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Design equation for predicting fire resistance of reinforced concrete beams

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ABSTRACT

An approach for evaluating the fire resistance of reinforced concrete (RC) beams is presented in this paper. A macroscopic finite element model is applied to study the influence of various parameters on the fire resistance of RC beams. Data from parametric studies is utilized to develop a simplified expression for evaluating the fire resistance of an RC beam as a function of influencing parameters. The validity of the proposed approach is established by comparing the fire resistance predictions with those obtained from finite element studies as well as from fire resistance tests. Predictions from the proposed equation are also compared with fire resistance estimates from current codes of practice. The applicability of the approach to design situations is illustrated through a numerical example. The proposed rational approach expresses fire resistance in terms of conventional structural and material design parameters, and thus facilitates easy evaluation of fire resistance. The proposed approach provides better estimates than those from current codes of practice and thus can be used to evaluate the fire resistance of RC beams with an accuracy that is adequate for design purposes.

1. Introduction

Reinforced concrete (RC) beams function as critical load bearing structural members in a building, and hence the provision of appropriate fire resistance is one of the major design requirements in buildings. The basis for this requirement can be attributed to the fact that when fire suppression and control systems fail, structural integrity is the last line of defense. The current method of evaluating fire resistance of RC beams is based on prescriptive approaches, and is usually a function of concrete cover thickness and beam width [1]. These tabulated fire resistance ratings specified in current standards are derived from standard fire resistance test data and do not account for critical factors such as load level, fire scenario and support conditions. Alternately, a detailed nonlinear thermal and structural analysis can be used, however this requires significant expertise and effort.

In lieu of tabulated rating and detailed analysis, simplified calculation methods can be applied for evaluating fire resistance of RC beams. This paper presents the development of a rational simplified approach for predicting the fire resistance of RC beams. The approach is derived based on a large set of parametric studies on RC beams and the analysis was carried out using a microscopic finite element model. The proposed approach accounts for critical parameters that influence the fire resistance of RC beams. The validity of the proposed approach is established by comparing the predicted fire resistance values with those obtained from numerical studies as well as from fire resistance experiments. The predictions from the proposed method are also compared with the fire resistance values obtained from ACI, Eurocode and Australian code provisions.

2. Research significance

The fire resistance of an RC beam depends on a number of factors including fire scenario, sectional characteristics, load level, geometric properties and support conditions. The current fire resistance provisions in codes and standards are prescriptive and do not account for many of these factors. The purpose of this study is to quantify the influence of these parameters on the fire resistance of RC beams and to develop a simplified approach for fire resistance design of RC beams under a performance-based code environment.

3. Current methodology

Provisions for evaluating the fire resistance of RC beams are generally specified in codes and standards such as ACI 216.1 [1], Eurocode 2 [2] and AS 3600 [3]. These provisions are based on standard fire tests, and thus they are prescriptive and do not

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List of Notations

b	Beam width
E_c	Modulus of elasticity of concrete
H	Total depth of concrete section
k_r	Axial restraint stiffness
L	Length of the beam
R	Fire resistance of a reinforced concrete beam
R_0	Initial estimate of fire resistance
SF	Section factor = heated parameter/cross sectional area
Y	Location of axial restraint force from the top most fibers of the concrete section
χ	Number of corner bars/total number of bars.
ϕ_{ag}, ϕ_{st} and ϕ_{cs}	Factors in evaluating the fire resistance of reinforced concrete beams to account for aggregate type, concrete strength and structural conditions, respectively.
ϕ_{SS}	Structural modification factor for fire resistance of simply supported beams
ϕ_{AR}	Structural modification factor for fire resistance of axially restrained beams
ϕ_{ER}	Structural modification factor for fire resistance of eccentrically axially restrained beams
ϕ_{RR}	Structural modification factor for fire resistance of rotationally restrained beams
ϕ_{FR}	Structural modification factor for fire resistance of axially and rotationally restrained beams
ψ_0	Section characteristic factor
ρ	Steel ratio = area of tension steel/effective area of cross section
SS	Simply supported
AR	Axially restrained
ER	Eccentrically axially restrained
RR	Rotationally restrained
FR	Axially and rotationally restrained (fully restrained)

account for the critical parameters that govern the behavior of RC structural members under fire conditions. As an illustration, the failure of an RC beam in a standard fire test is based on the temperature attained in the steel reinforcement, under a predefined standard fire exposure, without any consideration of a number of critical factors such as realistic fire scenarios, loading and restraint.

In lieu of prescriptive methods, Eurocode 2 provides rational approaches (such as the isotherm) for evaluation of fire resistance. In isotherm method, although fire induced loss of strength in concrete and steel is accounted for based on simplified approximation, axial restraint, thermal gradient, and moment redistribution are not taken into consideration. In addition, these methods do not yield fire induced deflection as a function of time which is necessary for estimating the fire resistance of the beam based on deflection failure criteria.

Further, the application of fire provisions in different codes produces different and varying fire resistance values for similar RC beams. As an illustration, for a simply supported RC beam with a cross-sectional dimension of 300 × 500 mm and a concrete cover thickness of 40 mm (axis distance of 50 mm), the fire resistance based on ACI 216.1, Eurocode 2, and AS 3600 is 210, 110, and 110 min, respectively. For this beam ACI 216.1 predicts a much higher fire resistance than the other two codes. This clearly indicates that current fire provisions in codes of practice, particularly ACI 216.1, may not yield reliable fire resistance of RC beams. Further, these provisions are not applicable for estimating fire resistance under realistic fire scenarios, load conditions and restraint effects.

The fire resistance performance of an RC beam under a design fire exposure can be related to its fire resistance under standard fire exposure provided a time equivalency is established between standard and design fire scenarios. A state-of-the-art review by Kodur et al. [4] shows that there are few time equivalent methods and empirical formulae for RC members (beams). It was also shown that the time equivalent values predicted by existing methods and empirical formulae are generally unconservative and have an appreciable variation for the case of RC beams [4]. They concluded that the current methods for evaluating time equivalency for RC beams may not be fully reliable. To overcome this problem, Kodur et al. [4] suggested a more reliable and conservative semi-empirical approach for establishing time equivalency. Full details of this time equivalent approach can be found elsewhere [4].

4. Factors governing fire resistance

Detailed parametric studies were carried out, using a microscopic finite element model [5] to investigate the influence of various parameters on the fire resistance of RC beams. This model utilizes a stepwise approach, and at each time step the fire resistance analysis is carried out in three steps, namely, evaluating fire temperature, thermal, and structural analyses. The model accounts for fire induced spalling (through a hydro-thermal sub-model derived based on the principles of mechanics and thermodynamics), fire induced axial restraint, various strain components, cracking and tensile strength of concrete, and high temperature material properties. At each time step, fire induced axial restraint force is evaluated by satisfying compatibility, equilibrium and convergence criteria along the span of the beam. The numerical model was validated by undertaking fire tests on a number of RCs (two normal strength concrete and four high strength concrete) beams under standard and realistic loading, restraint and fire scenarios. Spalling predictions were also validated against measured values from fire tests. As an illustration, and due to space limitation, the validation of spalling predictions for one beam is presented in this paper. The beam, designated as B6, was tested by Dwaikat and Kodur [6] and experienced significant spalling. The beam has similar properties to those of beam B3 in Table 1 but with severe design exposure. Details of beam properties and fire exposure can be found in Ref. [6].

The predicted values and times of spalling are compared with the measured values for beam B6 in Fig. 1. It can be seen that there is good agreement between the predicted and measured values in the entire range of fire exposure. The measured start and end times of spalling were 5 and 30 min, and the predicted times were 6.5 and 122 min. These results indicate that the program predicts well the start time of spalling. In contrast, there is variation between the predicted and the measured end time of spalling. This variation might be acceptable since the model predictions indicate that small amount of spalling after 30 min of fire exposure. This can be attributed to the drying of concrete which reduces the developed pore pressure and hence decreases the spalling at later stages of fire. The model predicts a slightly higher extent of spalling, which is a conservative estimate. Overall, the spalling predictions match reasonably well with the measured spalling for RC beams. Further validation of the spalling model is presented in Appendix A. Full details of the overall numerical model and the spalling model including validation can be found elsewhere [5–8]. It should be noted that the validity of the spalling sub-model with in the fire resistance model can be further enhanced by validating against spalling measurements in RC beams. This requires carefully monitored fire tests on RC beams.

