1.1. Overview of Bulk Deformation Processes:

- Metal Forming Processes can be classified as
  1. Bulk Deformation processes, and
  2. Sheet metalworking processes

- Bulk Deformation processes are generally characterized by significant deformations and shape changes. The term bulk describes the workparts that have low area-to-volume ratio. Starting work shapes for these processes include cylindrical billets and rectangular bars.

The Basic Operations in Bulk Deformation are:

FIGURE 20.1 Basic bulk deformation processes: (a) rolling, (b) forging, (c) extrusion, and (d) drawing. Relative motion in the operations is indicated by v, and forces are indicated by F.
1.2: Overview of Engineering Stress-Strain:

- The Engineering Stress ($\sigma_e$) and strain ($e$) in a tensile test are defined relative to the original area and length of the test specimen, these values are of interest in design.

- A typical Engineering Stress-strain curve of a metallic specimen is shown below:

![Engineering Stress-strain Curve Diagram]

- The Engineering Stress at any point on the curve is

$$\sigma_e = \frac{F}{A_o}$$

Where:

- $F = \text{applied force in the test (variable)} \ (N)$
- $A_o = \text{original area of specimen} \ (\text{mm}^2)$

- The Engineering Strain ($e$) at any point in the test is

$$e = \frac{L - L_o}{L_o}$$

(2)
Where:

\[ L = \text{gage length at any instant during the elongation} \]
\[ L_0 = \text{original gage length} \]

- In the elastic region, the relation between stress and strain is given by Hooke's law:

\[ \sigma_e = E \epsilon_e \]

Where:

\[ E = \text{modulus of elasticity, which is a measure of the stiffness of a material} \]

- \( Y_0 \) is the yield point (yield strength) at which plastic deformation starts.

- \( UTS \) is the ultimate tensile strength of the material and given by

\[ UTS = \frac{F_{max}}{A_0} \]

Note that at the ultimate point necking (localized reduction in area) starts. And the stress at fracture is known as fracture stress.

- \( \epsilon_f \) is the strain that the material can endure before failure and it is a measure of the ductility of the metal. Ductility is the ability of a material to plastically strain without fracture.

13. True Stress - Strain:

- Since in the test the area of the specimen becomes increasingly smaller, the True Stress is introduced and calculated by using the actual (instantaneous)
area rather than using the original area of the specimen. That is
\[ \varepsilon = \frac{F}{A} \]

where:
- \( F \) = force
- \( A \) = actual (instantaneous) area resisting the load

Similarly, during the extension the gage length changes considerably. For this reason, the definition of true strain (\( \varepsilon \)) is introduced. In this definition, the value of the true strain is obtained by dividing the total elongation into small increments, calculating the engineering strain for each increment, and then adding up the strain values, that is

\[ \varepsilon = \frac{L_1 - L_0}{L_0} + \frac{L_2 - L_1}{L_1} + \frac{L_3 - L_2}{L_2} + \ldots \]

In the limit, the true strain is

\[ \varepsilon = \int_{L_0}^{L} \frac{dL}{L_0} = \ln \frac{L}{L_0} \]

where:
- \( L \) = instantaneous length at any moment during elongation.

If we plot the Engineering stress-strain curve and the true stress-strain curve, obtained from the same test, on one figure then we get the following picture:
* As one can see from the above figure, in the elastic region Hooke’s law can still be used to relate true stress to true strain, that is
\[ \sigma_e = \sigma = \varepsilon E \] (in the elastic region)

* In the plastic region the values of true strain and engineering strain diverge, and can be related to each other by:
\[ \varepsilon = \ln (1+\varepsilon) \]

Similarly, true stress and engineering stress can be related by:
\[ \sigma = \sigma_e (1+\varepsilon) \]

* It is seen that the metal is becoming stronger as strain increases, this property is called strain hardening and it is important in metal forming processes.

- Now, if the portion of the true stress-strain curve representing the plastic region (the flow curve) was plotted using \( \log \sigma \) and \( \log \varepsilon \), the result would be a linear relationship
as shown below:

\[
\begin{align*}
\log k & \quad \log \varepsilon \\
\end{align*}
\]

\[ y = mx + c \]

or,

\[ \log \varepsilon = n \log \varepsilon + \log k \]

\[ \Rightarrow k = k \varepsilon^n \]

which implies that the relation between \( \varepsilon \) and \( \varepsilon \) in the plastic region is expressed as:

\[ k = k \varepsilon^n \]

( the flow curve)

Where:

\[
\begin{align*}
   k &= \text{strength coefficient} \quad (k = \frac{\varepsilon}{\varepsilon = 1}) \\
   n &= \text{strain hardening exponent}
\end{align*}
\]

* The value of \( n \) is directly related to the metal's tendency to work harden.

