

The main conclusions of the work indicated that the three methods give quite similar results for the analyzed robot. Therefore, provided that well excited trajectories are used in the identification, any of these methods can be used in a deterministic framework.

5.2 Dynamic model reduction

If the geometry of robot parts is taken into account, some rigid body base parameters have zero values or values close to it. Consider for instance the 3-RPS robot (Fig. (1)) where the links connected to the base have a cylindrical geometry. It can be supposed – with a degree of certainty that the gravity center of these links lies on an axis parallel to the actuator movement. In this case, the corresponding axes of the local reference system attached to the body is in (y) direction, so the parameter related to the (x) position of the gravity center can be expected to have values close to zero. The same assumption is applied to links connected to the moving platform. Therefore, parameters 1, 4, 14, 18, 20, 24 from Table (1) can be removed. Moreover, it is possible to consider as well the form of the platform which is circular and flat, and by doing so parameters 7, 8, 10, 13 can also be removed. This reduced model is highlighted by (*) in Table (1). Another simplification can be applied if the parallel manipulators symmetry is considered. In Table (1) the rigid body base parameters of this case is highlighted by (**). It is important to mention that the columns of the observation matrix, associated with base parameters that consider the symmetry, have to be added in order to develop a model which properly describes the dynamic behavior of the robot. Table (2) summarizes the different models that have been proposed here.

5.3 Identifiability of the base parameters

When a direct parameter identification process is experimentally performed, two sources of error become apparent. On the one hand, not all the aspects of the robot can be modeled in detail (modeling discrepancies). On the other hand, noise in measurements is present. These errors lead to the fact that not all the base parameters can be properly identified. This apparently occurs when the independent contribution of some parameters to the generalized forces is smaller than the measurement noise or the modeling discrepancies.

The study of which parameters can be properly identified can be established on analyzing the relative standard deviation (σ_{pi}) of each parameter (Khalil & Dombre, 2002b) along with considering the physical feasibility (Yoshida & Khalil, 2000). Physical feasible base parameters with σ_{pi} < 15% are considered properly identified.

Model	Nº of Rigid Body Parameters	Parameter removed from Table (1)
1	25	-
2	15	1, 4, 7, 8, 10, 13, 14, 18, 20 and 24
3	9	1, 4, 7, 8, 10, 13, 14, 15, 16, 19, 18, 20, 21, 22, 24 and 25

Table 2. Different rigid body models for the dynamic parameter identification process of the 3-DOF RPS parallel manipulator.

6. Application to a 3-RPS robot

In this section, the results of the identification process, implemented by the authors over a 3-RPS parallel manipulator, are presented. In the first part, the approach based on considering the physical feasibility in the identification process is presented. In the second part of the section, the identifiability of the dynamic parameter for the 3-RPS robot is evaluated. In both sections, the identification process is validated using a simulated manipulator built by making use of the ADAMS dynamic simulation program. After that, it is applied over a real one constructed at the Polytechnic University of Valencia, see Fig. (1).

6.1 Identification considering the physical feasibility
Because of the noise in the input data and/or the discrepancies between the actual parallel robot and the dynamic model used in the identification process, some of the inertial parameters obtained using LSM methods result physically unfeasible. Thus, the necessity for a constrained optimization process to ensure physical feasibility appears clearly. In this subsection, the results are shown as a comparison between the original actuator forces and those calculated using the identified dynamic parameters in the case of; a) linear friction models and. b) nonlinear friction models.

The dynamic model of this manipulator, trajectory optimization and identification process were built in FORTRAN programming language with the aid of the NAG library and the NLPQL Sequential Quadratic Programming subroutine (Schittkowski, 2000).