Based on results from previous qualitative studies [5,9,10], it was found that fire scenario, load ratio, section characteristics,

Table 1
Properties of beams used in equation validation and comparison with current codes.

Beam designation	B1	B3	BD1	BL1	BL2	BL3	BL4	BL5	BL6	BCS1	BCS2	BCS3	BCS4
Description	tested by Dwaikat and Kodur [6]		tested by Dotreppe and Franssen [21]	tested by Lin et al. [20]						analyzed beams using the numerical model			
Cross-section (mm)	254 × 406		200 × 600	305 × 355						300 × 900	400 × 800	700 × 400	600 × 600
Length (m)	3.66		6.5	6.1						11.7	14.4	3.2	10.8
Reinforcement	3 ϕ 19 mm bottom bars		3 ϕ 22 mm bottom bars	4 ϕ 19 mm bottom bars						5 ϕ 20 mm bottom bars	3 ϕ 35 mm bottom bars	6 ϕ 20 mm bottom bars	8 ϕ 20 mm bottom bars
	2 ϕ 13 mm top bars		2 ϕ 12 mm (#4) top bars	2 ϕ 19 mm top bars	6 ϕ 19 mm bottom bars					2 ϕ 14 mm top bars	2 ϕ 14 mm top bars	3 ϕ 14 mm top bars	3 ϕ 14 mm top bars
f'_c (Mpa)	52	91	15*	30						40	60	30	50
f_y (MPa)	450	450	300*	435.8						413	413	413	413
Loading ratio	0.54	0.54	0.263*	0.42	0.45	0.56	0.35	0.43	0.4	0.7	0.5	0.5	0.3
Concrete cover thickness (mm)	40			25 (bottom)						40	60	40	50
				38 (side)									
Aggregate type	Carbonate		Siliceous*	Carbonate						Siliceous			
Support conditions**	SS				RR					SS	AR with ax =		
											27.60%	9.10%	19.80%
Measured fire resistance (min)	180	160	120	80	216	183	271	210	243	Not applicable			

* Values based on correspondence [22]
** SS = simply supported, RR = rotationally restrained, AR = axially restrained, ax = ratio of axial restraint stiffness to the axial stiffness of the beam.

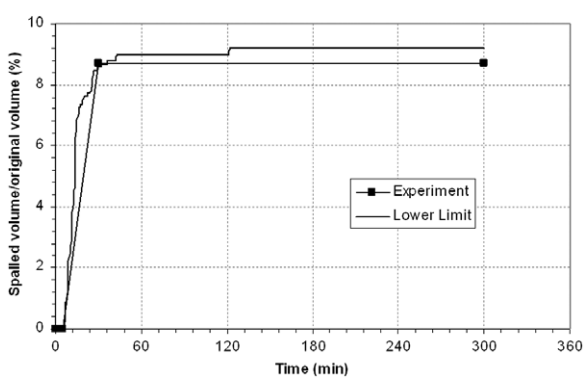

Fig. 1. Measured and predicted extent of spalling as a function of time for beam B6.

axial restraint stiffness, span-to-depth ratio, location of axial restraint, rotational restraint, aggregate type, spalling and failure criteria have an influence on the fire resistance of RC beams. These parameters were selected for parametric studies. The high temperature thermal and mechanical properties derived from the detailed state-of-the-art review by Kodur et al. [11] were used in the parametric studies.

In the parametric studies, a large number of NSC and HSC beams with four different cross-sections were analyzed. The reinforcement ratio in these beams (ratio of area of tension steel to the effective area of the beam cross-section) was in the range of 0.6%–1.7%, which lies within the minimum and maximum limits specified in ACI 318 [12]. The compressive strength of concrete was varied from 30 to 100 MPa, while the yield strength of rebars was assumed to be 413 MPa (Grade 60 steel bars). Properties of the four cross-sections of the beams are given in Table 2. Five different support conditions, namely; simply supported (SS), axially restrained (AR), eccentrically axially restrained (ER), rotationally restraint (RR) and axially and rotationally restrained (FR), were analyzed.

The parameters varied in the analysis include four different concrete cross-sections, three load ratios (LR) (30%, 50% and 70%), two concrete types (NSC and HSC), two aggregate types (carbonate and siliceous aggregate), three span-to-depth ratios (8, 13, and 18), three degrees of axial restraint stiffness (k_r) (0, 50 and 200 kN/mm), five locations (Y/H) of the axial restraint (0.3, 0.4, 0.5, 0.6 and 0.7) and five fire scenarios (two standard and three design fires). The location of axial restraint is measured from the topmost fiber of the beam cross-section as a ratio of the total depth of the beam. The fire scenarios consisted of two standard fire exposures (ASTM E119a standard fire [13] and ASTM E1529 hydrocarbon fire [14]) and three design fire exposures (Fire I, Fire II and Fire III). The time temperature curves for the five fire scenarios are given in Fig. 2. According to ACI 318 [12], the depth of an RC beam is to be larger than $L/21$ to satisfy high temperature deflection requirements. In the analysis, span-to-depth ratio was limited to a maximum value of 18 due to the fact that the design of RC beams is generally controlled by strength failure and thus larger beam depths are commonly used in practice.

Each of the analyzed beams is designated by five characters arranged from left to right as follows:

- Concrete type (N for NSC, and H for HSC),
- Aggregate type (S for siliceous aggregate and C for carbonate aggregate),
- Cross-section number (1, 2, 3 or 4 for the four sections detailed in Table 2),
- Support conditions (S for simply supported, A for axially restrained, R for rotationally restrained and AR for axially and rotationally restrained beams),
- Span-to-depth ratio (S representing small span-to-depth ratio of 8, M representing medium span-to-depth ratio of 13, and L representing large span-to-depth ratio of 18).

Table 2
Properties for concrete cross-sections used in the analysis.

Property	Cross-section #			
	1	2	3	4
Cross-section (mm) (in.)	300 (12) × 900 (36)	400 (16) × 800 (32)	700 (28) × 400 (16)	600 (24) × 600 (24)
Reinforcement for simply supported and axially restrained	2 ϕ 14 mm (#5) top bars 5 ϕ 20 mm (#6) bottom bars	2 ϕ 14 mm (#5) top bars 3 ϕ 35 mm (#11) bottom bars	3 ϕ 14 mm (#5) top bars 6 ϕ 20 mm (#6) bottom bars	3 ϕ 14 mm (#5) top bars 8 ϕ 20 mm (#6) bottom bars
Reinforcement for rotationally restrained beams	5 ϕ 20 mm (#6) top bars 4 ϕ 20 mm (#6) bottom bars	3 ϕ 35 mm (#11) top bars 3 ϕ 30 mm (#9) bottom bars	6 ϕ 20 mm (#6) top bars 4 ϕ 20 mm (#6) bottom bars	8 ϕ 20 mm (#6) top bars 5 ϕ 20 mm (#6) bottom bars
f_c' (MPa) (ksi) (for NSC beams)	40 (5.8)	60 (8.7)	30 (4.36)	50 (7.26)
f_c' (MPa) (ksi) (for HSC beams)	100 (14.5)	100 (14.5)	100 (14.5)	100 (14.5)
f_y (MPa) (ksi)	413 (60)	413 (60)	413 (60)	413 (60)
Concrete cover thickness (mm) (in.)	40 (1.6)	60 (2.4)	40 (1.6)	50 (2)

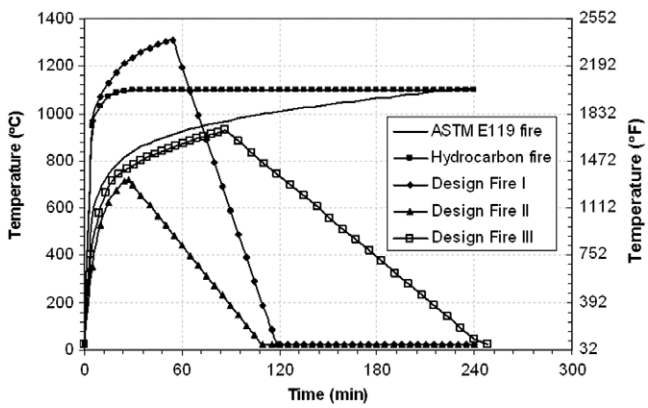

Fig. 2. Time-temperature curves for fire scenarios used in the analysis.

For example, designation NC2AM indicates a beam made of NSC with siliceous aggregate, of cross-section type '2', axially restrained, and has a medium span-to-depth ratio of 13. A sixth character in the form of a numeral is used in the designation of some beams, where the first five characters of the designation were repeated.

