Digital Image Processing

Chapter 4:
Image Enhancement in the Frequency Domain

Image Enhancement in Frequency Domain

- **Objective:**
  - **To understand the Fourier Transform and frequency domain and how to apply to image enhancement.**
- **Fourier Transform.**
- **Low pass filters.**
- **High pass filters**

Transform Operation

- Fourier: a periodic function can be represented by the sum of sines/cosines of different frequencies,multiplied by a different coefficient (Fourier series).

Fourier Transform: Definitions

1-D

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi ux} dx$$

$$f(x) = \int_{-\infty}^{\infty} F(u) e^{j2\pi ux} du$$

2-D

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx\,dy$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v) e^{j2\pi(ux+vy)} du\,dv$$

2-D Fourier Transform: Properties

- Linearity:
  $$\alpha f(x,y)+\beta g(x,y)\longleftrightarrow \alpha F(u,v)+\beta G(u,v)$$
- Convolution:
  $$f(x,y)*g(x,y)\longleftrightarrow F(u,v)\,G(u,v)$$
- Multiplication:
  $$f(x,y)g(x,y)\longleftrightarrow F(u,v)*G(u,v)$$
- Shift:
  $$f(x\pm x_o,y\pm y_o)\longleftrightarrow F(u,v)e^{\pm j2\pi(ux_o+vy_o)}$$
- Modulation:
  $$f(x,y)e^{\pm j2\pi(u_o x+v_o y)}\longleftrightarrow F(u\mp u_o,v\mp v_o)$$

1-D Discrete Fourier Transform (DFT)

- Suppose $\{f(0), f(1), \ldots, f(M-1)\}$ is a sequence/vector/1-D image of length M. Its M-point DFT is defined as
  $$F(u)=\sum_{x=0}^{M-1} f(x)e^{-j\frac{2\pi}{M}ux}, u=0,1,2,\ldots\ldots, M-1$$
- Inverse DFT
  $$f(x)=\frac{1}{M}\sum_{u=0}^{M-1}F(u)e^{j\frac{2\pi}{M}ux}, x=0,1,2,\ldots\ldots, M-1$$
- Recall: $e^{j\theta}=\cos\theta+j\sin\theta$

1-D DFT: Example

- Example: Let $f(x) = \{1, -1, 2, 3\}$. (Note that M=4.)

$$F(0) = \sum_{x=0}^{3} f(x) e^{-j\frac{2\pi}{4}x*0} = 5$$

$$F(1) = \sum_{x=0}^{3} f(x) e^{-j\frac{2\pi}{4}x*1} = -1 + 4j$$

$$F(2) = \sum_{x=0}^{3} f(x) e^{-j\frac{2\pi}{4}x*2} = 1$$

$$F(3) = \sum_{x=0}^{3} f(x) e^{-j\frac{2\pi}{4}x*3} = -1 - 4j$$

Magnitude, Phase and Power Spectrum

$$F(u) = R(u) + jI(u)$$

Magnitude: $|F(u)| = \sqrt{R^2(u) + I^2(u)}$

Phase: $\phi(u) = \tan^{-1}\left(\frac{I(u)}{R(u)}\right)$

Power Spectrum: $P(u) = |F(u)|^2$

2-D Discrete Fourier Transform (DFT): Definition

DFT

$$F(u,v) = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} f(x,y) e^{-j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$u = 0,1,2,\ldots,M-1, \quad v = 0,1,2,\ldots,N-1$$

IDFT

$$f(x,y) = \frac{1}{MN} \sum_{u=0}^{M-1} \sum_{v=0}^{N-1} F(u,v) e^{j2\pi\left(\frac{ux}{M}+\frac{vy}{N}\right)}$$

$$x = 0,1,2,\ldots,M-1, \quad y = 0,1,2,\ldots,N-1$$

Magnitude, Phase and Power Spectrum

$$F(u,v) = R(u,v) + jI(u,v)$$

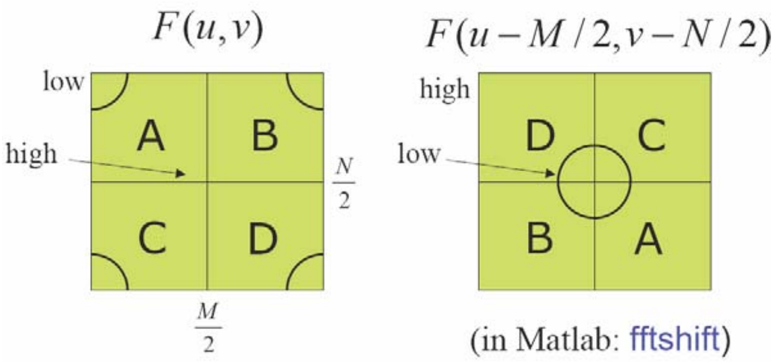
Magnitude: $|F(u,v)| = \sqrt{R^2(u,v) + I^2(u,v)}$