* Typical values of \( k \) and \( n \) for selected metals are given in the table below:

<table>
<thead>
<tr>
<th>Material</th>
<th>Strength coefficient, ( K )</th>
<th>Strain hardening exponent, ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminum, pure, annealed</td>
<td>25,000 lb/in.² (175 MPa)</td>
<td>0.20</td>
</tr>
<tr>
<td>Aluminum alloy, annealed</td>
<td>35,000 lb/in.² (240 MPa)</td>
<td>0.15</td>
</tr>
<tr>
<td>Aluminum alloy, hardened by heat treatment</td>
<td>60,000 lb/in.² (400 MPa)</td>
<td>0.10</td>
</tr>
<tr>
<td>Copper, pure, annealed</td>
<td>45,000 lb/in.² (300 MPa)</td>
<td>0.50</td>
</tr>
<tr>
<td>Copper alloy: brass</td>
<td>100,000 lb/in.² (700 MPa)</td>
<td>0.35</td>
</tr>
<tr>
<td>Steel, low C, annealed</td>
<td>75,000 lb/in.² (500 MPa)</td>
<td>0.25</td>
</tr>
<tr>
<td>Steel, high C, annealed</td>
<td>125,000 lb/in.² (850 MPa)</td>
<td>0.15</td>
</tr>
<tr>
<td>Steel, alloy, annealed</td>
<td>100,000 lb/in.² (700 MPa)</td>
<td>0.15</td>
</tr>
<tr>
<td>Steel, stainless, austenitic, annealed</td>
<td>175,000 lb/in.² (1200 MPa)</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Compiled from [9], [10], [11], and other sources.

*Values of \( K \) and \( n \) will vary according to composition, heat treatment, and work hardening.
There are three basic forms of stress-strain relationship that describe the behavior of nearly all types of solid materials, these are as shown below:

(a) Perfectly Elastic: This type of material fractures rather than yielding to plastic flow. Brittle material such as ceramics, many cast irons, and thermosetting polymers fall into this category. These materials are not good candidates for forming operations.

(b) Elastic and perfectly plastic: Once the yield strength \( Y \) is reached, the material deforms plastically at the same stress level. The flow curve is given by \( K = Y \) and \( n = 0 \). Metals behave in this fashion when they have been heated to sufficiently high temperature that they recrystallize rather than strain hardened during deformation.

(c) Elastic and strain hardening: Continued plastic deformation requires an increasing stress given by a flow curve whose \( K > Y \) and \( n > 0 \). The flow curve is generally represented as a linear function on a natural logarithmic plot.

(7)
1.4. Compressive Properties:

A compression test applies a load that squeezes a cylindrical specimen between two plates, as shown in the figure below:

![Compression Test Diagram](image)

The engineering stress in the compression test is given by:

\[ \sigma_e = \frac{F}{A_0} \quad \text{(will be negative)} \]

and the engineering strain is given by:

\[ e = \frac{h - h_0}{h_0} \]

The value of \( e \) will be negative. The negative sign is usually ignored when expressing values of compression stresses and strains.

Similarly, the true stress is

\[ \sigma = \frac{F}{A} \]

and the true strain is given by

\[ \varepsilon = \ln \frac{h_0}{h} \quad \text{(negative sign is ignored)} \]
Typical Engineering Stress-strain curve for a compression test is shown below:

![Stress-strain curve](image)

- In the case of using true stress-strain, the data obtained from tensile test and compression test are nearly identical. Therefore, tensile test data can apply with equal validity to a compression operation. Compression operations in metal forming are very common such as rolling.

1.58 Flow Stress, and Average flow stress:

- The flow curve describes the stress-strain relationship in the region in which metal forming takes place. In fact, the flow curve indicates the flow stress \( (Y_f) \) of the metal which is the value of stress required to keep the metal flowing, that is

\[
Y_f = k \varepsilon^n
\]
Where: $ Y_f = \text{flow stress, and it is max. when } \varepsilon \text{ is max.} $

In practical applications, rather than using the instantaneous flow stress $Y_f$ we usually use the average flow stress $\overline{Y_f}$ which is obtained as

\[
\overline{Y_f} = \frac{\int_{\varepsilon}^{\varepsilon_{\text{max}}} Y_f \cdot d\varepsilon}{(\varepsilon - \varepsilon_0)}
\]

or

\[
\overline{Y_f} = \frac{k \varepsilon^n}{1+n}
\]

where:
- $\overline{Y_f} = \text{average flow stress}$
- $\varepsilon = \text{maximum strain value during the deformation process.}$

*In the bulk deformation processes, given values of $k$ and $n$ for the work material, a method of computing final strain will be developed for each process. Based on this strain, $\overline{Y_f}$ can be used to determine the average flow stress to which the metal is subjected during the operation.*
Temperature In Metal Forming:

- The flow curve is a valid representation of stress-strain behavior of a metal during plastic deformation, particularly for cold working operations. For any material, k and n depend on temperature. In fact, there are three temperature ranges for metal working:

  * Cold Working: In which metal forming is performed at room temperature or slightly above.