Simply supported or axially restrained beams were discretized into 40 segments, while every rotationally restrained beam was discretized into 80 segments. This is mainly because rotational restraint beams require larger number of segments to obtain the same level of accuracy due to the large variation of curvature along the span of a rotationally restrained beam. The cross-section for each segment was idealized into elements such that the mesh size is 20 mm (0.8 in.) for NSC beams and 10 mm (0.4 in.) for HSC beams. Since fire induced spalling is a concern in HSC members a finer mesh was selected for HSC beams so as to obtain better spalling predictions.

The analysis was carried out in 2.5 min time increments until failure occurred in the beam. The failure time (fire resistance) for the analyzed beams is evaluated based on strength, deflection and rate of deflection limit state [15]. These three sets of failure criteria are defined as:

(1) *Strength failure*: when the beam is unable to resist the specified moments under fire conditions.
(2) *Deflection failure*: when the maximum deflection of the beam exceeds $L/20$ at any fire exposure time.
(3) *Rate of deflection failure*: when the rate of deflection exceeds $\frac{L^2}{9000d}$ (mm/min)

where L = span length of the beam (mm), and d = effective depth of the beam (mm).

The minimum failure times computed based on these three limit states is considered to be the fire resistance of the beam. Rebar temperature limit state is not considered in the analysis because this failure criterion is prescriptive and does not represent the actual failure of the beam. To illustrate the effect of various parameters on the fire resistance, some of the results from the parametric studies are given in Table 3. Full results of the analysis, with a detailed discussion, are presented by Dwaikat [16].

The results presented in Table 3 clearly indicate that fire resistance depends on a number of factors, namely, load ratio, cross-sectional dimensions, axial restraint stiffness, span-to-depth ratio, location of axial restraint, rotational restraint, type of aggregate and concrete type (spalling).

Details of the effect of these factors on the overall fire response of RC beams can be found elsewhere [16]. Results from the parametric studies show that the main factors that strongly influence the fire resistance of RC beams are fire scenario, load ratio, span-to-depth ratio, location of axial restraint, and rotational restraint. Factors that have a moderate influence on the fire resistance of RC beams are sectional dimensions (concrete cover thickness and beam width), aggregate type (carbonate or siliceous aggregate), degree of axial restraint, concrete strength, spalling, and failure criteria.

Results from the parametric studies are utilized to develop a design approach for predicting the fire resistance of RC beams. Development of a design approach for fire resistance is quite involved due to the large number of parameters and the interdependency among them, and the details are presented below.

5. Development of design approach

5.1. General approach

As discussed earlier, the fire resistance of an RC beam is influenced by a number of parameters including fire scenario. The existing fire resistance design approaches for RC beams are generally prescriptive and do not account for many of these factors. In addition, it becomes highly complex to account for the large number of fire scenarios that may occur in a building compartment. This can be avoided by using a simple, conservative and reliable approach for establishing time equivalency between standard and design fire scenarios.

A literature review by Kodur et al. [4] indicated that there are few methods for establishing time equivalency between standard and design fires. The review showed that these methods, developed for protected steel member, may not be fully applicable for RC members. Thus, a rational approach for evaluating fire resistance of RC beams is developed here.

Contrary to the current prescriptive approaches, the goal of the proposed approach is not to determine the fire resistance of an RC

Table 3
Summary of the fire resistance values for the analyzed beams.

Studied parameter	Beam designation*	*LR* (%)	*Y*/*H*	k_r (kN/mm) (kip/in.)	Fire resistance based on failure criterion (in minutes)		
					Strength	Deflection	Rate of deflection
Section characteristics	NS1SM	50	0.5	0	135	123	115
	NS2SM	50	0.5	0	215	192	189
	NS3SM	50	0.5	0	183	165	166
	NS4SM	50	0.5	0	233	217	218
	NC1AM	50	0.5	50 (285.8)	143	NF	138
	NC2AM	50	0.5	50 (285.8)	228	227	223
	NC3AM	50	0.5	50 (285.8)	195	NF	192
	NC4AM	50	0.5	50 (285.8)	253	NF	249
Load ratio	NS1SS1	30	0.5	0	168	160	143
	NS1SS2	50	0.5	0	135	128	115
	NS1SS3	70	0.5	0	108	105	93
	NC3AM1	30	0.5	50 (285.8)	258	NF	254
	NC3AM	50	0.5	50 (285.8)	195	NF	192
	NC3AM2	70	0.5	50 (285.8)	145	NF	NF
Aggregate type	NS2SM	50	0.5	0	215	192	189
	NC2SM	50	0.5	0	283	245	244
	NS4AM	50	0.5	50 (285.8)	198	NF	194
	NC4AM	50	0.5	50 (285.8)	253	NF	249
Span-to-depth ration	NS1AS	50	0.5	50 (285.8)	200	NF	194
	NS1AM	50	0.5	50 (285.8)	110	NF	106
	NS1AL	50	0.5	50 (285.8)	75	74	70
Degree of axial restraint	NS1SS2	50	0.5	0	135	128	115
	NS1AS	50	0.5	50 (285.8)	200	NF	194
	NS1AS1	50	0.5	100 (571.6)	210	NF	206
	NS1SL1	50	0.5	0	135	117	115
	NS1AL	50	0.5	50 (285.8)	75	74	70
	NS1AL3	50	0.5	100	63	61	56
Location of axial restraint	NS1AS1	50	0.3	50 (285.8)	123	NF	115
	NS1AS2	50	0.4	50 (285.8)	143	NF	137
	NS1AS	50	0.5	50 (285.8)	200	NF	194
	NS1AS4	50	0.6	50 (285.8)	378	NF	373
	NS1AS5	50	0.7	50 (285.8)	588	NF	582
	NS1AL1	50	0.3	50 (285.8)	98	96	91
	NS1AL2	50	0.4	50 (285.8)	90	88	83
	NS1AL	50	0.5	50 (285.8)	75	74	70
	NS1AL4	50	0.6	50 (285.8)	80	NF	77
	NS1AL5	50	0.7	50 (285.8)	110	NF	106
Rotational restraint	NS2SM	50	0.5	0	215	192	189
	NS2RM	50	0.5	0	363	NF	NF
	NS4AM	50	0.5	50 (285.8)	198	NF	194
	NS4ARM	50	0.5	50 (285.8)	498	NF	NF
Fire scenario	NC1SL1	50	0.5	0	170	146	149
	NC1SL2	50	0.5	0	133	113	113
	NC1SL3	50	0.5	0	NF	105	105
	NC1SL4	50	0.5	0	NF	NF	NF
	NC1SL5	50	0.5	0	NF	NF	NF
	NC1AS1	30	0.5	200 (1143)	428	NF	425
	NC1AS2	30	0.5	200 (1143)	425	NF	423
	NC1AS3	30	0.5	200 (1143)	NF	NF	NF
	NC1AS4	30	0.5	200 (1143)	NF	NF	NF
	NC1AS5	30	0.5	200 (1143)	NF	NF	NF
Concrete type and spalling	NS1SM	50	0.5	0	135	123	113
	HS1SM	50	0.5	0	130	118	105
	NS1AM1	50	0.5	200 (1143)	98	NF	95
	HS1AM	50	0.5	200 (1143)	80	NF	75
	NS1RM	50	0.5	0	318	307	298
	HS1RM	50	0.5	0	120	NF	NF

N/*H*—HSC/NSC *S*/*C*—siliceous/carbonate aggregate 1/2/3/4—cross-sectional size (refer to Table 2 for cross-sectional sizes) *S*/*A*/*R*/AR—simply suppurated/axially restrained/rotationally restrained/axially and rotationally restrained S/M/L—small/medium/large span-to-depth ratio.

beam under design fire exposure, but to determine if an RC beam can survive a design fire scenario. This is achieved in two steps, namely:

- Evaluating the fire resistance of the RC beam (R) under standard fire exposure,
- Determining the time equivalent (t_e) under a design fire scenario utilizing equivalent energy concept, and then checking failure of the beam (under a design fire exposure) which is said to occur when:

$$t_e > R \Rightarrow \text{Failure} \quad (1)$$

where:
R = fire resistance of RC beam, and
t_e = time equivalent.