Phase: $\phi(u,v) = \tan^{-1}\left(\frac{I(u,v)}{R(u,v)}\right)$

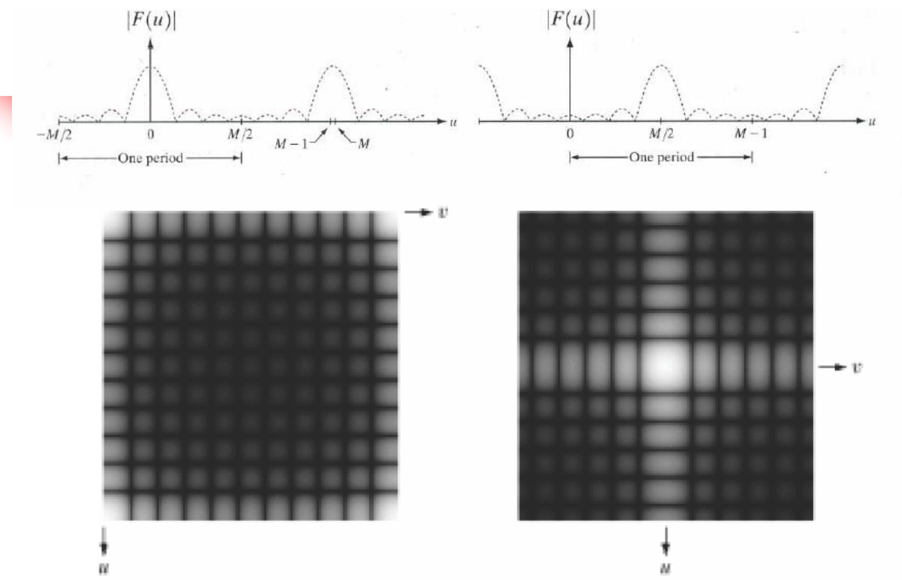
Power Spectrum: $P(u,v) = |F(u,v)|^2$

Displaying the 2-D DFT

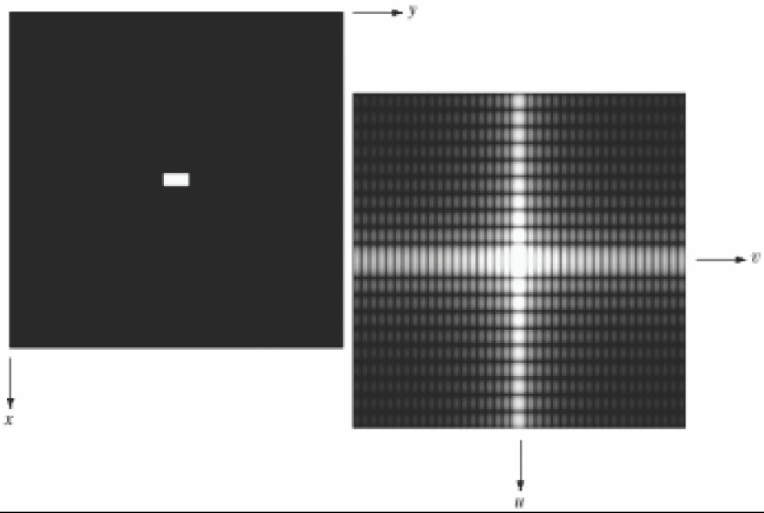
$$f(x,y)(-1)^{x+y} \longleftrightarrow F(u-M/2, v-N/2)$$



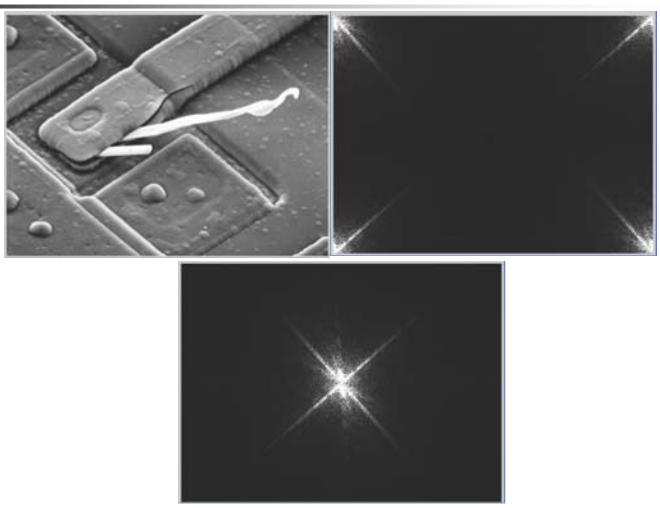
(in Matlab: fftshift)