Simulated Robot
In the simulated robot, nonlinear friction model is considered at all the joints of the robot. It can be represented by the following relation,

$$F(v) = F_C + (F_S - F_C)e^{|v/v_S|^{\delta s}} + F_v v \qquad (32)$$

Where F_C, F_S and F_v are the Coulomb, static, and viscous friction coefficients, respectively, v_S is the Stribeck velocity and δ_s is the stiction transition velocity. This model consists of five parameters and captures the Coulomb, static, viscous and Stribeck friction forces (Olsson et al., 1998). After calculating the external original forces, errors are introduced assuming a normal distribution producing the perturbed forces. Now, based on these perturbed forces,

identification is carried out considering asymmetric linear friction models for all joints, using LSM and optimization, and symmetric nonlinear friction models using optimization. The corresponding identification errors are shown in Table (3). where ε_{RA} is the Relative Absolute Error and is defined as,

$$\varepsilon_{RA} = \frac{\sum_i \left| \tau_{idnt_i} - \tau_i^* \right|}{\sum_i \left| \tau_i^* - \overline{\tau^*} \right|} \tag{33}$$

where, τ^* and τ_{idnt} are the actual applied force and those calculated using the dynamic model applying identified dynamic parameters, and $\overline{\tau^*}$ is the average of τ^*.

ε_{RA} (%)	Perturbed	Original
Linear (LSM)	11.96	8.26
Linear (Opt.)	12.44	8.85
Nonlinear (Opt.)	11.02	7.28

Table 3. Identification errors based on simulated manipulator.

As can be seen in Table (3), considering the case where linear friction models were used in the identification process, when the physical feasibility had been ensured, i.e. identification by optimization, the error increased. On the contrary, when this was accompanied by the nonlinear friction models, the results were improved considering both the perturbed and the original forces. Note that in this step, identification was carried out simulating both types of the mentioned identification process error sources.

Actual robot

Now, after verifying the identification process over the simulated manipulator, the results are shown in detail considering the identification of the dynamic parameters of the actual manipulator. Starting with the optimization process for the exciting trajectory and changing the initial estimations, different optimized trajectories were obtained. An example of such an optimized trajectory is that one presented previously in Fig. (2). Hereafter, the identified dynamic parameters were obtained basing on anther optimized trajectory with a corresponding condition number of 638.

A PID controller is used in order to determine the control actions. The control actions were applied with a frequency of 100Hz, at which measurement were also taken. The total duration of the optimized trajectory is 7.5s. Trajectories were repeated several times, the applied control actions were averaged and then a second order lowpass digital Butterworth filter was applied. For the identification process, 75 configuration points are extracted every 0.1s.

When the LSM was used in the identification process, a non physically feasible base parameters were found. Hence, the identification process was held using the nonlinear constrained optimization process where the physical feasibility of the obtained inertial parameters was ensured. Fig. (3) shows a graphical comparison between the actual forces and those calculated using the dynamic parameters identified by LSM and optimization considering asymmetric linear friction models at the prismatic joints.

In order to justify the use of nonlinear friction models in the identification process rather than those which are linear, as the friction phenomenon in the considered joints has this tendency, a thorough error comparison was made. This is established considering three different sets of dynamic parameters identified by: LSM, optimization in the linear case if linear friction models are considered and optimization in the nonlinear case. In order to make an overall judgment, error comparison takes place over the same trajectory used in the identification process (that one of a condition number of 638) and others which are excited, including the low velocity one. The resulted calculated error in each case is shown in Table (4).

As can be observed from Table (4), considering identification by the optimization case and excluding the trajectory used in the identification process, the errors in the predicted applied forces considering the identified nonlinear friction models are lower than those corresponding to the linear friction ones for all trajectories. This shows that the dynamic model that includes nonlinear friction models has a better overall response. On the other hand, the dynamic parameters obtained by the LSM give the lowest error for all of the test trajectories. However, the calculated errors using the identified parameters found by means of the test trajectories became bigger, and almost doubled, contrary to those calculated by optimization, which kept the same order. Furthermore, the identified dynamic parameters using the optimization process are physically feasible.