The proposed approach is derived by giving full consideration to critical parameters, including fire scenario, load ratio, and restraint

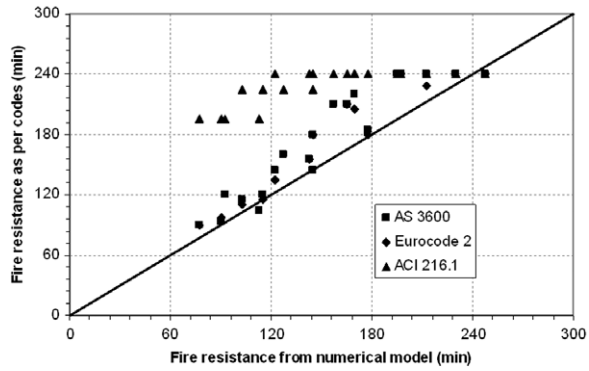

Fig. 3. Initial estimates of fire resistance (in minutes) based on beam width and concrete cover thickness (R_0).

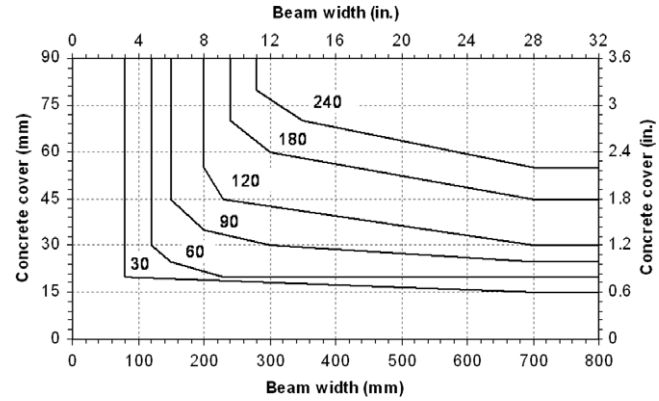

Fig. 4. Initial estimates of fire resistance (minutes) based on beam width and concrete cover thickness (R_0) (reproduced from AS 3600 [3]).

effects, that influence the fire resistance of RC beams. Influencing parameters such as section characteristics, load ratio, axial restraint, rotational restraint, span-to-depth ratio, and aggregate type are accounted for in developing the fire resistance equation under standard fire exposure. In the second step, the fire scenario is accounted for in evaluating the time equivalent.

5.2. Equation for fire resistance under standard fire conditions

A simplified expression is developed for evaluating the fire resistance of an RC beam under standard fire exposure. The equation is developed in a two sub-step process wherein fire resistance is first estimated based on thermal considerations and then correction factors are applied to account for other factors influencing fire resistance.

- The first sub-step is to estimate an initial value of fire resistance, R_0, based on the thermal consideration by applying the rebar temperature failure criteria. For estimating initial fire resistance (R_0), the provisions in current codes can be utilized since fire resistance in these codes (such as ACI 216.1, Eurocode 2 and AS 3600) are mainly derived based on rebar temperature failure criteria. To develop a base value for estimating R_0, twenty simply supported RC beams were analyzed. The beams were assumed to be made of four cross-sections as given in Table 2. For each cross-section, five values of concrete cover thickness to rebar, namely, 30, 40, 50, 60 and 70 mm, were assumed in the analysis. The fire resistance for the analyzed beams was computed based on rebar temperature failure criteria, as well as by applying the prescriptive provisions in three codes of practice (ACI 216.1, Eurocode 2 and AS 3600). The results are plotted in Fig. 3. It can be seen from the figure that the fire resistance computed based on ACI 216.1 is higher than the predicted value (from the numerical model) for most of the analyzed beams. It can also be seen that ACI 216.1 predicts the same fire resistance for many of the beams having different concrete cover thickness and cross-sectional dimensions. It should also be noted that ACI 216.1 specifies a fire resistance rating at a large time increment (1, 2, 3, and 4 h) and may not provide reliable fire resistance with finer time increments. Thus, ACI 216.1 provisions may not provide a reasonable estimate for R_0.
  Fig. 3 also shows that the Eurocode 2 and Australian code predictions are similar. However, the Australian code provisions specify different combinations of concrete cover thickness and beam widths to obtain a required fire resistance rating. Further, similar to ACI 216.1, Eurocode 2 provisions specify fire resistance rating at relatively large time increment (30 min) compared to the Australian code. Thus, the graph in the Australian code, reproduced in Fig. 4, is selected to obtain an initial estimate of fire resistance, R_0. It should be noted that R_0 is an estimate and any graph similar to that in Fig. 4 can be utilized.
- The second step in computing fire resistance is to apply modification factors to estimated R_0 values to account for other critical factors that influence fire resistance of the beam. Three modification factors are to be applied, namely:
  - A 'structural factor' to account for the influence of support conditions (SS, AR, ER, RR, and FR), load ratio, location of axial restraint, and span-to-depth ratio.
  - An 'aggregate factor' to account for the influence of aggregate (siliceous or carbonate) in concrete.
  - A 'strength factor' to account for the influence of concrete strength and spalling.

Data generated from parametric studies is utilized to develop a relationship for the three modification factors using a regression analysis (presented below). The fire resistance data for the analyzed NSC beams, made of siliceous aggregate concrete, is used to establish relationships for the structural factor. The rest of the data is used to compute the aggregate and strength factors. Analysis results of the beams were randomly divided into two sets of which one set is used for regression analysis (calibration) and the other set is used for verification (validation).

Results from the regression analysis indicate that the fire resistance of an RC beam can be expressed as:

$$R = \phi_{st}\phi_{ag}\phi_{cs}R_0 \tag{2}$$

where R = fire resistance of RC beam, R_0 = initial estimate of fire resistance (obtained from Fig. 4), ϕ_{st} = structural modification factor, ϕ_{ag} = aggregate modification factor, and ϕ_{cs} = strength modification factor.

Regression analysis indicates that ϕ_{st} mainly depends on support conditions, load ratio, axial restraint stiffness, location of axial restraint, span-to-depth ratio, and section characteristics. Based on the relationship between ϕ_{st} and these parameters, five empirical relationships were developed for the five support conditions (SS, AR, ER, RR and FR). For each support condition, the empirical relationships were developed such that its predictions had minimal variation from the values predicted by the finite element model. Regression analysis was carried out using statistical software 'SPSS' [17] to find functions that accurately fit the data for different support conditions. It was found that the residuals from the regression analysis are not uniformly distributed for AR and ER beams. Thus, transformation (logarithmic) is used for predicted and simulated values to reduce the non-uniformity of the residuals for these two cases (AR and ER). The regression analysis is repeated for AR and ER with the transformed values to find functions that best fit the data. The obtained functions of the structural modification factor (ϕ_{st}) for beams with each of the five support conditions were then simplified to arrive at the following equations:

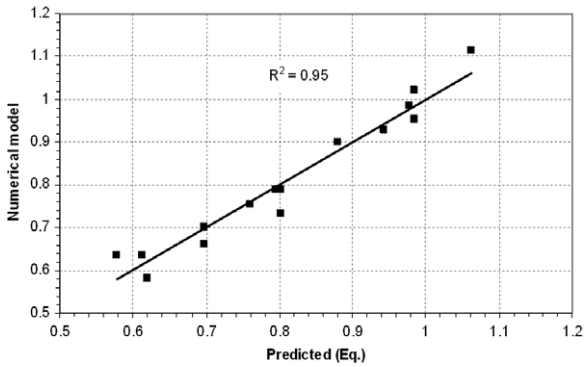

Fig. 5. Simulated versus predicted values of structural factor (ϕ_{SS}).