Example: 2-D DFT of a Rectangle



Example: 2-D DFT



Basic steps for filtering in the frequency domain

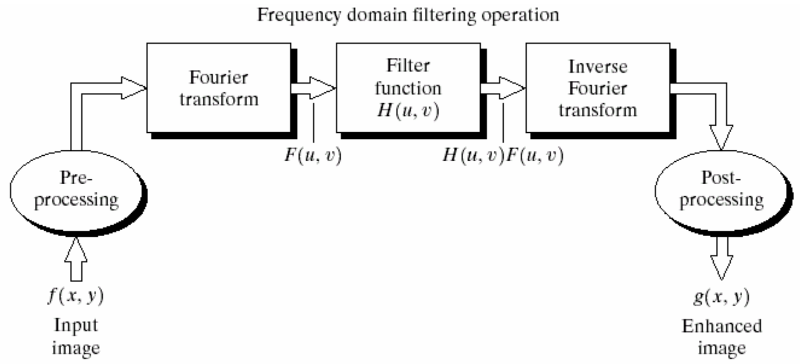


FIGURE 4.5 Basic steps for filtering in the frequency domain.

Basics of filtering in the frequency domain

1. multiply the input image by $(-1)^{x+y}$ to center the transform to u = M/2 and v = N/2
2. compute F(u,v), the 2-D DFT of the image from (1)
3. multiply F(u,v) by a filter function H(u,v)
4. compute the inverse DFT of the result in (3)
5. obtain the real part of the result in (4)
6. multiply the result in (5) by $(-1)^{x+y}$ to cancel the multiplication of the input image.

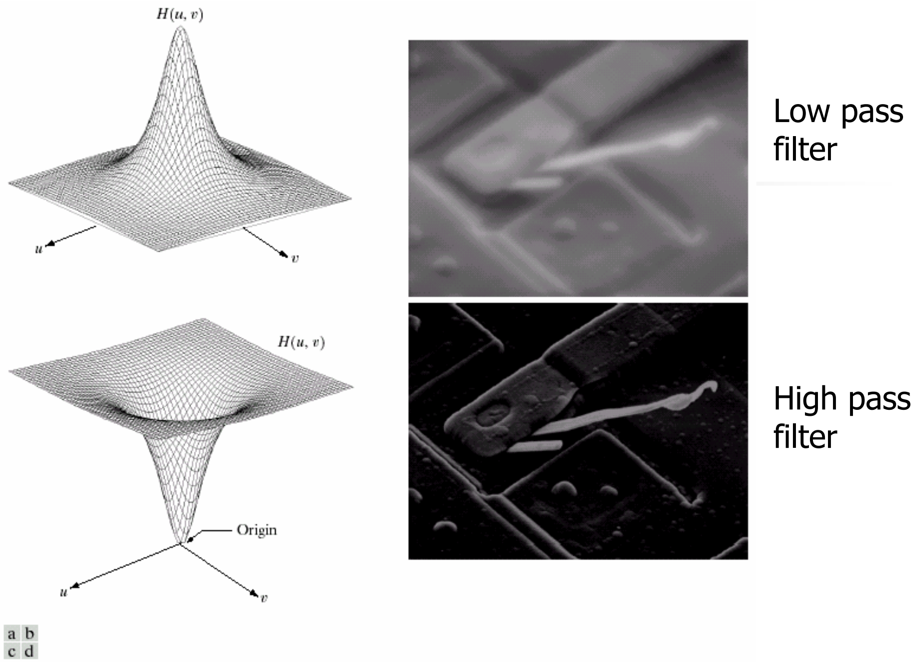


FIGURE 4.7 (a) A two-dimensional lowpass filter function. (b) Result of lowpass filtering the image in Fig. 4.4(a). (c) A two-dimensional highpass filter function. (d) Result of highpass filtering the image in Fig. 4.4(a).