6.2 Identifiability of the base parameters

The identification process, as has been pointed out in the previous section, has the ability to obtain a physical feasibility solution; however, the constrained optimization problem is cumbersome. This occurs because constraint equations are functions of the terms of the inertia tensor calculated with respect to the center of gravity of the corresponding body, and the linear relation between the base parameter vector and the physical parameters is not just one.

In addition, the solution of the nonlinear problem does not guarantee that the set of physical parameters found has been identified accurately. Another approach that can be used for parameter identification is to evaluate the physical feasibility after the identification process has been carried out (Yoshida et al., 1996) along with the statistical analysis of variances in the resulting parameters. Hence, two aspects are verified: Base Parameters with $\sigma_{pi} < 15\%$ and physical feasibility. If the parameter accomplishes these criteria, the parameter is considered properly identified.

For example, here, the identifiably of the dynamic parameters of a 3-RPS parallel robot is addressed considering a simulated manipulator whose inertial parameters have been obtained from the CAD models and the friction parameters has been obtained from an indirect parameter identification process performed by the authors (Farhat et al., 2006). Noise was added to the generalized forces as well as the independent generalized coordinates and their time derivates.

The three models previously introduced in Table (2) were used in the identification process. Friction was identified using symmetrical linear models that include Coulomb and viscous frictions. When the parameters identified by using Model 1 were analyzed, only 4 of the 34 parameters, including friction and rotor and screw inertias, had σ_{pi} lower than 15%, and some of the identified parameters were physically unfeasible. This can be demonstrated in Table (5) (marked by *), where it can be seen the values of parameters of the simulated and the identified models, respectively.
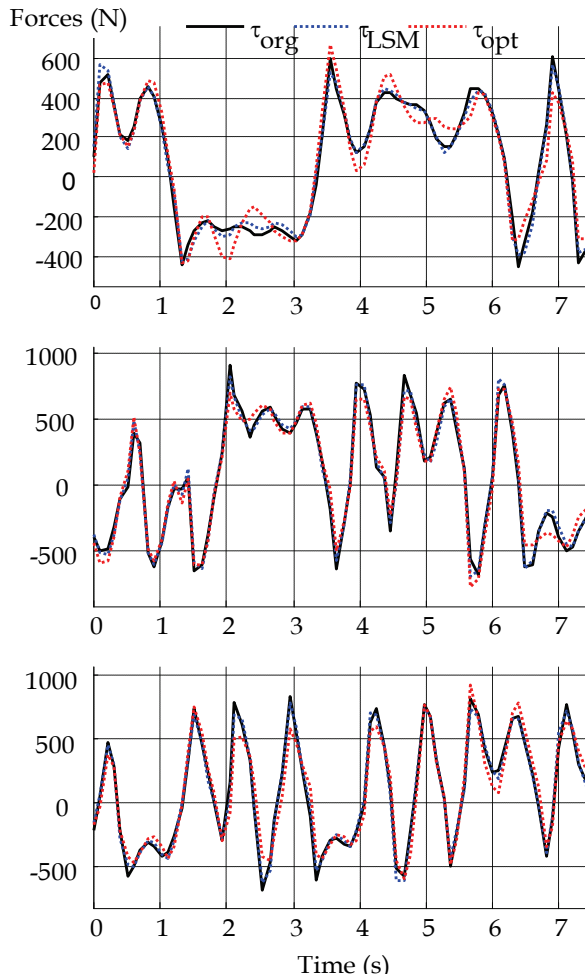
Fig. 3. Results from the LSM (τ_{LSM}) and the optimization process (τ_{opt}).

ε_{RA} (%)		Trajectory condition number					
Friction model		563	638	718	601	492	LowVel
Linear	LSM	12.8	9.48	14.6	17.9	17.9	15.5
Linear	Opt	26.1	20.5	24.4	23.9	23.9	19.2
Nonlin	Op	24.7	21.2	23.5	23.4	23.4	18.0

Table 4. Error comparison considering linear and nonlinear friction models.