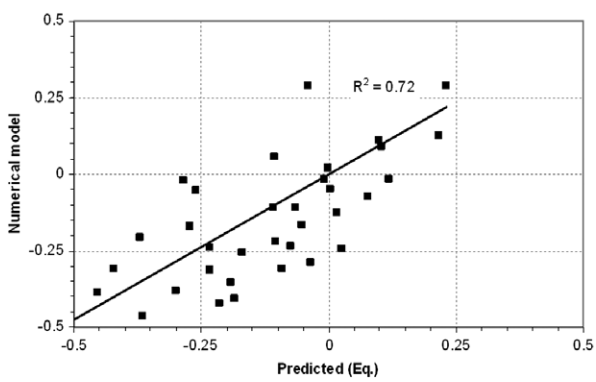

Fig. 6. Simulated versus predicted values of structural factor ($LN(\phi_{AR})$).

$$\phi_{SS} = 1.4 - LR - 0.02\psi_0 \quad (\psi_0 \text{ in meters}) \tag{3}$$
$$\phi_{SS} = 1.4 - LR - 0.006\psi_0 \quad (\psi_0 \text{ in feet})$$

$$\phi_{AR} = 0.9\phi_{SS} - 0.2\left(\frac{L}{H} - 14\right)(0.1 + \text{ax}) \tag{4}$$

$$\phi_{ER} = 0.8\begin{Bmatrix} 0.8\phi_{AR} + 0.2, & \frac{Y}{H} < 0.6 \\ \phi_{AR} + \left(18 - \frac{L}{H}\right)\left(\frac{Y}{H} - 0.6\right) & \frac{Y}{H} \geq 0.6 \end{Bmatrix} \tag{5}$$

$$\phi_{RR} = \phi_{SS} + 3 - 3.7LR \tag{6}$$

$$\phi_{FR} = \phi_{RR} + 0.5\phi_{AR}^2 \tag{7}$$

where $\phi_{SS}, \phi_{AR}, \phi_{ER}, \phi_{RR}$ and ϕ_{FR} = structural factors for SS, AR, ER, RR and FR beams, respectively, ψ_0 = section characteristic factor = $\frac{\chi}{SF\rho}$, ρ = steel ratio = area of tension steel/effective area of cross-section, SF = section factor = heated parameter/cross-sectional area, χ = number of corner bars/total number of bars, LR = load ratio, L = span of the beam, H = total depth of the beam, Y = location of axial restraint measured from the topmost fibers of the beam cross-section, ax = $\frac{k_r L}{E_c bH}$, k_r = axial restraint stiffness, b = beam width, and E_c = modulus of elasticity of concrete.

The simulated values of the structural factor (computed from the actual data assuming ϕ_{ag} and ϕ_{cs} to be 1.0 for siliceous aggregate and NSC, respectively) are plotted against the predicted values from the best fit equations in Figs. 5–9 for beams with different support configurations. The trends in these figures show that the best fit equation is sufficiently accurate in predicting the structural parameters. The coefficient of determination was found to be 0.95, 0.72, 0.79, 0.92 and 0.73 for the five equations, see Figs. 5–9. The low coefficient of determinations for AR, ER and FR support conditions can be attributed to large number of influencing parameters and the interactions among these parameters. Accounting for such a large number of parameters will make the equation very complex and unsuitable for design purposes.

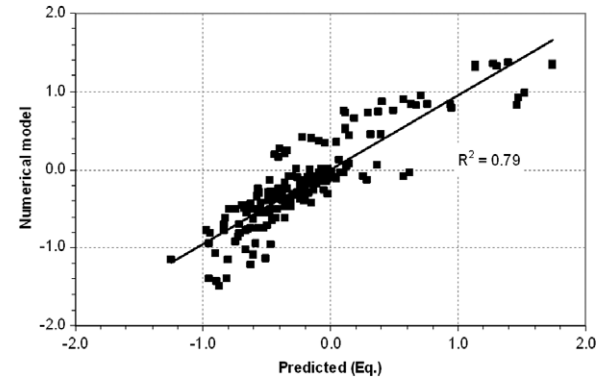

Fig. 7. Simulated versus predicted values of structural factor ($LN(\phi_{ER})$).

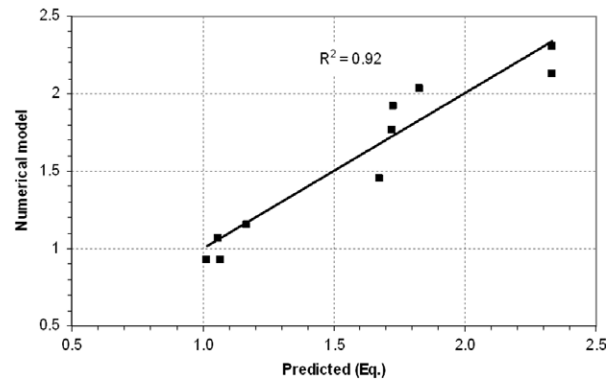

Fig. 8. Simulated versus predicted values of structural factor (ϕ_{RR}).

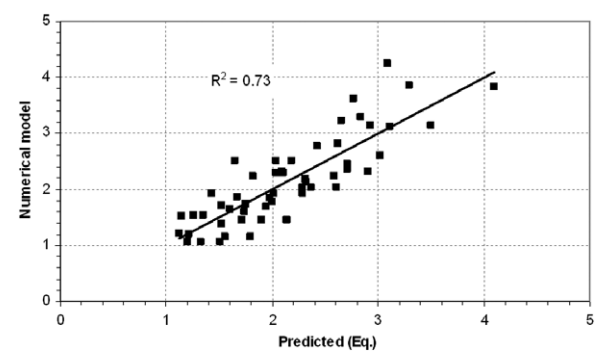

Fig. 9. Simulated versus predicted values of structural factor (ϕ_{FR}).

The second modification factor (ϕ_{ag}) is derived to take into account the influence of aggregate type. In practice, two types of aggregate are most commonly used in concrete mix, namely, siliceous and carbonate. In the regression analysis, siliceous aggregate concrete is taken as the base line and hence its aggregate factor is 1.0. Results from parametric studies indicate that beams made of carbonate aggregate concrete have about 20% higher fire resistance than those made of siliceous aggregate concrete. Further, previous fire resistance tests on RC structural members have shown that members made of carbonate aggregate concrete have 10%–20% higher fire resistance than those made of siliceous aggregate concrete [18]. Thus, for carbonate aggregate concrete, the aggregate factor is selected to be 1.2.

The simulated fire resistance values for carbonate aggregate RC beams are plotted versus the predicted values in Fig. 10. It can be seen from the figure that the predicted fire resistance values match well with the simulated values, and are conservative for

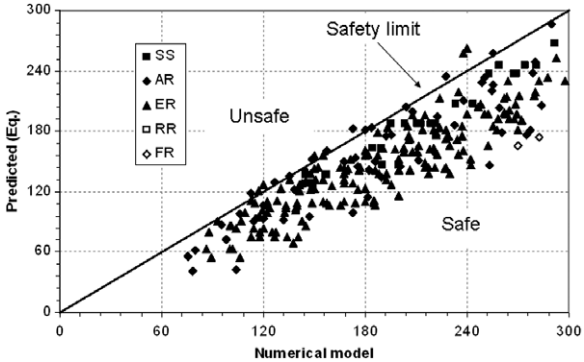

Fig. 10. Simulated versus predicted fire resistance (in minutes) for RC beams made of carbonate aggregate concrete.

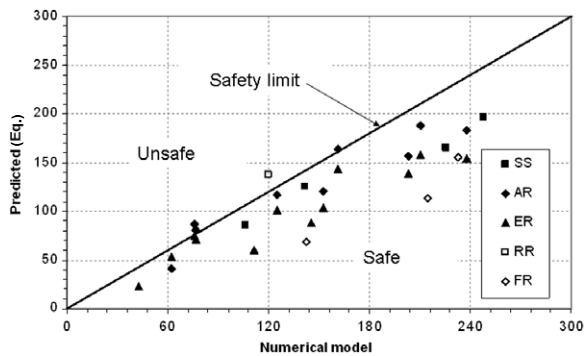

Fig. 11. Simulated versus predicted fire resistance (in minutes) for HSC beams.

most of the carbonate aggregate concrete beams. Only for few beams, predicted fire resistance values are unconservative, but are very close to the simulated values. This indicates that the proposed ϕ_{ag} factor is capable of accounting for aggregate influence on fire resistance of RC beams.

The third modification factor (ϕ_{cs}) to be accounted for in the fire resistance equation is the influence of concrete strength and fire induced spalling. In the regression analysis, the value of the material modification factor is assumed to be 1.0 for NSC beams. Results from parametric studies indicate that rotationally unrestrained (SS, AR and ER) HSC beams have 5%–25% lower fire resistance than NSC beams. The results also show that rotationally restrained (RR and FR) HSC beams have 40%–60% lower fire resistance than NSC beams. Previous studies of RC beams and columns have shown that HSC members have 20%–30% lower fire resistance than those made of NSC [6,19]. Thus, for HSC beams strength modification factors of 0.8 and 0.5 were selected for rotationally unrestrained and restrained beams, respectively. The reason for selecting two different values is that concrete strength (and spalling) has a large influence on the fire resistance of RR and FR beams [16]. The values are selected in such a way that the fire resistance predictions for the most of the analyzed HSC beams are conservative.