Lowpass Filter (LPF)

- Edges and sharp transitions in gray values in an image contribute significantly to high-frequency content of its Fourier transform.
- Regions of relatively uniform gray values in an image contribute to low-frequency content of its Fourier transform.
- Hence, an image can be smoothed in the Frequency domain by attenuating the high-frequency content of its Fourier transform. This would be a lowpass filter!
- For simplicity, we will consider only those filters that are real and radially symmetric.

Correspondence between filter in spatial and frequency domains

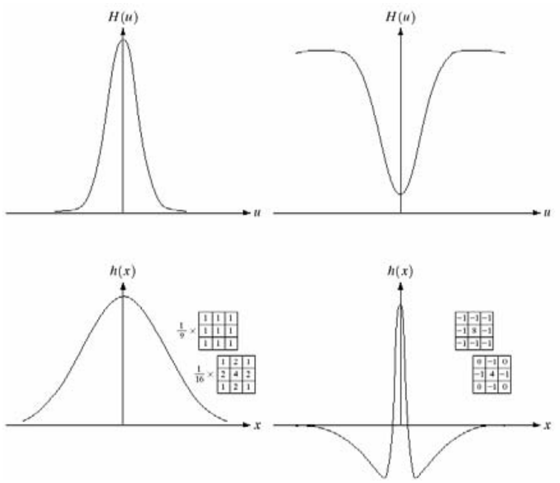


FIGURE 4.9 (a) Gaussian frequency domain lowpass filter. (b) Gaussian frequency domain highpass filter. (c) Corresponding lowpass spatial filter. (d) Corresponding highpass spatial filter. The masks shown are used in Chapter 3 for lowpass and highpass filtering.

Ideal Low Pass Filter

- Has transfer function

$$H(u,v)=\begin{cases}1 & if\ D(u,v)\le D_o\\ 0 & if\ D(u,v)>D_o\end{cases}$$

Where *D(u,v)* is the distance from point (*u,v*) from the origin of the frequency rectangle

- If the center is at (*M*/2,*N*/2)

$$D(u,v)=\sqrt{(u-M/2)^2+(v-N/2)^2}$$

Smoothing Frequency-domain filters: Ideal Lowpass filter (ILPF)

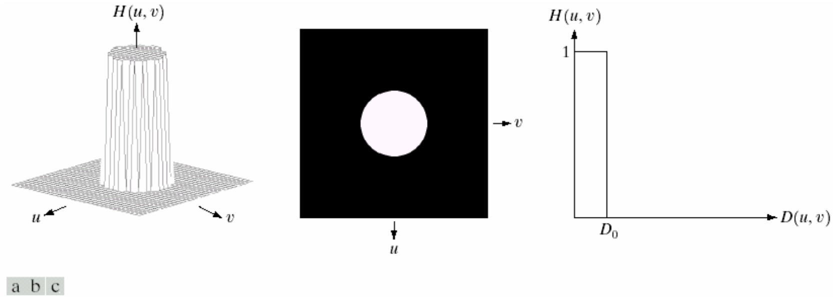


FIGURE 4.10 (a) Perspective plot of an ideal lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross section.

Image Power Circles

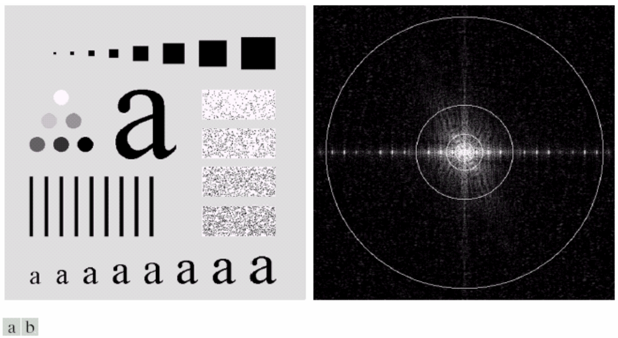


FIGURE 4.11 (a) An image of size 500 × 500 pixels and (b) its Fourier spectrum. The superimposed circles have radii values of 5, 15, 30, 80, and 230, which enclose 92.0, 94.6, 96.4, 98.0, and 99.5% of the image power, respectively.

Result of ILPF

•Notice the severe *ringing* effect in the blurred images, which is a characteristic of ideal filters. It is due to the discontinuity in the filter transfer function.