Results of the number of parameters properly identified, when model 1-3 was used in the identification process, are listed in Table (6). The table includes also the average relative error of the identified parameter relative to the exact parameter (ε_{AV}),

$$\varepsilon_{AV} = \frac{1}{n_p} \sum \left| \frac{\Phi_i - \hat{\Phi}_i}{\Phi_i} \right| \tag{34}$$

where Φ_i the exact values of the parameters and $\hat{\Phi}_i$ is the vector containing the identified parameters. An interesting fact is that despite that Model 1 achieving the lowest ε_{RA}, the corresponding ε_{AV} value was the highest. In addition, only 4 parameters were properly identified. The difference between models 2 and 3 in ε_{RA} was about 1.5%. As 12 parameters are identifiable, identification was performed using only these parameters (Model 4). This model includes 3 inertial parameters of the links related to the platform and marked by (†) in Table (1).

Parameter	Exact Values	Identified Values	σ_{pi} %
$I_{zz}(4)+I_{yy}(5)$	0.1555	-86.2016*	80.1596
$I_{zz}(6)+I_{yy}(7)$	0.1555	-66.6355*	62.0087
$Fv(1)$	3272.0	3296.73	2.5677
$Fc^+(1)$	227.96	1659.0181	53.3048
$Fc^-(1)$	228.04	-1210.55*	73.1859
$J_r(1)+J_S(1)$	483.10	505.03	13.8778

Table 5. Some of the base parameters identified using Model 1

Model	ε_{RA} %	ε_{AV} %	Number of Parameters
1	4.57	5.37	37/4
2	4.58	2.94	24/12
3	4.63	3.06	18/12
4	4.73	3.17	12/12

Table 6. Results of ε_{RA} and ε_{AV} from different models.

This result could indicate that, because of the topology of the parallel manipulator and in the presence of measurement noise, 12 parameters from which 3 are of the links inertial parameters can be used for modeling and simulating the 3-RPS parallel manipulator behavior.

Following the same procedure, a dynamic parameters identification process was applied over an actual parallel 3-RPS manipulator. The resulted ε_{RA} values and the number of parameters properly identified are shown in Table (7). Comparing Table (6) and Table (7), the level of ε_{RA} in the actual manipulator was doubled, but the numbers of parameter properly identified was found similar (12 for Model 4). The identified links base parameters of Model 1 and 4 are presented in Table (8) along with those of the simulated manipulator. As can be observed, the identified parameters of the actual manipulator using model 4 and the original CAD values of the simulated manipulator are comparable. Contrary to those identified using Model 1 where a significant difference appears.

The fact that 12 parameters can be properly identified is reasonable. On the one hand the topology itself of the parallel manipulator, does not allow finding well-excited trajectories. Additionally, some base parameters have a little contribution to the dynamic behavior of the

model; for example, during the movement the accelerations of the limbs are smaller than the platform. On the other hand, the friction of the linear actuator of the real manipulator was found to be high, this difficult even more the identifiability of the base parameters of the links.

Finally, Model 4 was validated. Parameters obtained from one trajectory were used to compute the forces for another one that had not been used for identification. Fig. (4) depicts this comparison. As can be seen, the estimated and measurements forces are very close.

Model	ε_{RA} %	Number of Parameter
1	8.40	37/2
2	8.43	24/9
3	8.53	18/12
4	8.62	12/12

Table 7. ε_{RA} from actual 3-RPS Manipulator.

Base Parameter	CAD	Real Manipulator Model 4	Real Manipulator Model 1
mx(3)+0.5774my(3) - 0.4563(m(3)+m(2))	-2.47	-2.59	1.16
m(5)-2.531my(3)+m(3)+m(2)	10.83	13.72	-3.29
m(7)+2.531my(3)	5.42	6.95	-0.557

Table 8. Rigid Body Base Parameters Model 1 vs Model 4.