The simulated fire resistance values (using the computer program) for HSC beams are plotted against the predicted values (based on Eq. (2)) in Fig. 11. The predicted fire resistance values are in good agreement with the simulated values and are conservative for most of the analyzed beams. Also, the fire resistance predictions are slightly unconservative for only few HSC beams and the predicted fire resistance values for these beams are very close to the simulated values. This indicates that the proposed ϕ_{cs} factor is capable of accounting for the influence of concrete strength on fire resistance calculations.

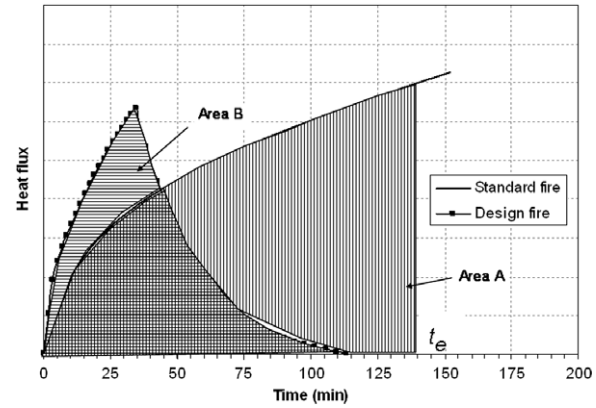

Fig. 12. Equivalent energy concept for standard and design fires.

5.3. Establishing fire resistance under design fires

The fire resistance performance of a beam under a design fire exposure can be related to its fire resistance under standard fire exposure (R) provided a time equivalency is established between standard and design fire scenarios. Kodur et al. [4] proposed a reliable and conservative semi-empirical approach for establishing time equivalency between standard and design fire scenarios. This approach is based on the concept of equivalent energy, which assumes that the fire severity, and thus the fire resistance, depends on the amount of energy transferred to the beam. Accordingly, two fires will have the same fire severity if they transfer the same amount of energy to an RC beam. The amount of energy transferred to an RC beam exposed to fire is related to the heat flux on the fire exposed boundaries of the beam, which involves heat transfer through convection and radiation. To arrive at equivalency, first the total area under the heat flux curve for the design fire (area B in Fig. 12) is computed. The area under the heat flux of a standard fire (area A in Fig. 12) is computed at various time steps. According to the proposed time equivalent method, the time at which area A (which varies as a function of time) equals area B is the time equivalent of the design fire. Full details on the time equivalency approach can be found elsewhere [4,16].

Once the time equivalency is established between standard and design fire scenarios, the failure of the beam is checked by comparing time equivalency with the fire resistance of the beam. The failure of the beam is considered to occur if the time equivalency of the design fire exceeds the fire resistance of the beam.

6. Validation of the proposed approach

The validity of the proposed approach in evaluating fire resistance of RC beams is established by comparing the predictions from the proposed approach with results from numerical studies and from fire resistance tests for both standard and design fire scenarios.

For validation of proposed equation, under standard fire exposure, results for the second set of beams (other than the one used for regression analysis) were used. The comparison is illustrated in Fig. 13 for beams with different support conditions (SS, AR, ER, RR and FR). In the figure, the 'safety limit' line represents the boundary where the predictions from proposed equation coincide with the simulated values. Any prediction above the 'safety limit' line indicates that the proposed expression overestimates the fire resistance and hence is unconservative.

It can be seen in Fig. 13 that predictions from the proposed equation compare well with the simulated values (using the numerical model), and are conservative for most of the beams. The

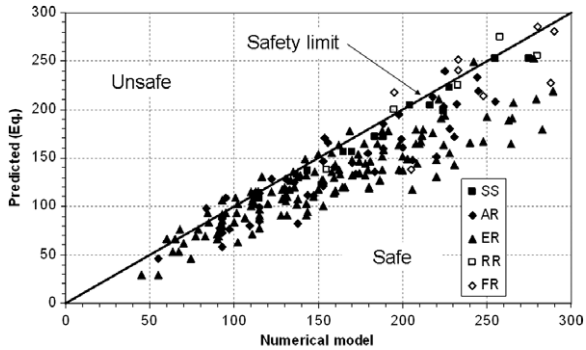

Fig. 13. Simulated versus predicted fire resistance (in minutes) for RC beams.

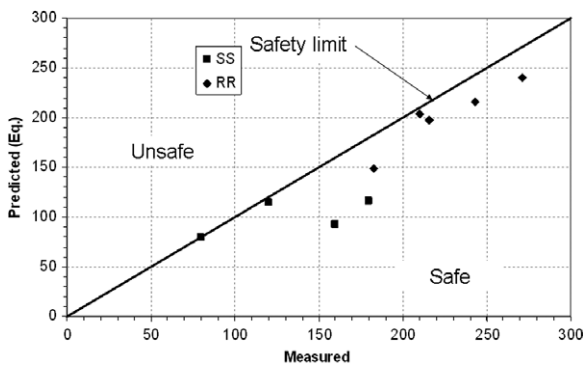

Fig. 14. Measured versus predicted fire resistance (in minutes) for RC beams.

predicted fire resistance values are unconservative for only few beams, most of which have fire resistance higher than 3 h. Also, some predictions from the equation are highly conservative mostly for fire resistance beyond 3 h. This may not be a major drawback since in most practical applications fire resistance ratings of 1–3 h is required for RC beams. Therefore, it is deemed that the proposed equation is capable of predicting the fire resistance of RC beams with a sufficient degree of accuracy.

The validity of the equation is also established by comparing the predictions from the proposed equation with a measured fire resistance from tests. Nine RC beams, selected from the literature, were used in the validation. Six of these RC beams are tested by Lin et al. [20], two beams are tested by the authors [6] and one beam tested by Dotreppe and Franssen [21]. Other tested beams reported in the literature are not used for the validation of the method due to the fact that most of these tests were undertaken on small scale specimens and some were aimed at evaluating the extent of spalling in HSC beams or the residual strength of RC beams. The nine beams include four simply supported and five rotationally restrained beams. The properties of the RC beams together with measured values of fire resistance (as reported from fire tests), are provided in Table 1. Fig. 14 shows the comparison of the predicted and measured fire resistance (using the proposed expression) to the measured fire resistance values. As before, the 'safety limit' line in this figure represents the boundary where the predictions from proposed equation coincide with the test measurements.

It can be seen in Fig. 14 that the predictions from the proposed equation compare well with the test results, and are conservative for most of the beams. Also, the predictions from the equation are highly conservative for beams B1 and B3 tested by Dwaikat and Kodur [6]. This can be attributed to the large variation in the high temperature properties for concrete and reinforcing steel which results in a large variation in the computed fire resistance. However, the computed fire resistance is conservative for these two beams. Overall, the fire resistance equations, presented here,

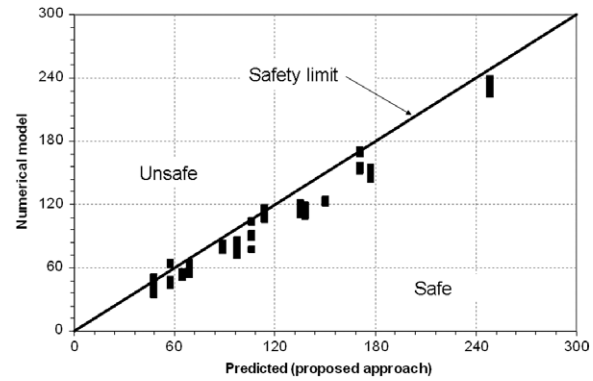

Fig. 15. Simulated versus predicted time equivalent (in minutes).

seem to produce reasonable fire resistance predictions for the nine tested beams.

The proposed energy-based time equivalent approach is validated by comparing the time equivalent predictions from this approach with those obtained from finite element analysis for 90 beam-fire combinations. Data for this validation are generated by analyzing beams with four different concrete cross-sections (described in Table 2), three load levels, fourteen design fire scenarios (which represent typical fire exposures in a building compartment), three span-to-depth ratios, and three values of axial restraint stiffness. Thirteen of the analyzed beams were assumed to be made of HSC with compressive strength 100 MPa. The remaining beams were assumed to be made of NSC. More details on the characteristics of the analyzed beams can be found elsewhere [16]. The main results from the analysis are summarized Fig. 15.

Fig. 15 shows the comparison of the predicted time equivalent (using the proposed method) with the simulated values from finite element analysis. It can be seen in Fig. 15 that the predictions from the proposed method compare well with the simulated values, and are conservative for most of the beams. The predicted time equivalent values are slightly unconservative for very few beams. In these cases the predicted values are close to the simulated values. Compared to the currently available time equivalent methods, the proposed method shows less variation [16]. Thus, the proposed equal energy method can be considered as a reliable tool for the estimation of the time equivalent for design fires.