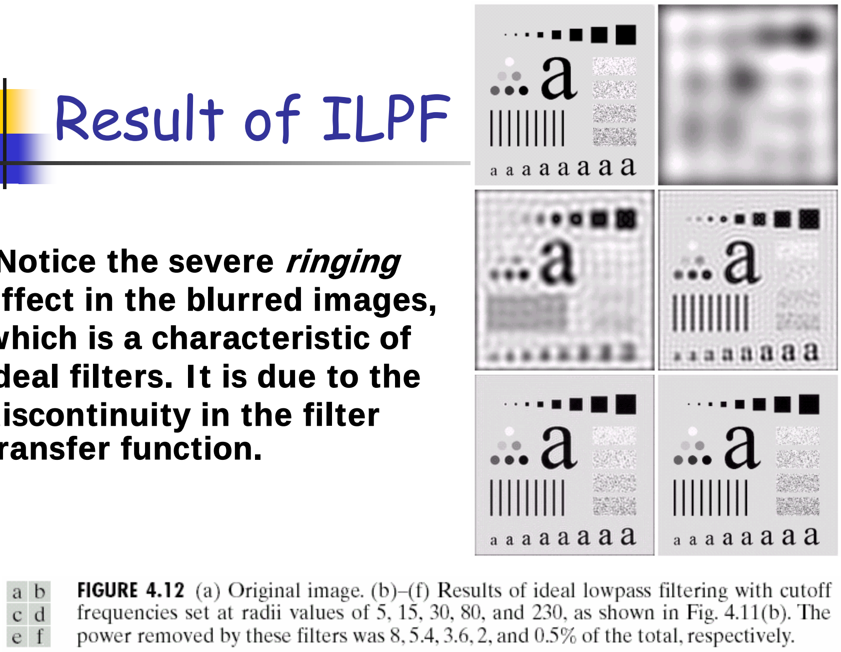


FIGURE 4.12 (a) Original image. (b)–(f) Results of ideal lowpass filtering with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). The power removed by these filters was 8, 5.4, 3.6, 2, and 0.5% of the total, respectively.

Example

•Ideal low pass filter is not practical, because it causes ringing effect.

How to avoid this ringing effect?

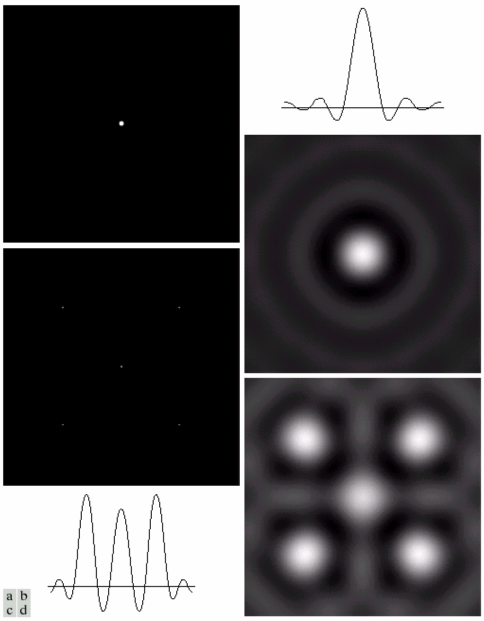


FIGURE 4.13 (a) A frequency-domain ILPF of radius 5. (b) Corresponding spatial filter (note the ringing). (c) Five impulses in the spatial domain, simulating the values of five pixels. (d) Convolution of (b) and (c) in the spatial domain.

Choice Of Cutoff Frequency in LPF

- The cutoff frequency *D0* of the ideal LPF determines the amount of frequency components passed by the filter.
- Smaller the value of *D0*, more the number of image components eliminated by the filter.
- In general, the value of *D0* is chosen such that most components of interest are passed through, while most components not of interest are eliminated.

Butterworth Lowpass Filter: BLPF

$$H(u,v) = \frac{1}{1+[D(u,v)/D_o]^{2n}}$$

•Frequency response does not have a sharp transition
•more appropriate for image smoothing
•not introduce ringing.