7. Conclusions and further research

In this chapter, the problem of the identification of inertia and friction parameters for parallel manipulators was addressed. In the first part of the chapter an overview of the identification process applied to parallel manipulators was presented. First, the dynamic model was obtained in a systematic way starting from the Gibbs-Appell equations of motion. This dynamic model was reduced to a subset of parameters called base parameters by means of SVD. After that and to ensure the minimal input/output error transmissibility, approaches to obtaining optimized trajectories that have to be used in the identification process were presented. In the second part of the chapter, a direct identification approach was implemented on a 3-DOF RPS parallel manipulator considering the physical feasibility of the identified inertial parameters. To this end, a procedure based on a nonlinear constrained optimization problem has been reviewed. In addition, nonlinear friction models were included in the dynamic formulation subjacent to the identification process. In the last part of the chapter, a study of the identifiability of the base parameters was presented. It based on both analyzing the relative standard deviation of each parameter and considering its physical feasibility. For this approach a simulated manipulator was necessary for studying and evaluating models used in the identification process. For future research, a systematical approach is expected to be found, based on statistical frameworks and physical feasibility, for studying the identifiability of the dynamic parameter without the necessity of a simulated manipulator. Concepts presented here for parameter identification of parallel

manipulators can be extended to other areas. For instance, vehicle components (Butz et al., 2000; Serban & Freeman, 2001; Chen & Beale, 2003; Sujan & Dubowsky, 2003) and ultimately the novel humanoid systems (Gordon & Hopkins, 1997; Silva et al., 1997; Kraus et al., 2005).
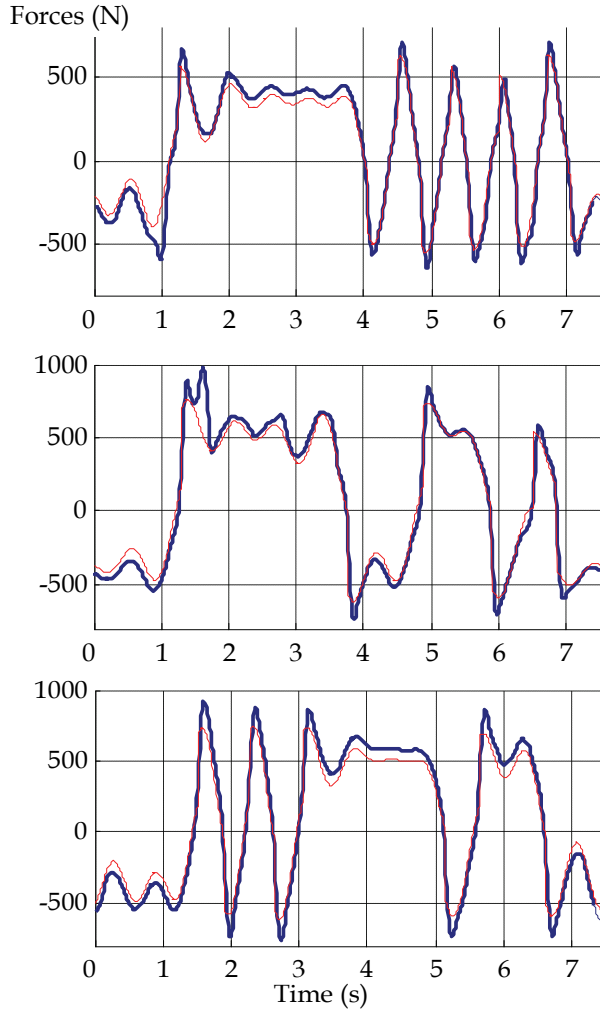
Fig. 4. Measurement Forces (-red) and Forces from Identified Parameter (–o blue)

8. Acknowledgements

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