Comparison with current codes

To further evaluate the proposed approach, fire resistance predictions from Eq. (2) are compared to those obtained from current code provisions for eleven RC beams. Nine of the selected beams are the ones used in the validation against fire tests results, and the remaining four represent typical RC beams used in practice. Details of the properties of the eleven beams are given in Table 1. Fig. 16 shows a comparison of the fire resistance obtained from current provisions in ACI 216.1 [1], Eurocode 2 [2] (tabulated data and the isotherm method) and AS 3600 [3], with those predicted by the proposed equation (Eq. (2)). The ACI code overestimates the fire resistance (significantly) for most of the RC beams. This is mainly due to the fact that ACI provisions give fire resistance ratings at 1, 2, 3 and 4 h. These ratings are mostly based on concrete cover thickness requirements and minimum section dimensions, but do not fully account for critical factors such as load ratio, span-to-depth ratio, and axial restraint effects. However, ACI 216.1 underestimates the fire resistance for the two rotationally restraint beams. This can be attributed to the fact that the tabulated fire resistance in ACI 216.1 does not fully account for the rotational restraint effect since the definition of support conditions (restrained or unrestrained) is not clearly addressed in the code provisions. Thus, it is not clear whether the restrained

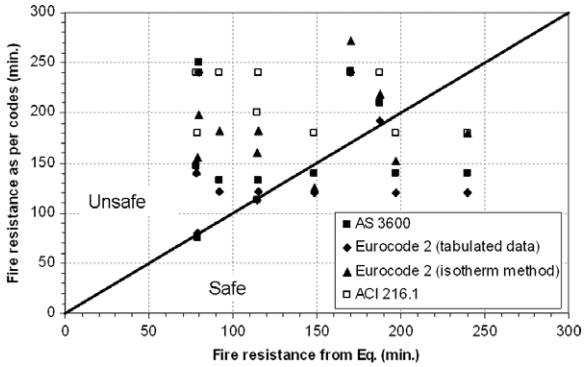

Fig. 16. Comparison of fire resistance predicted from proposed equation with provisions in current codes.

case refers to rotationally restrained, axially restrained or both rotationally and axially restrained.

It can be seen from Fig. 16 that predictions from both methods of Eurocode 2 are also unconservative for most of the beams, particularly axially restrained slender beams (with high span-to-depth ratio) because this important parameter is not accounted for in the current Eurocode methods. However, despite the fact that Eurocode specifications account for rotational restraint, the fire resistance predictions based on that code are overly conservative for some cases (particularly rotationally restrained beams). This can be mainly attributed to the fact that both methods in Eurocode 2 do not account for the interdependent influence of load ratio and rotational restraint on the fire resistance of RC beams.

Fig. 16 also shows that the Australian code predictions are similar to those from Eurocode 2 for all beams. This can be attributed to the fact the fire resistance provisions in both AS 3600 and Eurocode 2 are derived based on concrete cover thickness and beam width, and do not account for other governing parameters such as load ratio, restraint and span-to-depth ratio. Overall, the proposed equation provides better predictions of fire resistance than the current code estimates since it accounts for critical governing parameters.

7. Limitations

The proposed equation expresses fire resistance as a function of structural, material and fire parameters, and thus offers a convenient way for evaluating fire resistance. Since the proposed equation is based on results from numerical studies, it is necessary to set limits of applicability on the parameters such that they are within the range of values used for developing the equation. Overall, the proposed equation is valid for the following range of variables:

- Fire resistance (R) duration: 1–5 h.
- Type of fire exposure: ASTM E119 standard fire, equivalent standard fire scenarios such as ISO 834 standard fire, or any design fire that follows the specifications of Eurocode 1 [22] (or SFPE [23]).
- Span-to-depth ratio: 8–18.
- Specified 28 day compressive strength (f'_c): 30–100 MPa.
- Axial restraint ratio (ax): 0%–50%.
- Tension steel ratio (ρ): 0.7%–1.7%.
- Location of axial restraint (Y/H): 0.3–0.7.
- Aggregate type: Siliceous and carbonate aggregate.
- Section characteristics factor (ψ_0): 2.8–10.5 m.

8. Design applicability and implications

The proposed approach can be conveniently applied for evaluating the fire resistance of RC beams in design situations by applying the following procedure:

- Develop an initial estimate of fire resistance (R_0) based on the concrete cover thickness and beam width.
- Compute the structural modification factor using Eqs. (3)–(6) or (7) based on the boundary conditions of the beam.
- Determine the aggregate and material modification factors.
- Apply modification factors to R_0 and compute fire resistance (R) of the beam using Eq. (2), for a standard fire exposure.
- To estimate fire resistance under design fire, compute the time equivalent between design fire and standard fire scenarios.
- Check the failure of the beam using the following expression:

  time equivelant > fire resistance ⇒ Failure.

Application of these steps for evaluating the fire resistance of RC beams under a standard fire scenario is illustrated through a worked example in Appendix B.

The proposed approach provides a convenient way of obtaining fire resistance of RC beams, and thus can be used for evaluating fire resistance in lieu of full-scale standard fire resistance tests or detailed numerical analysis. It incorporates significant parameters that influence the fire resistance of RC beams. Thus, the proposed approach provides a simple but rational approach for evaluating the fire resistance of an RC beam under realistic loading, fire and restraint conditions. Also, the proposed approach expresses fire resistance in terms of commonly used structural parameters and thus can easily be integrated into structural design. The approach facilitates a rational methodology for achieving desired fire resistance by varying parameters such as beam dimensions, length and load. The applicability of the proposed equation in a design simulation is illustrated through a solved numerical example, presented in Appendix B. The proposed equation provides a better estimate of fire resistance than those predicted by current codes of practice. Therefore, this approach can be considered for incorporation in codes and standards.

9. Conclusions

Based on the results of this study the following conclusions can be drawn:

- Fire scenario, load ratio, span-to-depth ratio, location of axial restraint and rotational restraint have a significant influence on the fire response of RC beams. Concrete cover thickness, beam width, aggregate type, axial restraint stiffness, and concrete strength and spalling have a moderate influence on the fire response of beams. Strength of reinforcement and ratio of tension steel do not significantly influence the fire resistance of RC beams.
- The proposed fire resistance approach accounts for critical governing parameters and thus is capable of predicting fire resistance of NSC and HSC beams under realistic fire, loading and support conditions.
- The proposed energy based time equivalent approach is capable of predicting time equivalent of design fires with an accuracy that is sufficient for design purposes.
- In the proposed approach, fire resistance is expressed in terms of commonly used design parameters and therefore fire resistance calculations can be generally integrated into structural design.
- The proposed approach provides better fire resistance predictions than those obtained from current code provisions.

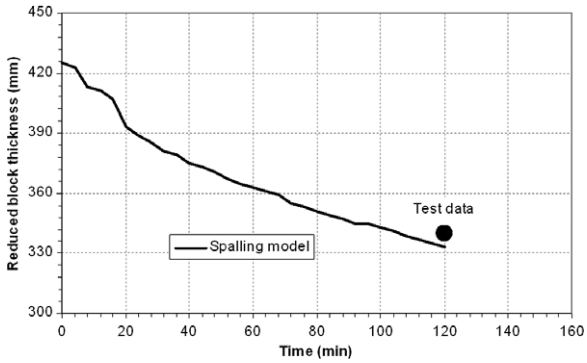

Fig. A.1. Measured and predicted block thickness as a function of time.

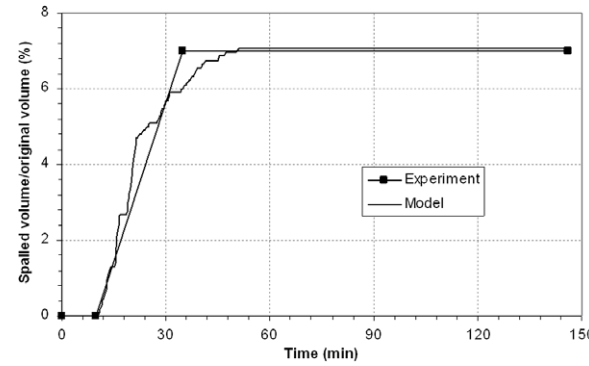

Fig. A.2. Measured and predicted extent of spalling as a function of time for beam B5.