•n : filter order
•$D0$: cutoff frequency

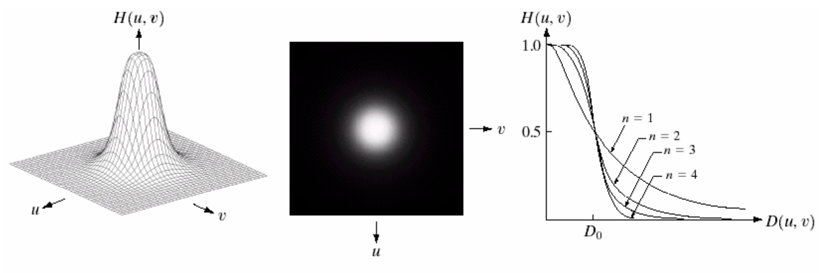
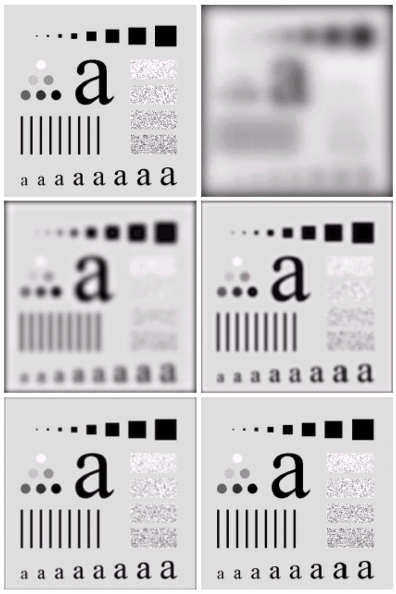


FIGURE 4.14 (a) Perspective plot of a Butterworth lowpass filter transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections of orders 1 through 4.

Example



a b
c d
e f

FIGURE 4.15 (a) Original image. (b)–(f) Results of filtering with BLPFs of order 2, with cutoff frequencies at radii of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Fig. 4.12.

Gaussian Lowpass Filter: GLPF

$$H(u,v) = e^{-D^2(u,v)/2D_0^2}$$

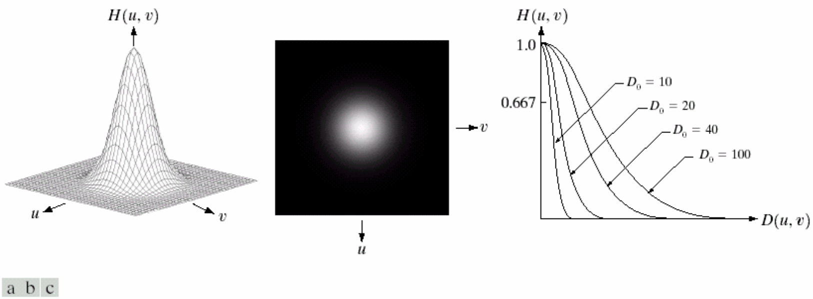


FIGURE 4.17 (a) Perspective plot of a GLPF transfer function. (b) Filter displayed as an image. (c) Filter radial cross sections for various values of D_0.

Example

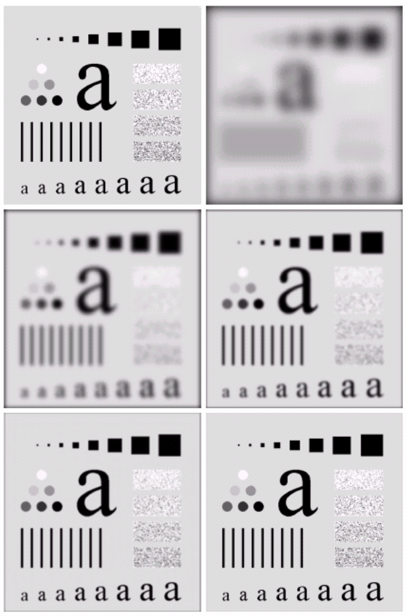


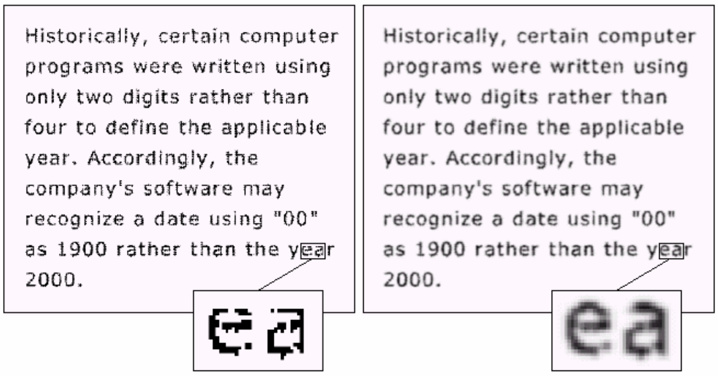
FIGURE 4.18 (a) Original image. (b)–(f) Results of filtering with Gaussian lowpass filters with cutoff frequencies set at radii values of 5, 15, 30, 80, and 230, as shown in Fig. 4.11(b). Compare with Figs. 4.12 and 4.15.

a b
c d
e f

Example

a b

FIGURE 4.19 (a) Sample text of poor resolution (note broken characters in magnified view). (b) Result of filtering with a GLPF (broken character segments were joined).