Acknowledgements

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Appendix A

The validity of the spalling sub-model is also established by comparing the quantity of spalled concrete with the measured spalling values in the fire tests conducted by Bilodeau et al. [24]. In their tests, six concrete blocks of 610 × 425 × 770 mm size were exposed to ASTM E1529 hydrocarbon fire for two hours in a furnace. Only one face of each tested block was exposed to fire, while all the other faces are covered with insulation. Of the six blocks, Block 5, which was made without polypropylene fibers, had the largest amount of spalling, and thus this block is used for the validation of the model. The measured and predicted reduction in the block thickness (loss of cross-section) as a function of fire exposure time is presented in Fig. A.1 It should be noted that the extent of spalling was only measured at the end of the fire test and not throughout the test and thus, one measured value of reduced block thickness is plotted as shown in Fig. A.1. It should be noted that spalling was only measured at the end of the fire test and not throughout the test. Thus, the reduced block thickness was only measured at the end of the test as can be seen in Fig. A.1. The comparisons in Fig. A.1 show good agreement between predicted and measured concrete spalling.

Predictions from the spalling sub-model are also compared with spalling measurements for beams B5 (tested by Dwaikat and Kodur [6]). The beam has similar properties to those of beam B3 in Table 1 but with severe design fire exposure. Details of beam properties and fire exposure can be found in Ref. [6]. The predicted quantities and times of spalling are compared with the measured values for beam B5 in Fig. A.2. It can be seen that there is good agreement between the predicted and measured values in the entire range of fire exposure. The model predicts spalling to start at about 11 min and stop at 50 min after fire exposure. The visual observations in the test indicated that spalling started and stopped at 10 and 35 min after fire exposure, respectively. These results indicate that the program predicts well the start time of spalling. In contrast, there is variation between the predicted and the measured end time of spalling. This variation might be acceptable since the mode predictions indicate that small amount of spalling after 35 min for this beam. This can be attributed to the drying of concrete which reduces the developed pore pressure and hence decreases the spalling at later stages of fire.

The overall fire resistance macroscopic finite element model has been validated by comparing temperatures, deflections, spalling and fire resistance values against measured values in fire resistance tests [5–8]. Good agreement was obtained under various fire, loading and restraint scenarios.

Appendix B. Numerical example

To illustrate the applicability of the proposed equation in design practice, the fire resistance of an RC beam is evaluated. Details of the properties of the beam are shown in Fig. B.1. The beam has a 400 × 600 mm rectangular cross section and span length of 5 m. The beam is reinforced with 6 ϕ 20 mm as tension reinforcement. The clear concrete cover thickness to the rebars is 50 mm. The beam is assumed to be fabricated with carbonate aggregate concrete having a compressive strength of 40 MPa and a modulus of elasticity of 30 GPa. The beam is loaded with a uniformly distributed dead load of 40 kN/m and a live load of 20 kN/m. The fire resistance calculations from the proposed equation are carried out for two support conditions, namely; simply supported and axially restrained with an axial restraint stiffness of 300 kN/mm. The results from the proposed equation are compared with fire resistance values from current codes and standards.

Proposed method

Compute R_0:

Concrete cover = 50 mm (2 in.)

beam width = 400 mm (16 in.) $\Rightarrow R_0 = 157$ min.

Compute load ratio:

First compute the nominal room temperature load carrying capacity of the beam:

$$M_n = A_s f_y \left(d - \frac{A_s f_y}{1.7 f_c' b}\right)$$

$$A_s = \frac{6 \times 20^2 \times \pi}{4} = 1885 \text{ mm}^2$$

$$d = 600 - 50 - 20/2 = 540 \text{ mm}$$

hence, $M_n = 1885 \times 400 \left(540 - \frac{1885 \times 400}{1.7 \times 40 \times 400}\right) \times 10^{-6} = 386.3$ kN m.

The room temperature capacity from different codes will be very similar due to the fact that the values of all strength reduction factors used in computing the room temperature capacity should be assumed 1.0.

$$w_n = \frac{8M_n}{L^2} = \frac{8 \times 386.3}{5^2} = 123.6$$

Load under fire = $1.2 \times 40 + 0.5 \times 20 = 58$ kN/m

$$\text{Load ratio} = LR = \frac{58}{123.6} = 0.47.$$

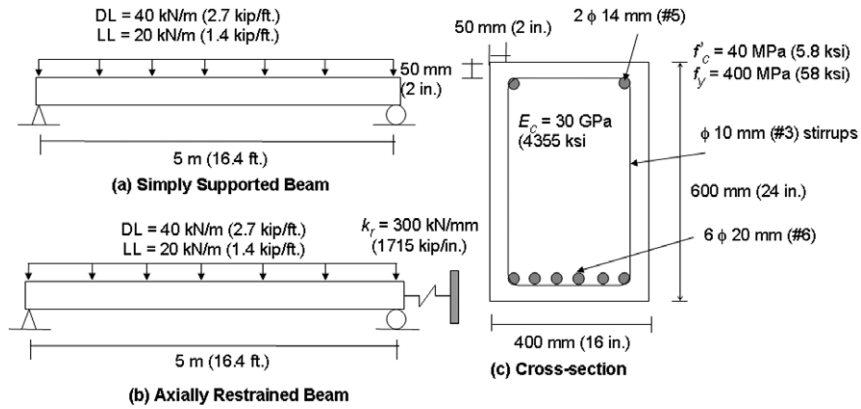

Fig. B.1. Cross-section and elevation of RC beam used in the numerical example.

Table B.1
Prediction of fire resistance values for analyzed beam using proposed method and different codes.

Support conditions	Fire resistance (min)			
	Proposed equation	ACI	Eurocode	Australian code
Simply supported	153	240	156	157
Axially restrained	204	>240	NA[a]	NA

[a] Not applicable.

Calculate ψ_0:

$$\rho = \frac{6 \times 10^2 \pi}{400 \times 540} = 0.0087$$

$$\chi = \frac{2}{6} = 0.333$$

$$SF = \frac{2 \times 0.6 + 0.4}{0.6 \times 0.4} = 6.667\ \mathrm{m}^{-1}$$

$$\psi_0 = \frac{0.333}{6.667 \times 0.0087} = 5.7\ \mathrm{m}.$$

Standard fire resistance for simply supported RC beam (R_{SS}):

$$\mathrm{Rss} = \phi_{SS}\phi_{ag}\phi_{cs}R_0.$$

Carbonate aggregate, $\phi_{ag} = 1.2$.

NSC, $\phi_{cs} = 1.0$.

$$\phi_{SS} = (1.4 - 0.47 - 0.02 \times 5.7) = 0.816.$$

Hence, $\mathrm{Rss} = 0.816 \times 1.2 \times 1 \times 157 = 153$ min.

Standard fire resistance for axially restrained RC beam (R_{AR}):

$$\mathrm{Rss} = \phi_{AR}\phi_{ag}\phi_{cs}R_0$$

$$\mathrm{ax} = \frac{kL}{AE} = \frac{300000 \times 5}{0.4 \times 0.6 \times 30 \times 10^6} = 0.2083$$

$$\frac{L}{H} = \frac{5}{0.6} = 8.333$$

$$\phi_{AR} = 0.9\phi_{SS} - 0.2\left(\frac{L}{H} - 14\right)(0.1 + \mathrm{ax})$$

$$\phi_{AR} = 0.9 \times 0.816 - 0.2\,(8.333 - 14)\,(0.1 + 0.2083) = 1.084$$

$$R_{AR} = 1.084 \times 1.2 \times 1 \times 157 = 204\ \mathrm{min}.$$

Eurocode method [2]

Axis distance $= 50 + 20/2 = 60$ mm beam width $= 400$ mm.

Based on Table 5.5 in Eurocode 2 [2], the fire resistance of this beam is 156 min.

It should be noted that Eurocode 2 does not have clear guidance for evaluating the fire resistance of axially restrained beams.

Australian code method [3]

Concrete cover $= 50$ mm beam width $= 400$ mm.

Based on Figure 5.4.2(A) in the Australian Code [3], the fire resistance for this beam is 157 min.

Similar to Eurocode 2, the Australian code does not have clear guidance for evaluating the fire resistance of axially restrained beams.

ACI method (*ACI* 216.1 [1])

Concrete cover $= 50$ mm beam width $= 400$ mm

Fire resistance for simply supported beam $= 240$ min

Fire resistance for axially restrained beam > 240 min.

Summary

Based on the above calculation the fire resistance calculated from various codes and the proposed equation for standard fire exposure is shown in Table B.1.

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