Example

a b c

FIGURE 4.20 (a) Original image (1028 × 732 pixels). (b) Result of filtering with a GLPF with $D_0 = 100$. (c) Result of filtering with a GLPF with $D_0 = 80$. Note reduction in skin fine lines in the magnified sections of (b) and (c).

Example

a b c

FIGURE 4.21 (a) Image showing prominent scan lines. (b) Result of using a GLPF with $D_0 = 30$. (c) Result of using a GLPF with $D_0 = 10$. (Original image courtesy of NOAA.)

High Pass Filter

- Edges and sharp transitions in gray values in an image contribute significantly to high-frequency content of its Fourier transform.
- Regions of relatively uniform gray values in an image contribute to low-frequency content of its Fourier transform.
- Hence, image sharpening in the Frequency domain can be done by attenuating the low-frequency content of its Fourier transform. This would be a high-pass filter!
- For simplicity, we will consider only those filters that are real and symmetric.

Sharpening Frequency Domain Filter:

Ideal highpass filter

$$H(u,v) = \begin{cases} 0 & \text{if } D(u,v) \le D_0 \\ 1 & \text{if } D(u,v) > D_0 \end{cases}$$

Butterworth highpass filter

$$H(u,v) = \frac{1}{1 + [D_0 / D(u,v)]^{2n}}$$

Gaussian highpass filter

$$H(u,v) = 1 - e^{-D^2(u,v)/2D_0^2}$$

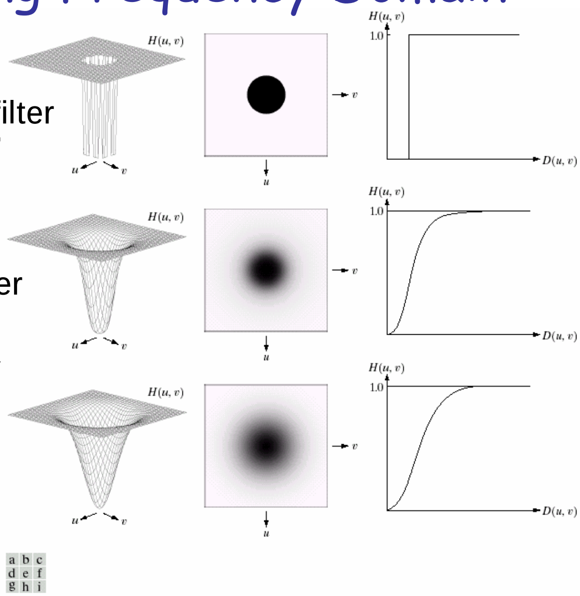


FIGURE 4.22 Top row: Perspective plot, image representation, and cross section of a typical ideal highpass filter. Middle and bottom rows: The same sequence for typical Butterworth and Gaussian highpass filters.

High Pass Filters (HPF)

- Butterworth:
  - Frequency response does not have a sharp transition as in the ideal HPF.
  - This is more appropriate for image sharpening than the ideal HPF, since this not introduce ringing.
- Gaussian:
  - The parameter D_0 measures the spread of the Gaussian curve. Larger the value of D_0, larger the cutoff frequency.
  - No ringing effect

Spatial representation of Ideal, Butterworth and Gaussian highpass filters

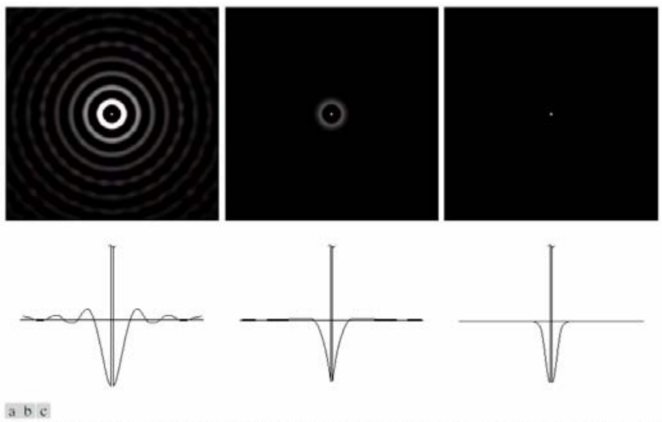
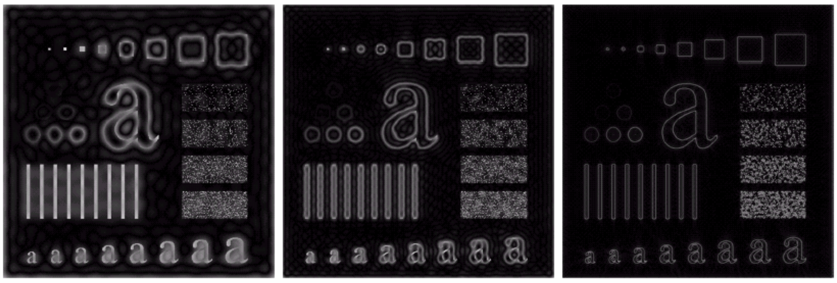


FIGURE 4.23 Spatial representations of typical (a) ideal, (b) Butterworth, and (c) Gaussian frequency domain highpass filters, and corresponding gray-level profiles.

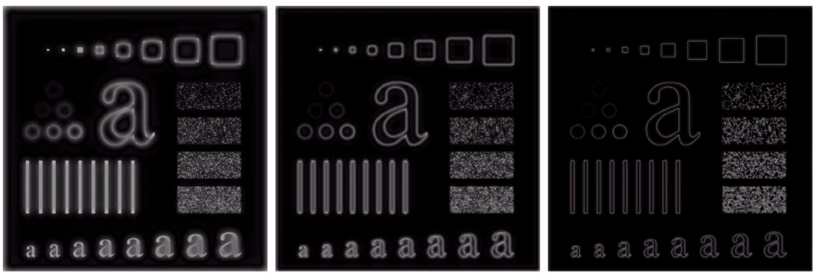
Example: result of IHPF



a b c

FIGURE 4.24 Results of ideal highpass filtering the image in Fig. 4.11(a) with $D_0 = 15$, 30, and 80, respectively. Problems with ringing are quite evident in (a) and (b).

Example: result of BHPF



a b c

FIGURE 4.25 Results of highpass filtering the image in Fig. 4.11(a) using a BHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. These results are much smoother than those obtained with an ILPF.

Example: result of GHPF

a b c

FIGURE 4.26 Results of highpass filtering the image of Fig. 4.11(a) using a GHPF of order 2 with $D_0 = 15$, 30, and 80, respectively. Compare with Figs. 4.24 and 4.25.

Unsharp Masking and High-Boost Filtering in the Frequency Domain

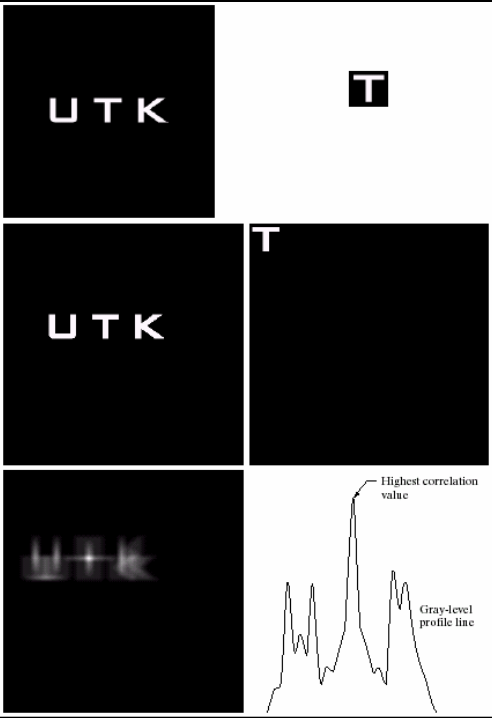
- Unsharp Masking: $f_s(x,y) = f(x,y) - f_{lp}(x,y)$

$$F_s(u,v) = F(u,v) - F_{lp}(u,v) = (\underbrace{1 - H_{lp}(u,v)}_{H_{hp}(u,v)})F(u,v)$$

- High-Boost Filtering: $f_{hb}(x,y) = Af(x,y) - f_{lp}(x,y)$ $(A \geq 1)$

$$F_{hb}(u,v) = AF(u,v) - F_{lp}(u,v) =$$

$$= (A - H_{lp}(u,v))F(u,v) = (A - 1 + H_{hp}(u,v))F(u,v)$$



a b
c d
e f

FIGURE 4.41 (a) Image. (b) Template. (c) and (d) Padded images. (e) Correlation function displayed as an image. (f) Horizontal profile line through the highest value in (e), showing the point at which the best match took place.