  * Warm Working: In which metal forming is performed within a temperature range $0.3T_m < T < 0.5T_m$ where $T_m$ is the melting temperature.

  * Hot Working: In which metal forming is performed within a temperature range $0.5T_m < T < 0.75T_m$.

- At the temperature of hot working, flow stress depends on strain rate. Strain rate is directly related to the speed of deformation, in a tensile test the strain rate is defined as:

  $$\dot{\varepsilon} = \frac{v}{h}$$

  Where:

  $\dot{\varepsilon}$ = true strain rate (1/s)

  $h$ = instantaneous length of the workpiece

  $v$ = Speed of the clamp

(11)
The effect of strain rate on strength \( (Y_f) \) is known as strain-rate sensitivity. This effect on \( Y_f \) is given by the relation

\[
Y_f = C \varepsilon^m
\]

where

- \( C \) = strength constant,
- \( m \) = strain rate sensitivity exponent.

The effect of strain rate on \( Y_f \) can be seen on the following figure:

**FIGURE 20.4** (a) Effect of strain rate on flow stress at an elevated work temperature. (b) Same relationship plotted on log-log coordinates.

The effect of temperature on \( m \) and \( C \) is shown in the following figure. Increasing temperature decreases the value of \( C \) and increases the value of \( m \).

**FIGURE 20.5** Effect of temperature on flow stress for a typical metal. The constant \( C \) in Eq. 20.4, indicated by the intersection of each plot with the vertical dashed line at strain rate = 1.0, decreases, and \( m \) (slope of each plot) increases with increasing temperature.
In fact, in our study of the various bulk deformation processes we shall neglect the effect of strain rate in analyzing forces and power. This neglect is a reasonable assumption for cold working, warm working, and for hot working operations at relatively low deformation speeds.

1.7.8 Friction in Metal Forming:

Friction in metal forming arises because of the close contact between the tool and work surfaces and the high pressures that drive the surfaces together in these operations.

If the coefficient of friction becomes large enough, a condition known as sticking occurs. Sticking is the tendency for the two surfaces in relative motion to stick to each other rather than slide. This means that the friction stress \( f_s \) between the surfaces exceeds the shear flow stress \( f_s = 0.5Y_f \) of the work metal, thus causing the metal to deform by a shear process beneath the surface rather than slip at the surface.
Rolling:

Rolling is a deformation process in which the thickness of the work is reduced by compressive forces exerted by two opposing rolls. As shown in the following figure, the rolls rotate to pull and simultaneously squeeze the work between them.

FIGURE 21.1  The rolling process (specifically, flat rolling).

Some of the steel products made in a rolling mill are shown below. A bloom has 6x6 in cross section, while billet 15x15 in.

FIGURE 21.2  Some of the steel products made in a rolling mill.

<table>
<thead>
<tr>
<th>Intermediate rolled form</th>
<th>Final rolled form</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bloom</td>
<td>Structural shapes</td>
</tr>
<tr>
<td>Slab</td>
<td>Rails</td>
</tr>
<tr>
<td>Billet</td>
<td>Plates, sheets</td>
</tr>
<tr>
<td></td>
<td>Coils</td>
</tr>
<tr>
<td></td>
<td>Bars, rods</td>
</tr>
</tbody>
</table>
1.8.1: Analysis of Flat Rolling:

- Flat rolling involves the rolling of slabs, strips, sheets, and plates. In flat rolling, the work is squeezed between two rolls so that its thickness (which is much smaller than its width) is reduced by an amount called the draft:

\[ d = t_o - t_f \]

- If the draft is expressed as a fraction of the starting block thickness, it is called reduction, \( \eta \):

\[ \eta = \frac{d}{t_o} \]

- Rolling increases the work width from \( w_o \) to \( w_f \), this is called spreading. (Spreading is neglected when \( w_o \gg t_o \)). That is \( w_o = w_f \).
— The inlet and outlet volume rates of material flow must be the same, that is

\[ t_0 \cdot \nu_i = t_f \cdot \nu_f \]

or

\[ t_0 \cdot \nu_i = t_f \cdot \nu_f \]

— It is fact that \( \nu_i < \nu_f < \nu_f \). The point where roll velocity equals work's, known as the neutral point N.

— The amount of slip between the rolls and the work can be measured by the forward slip \( S \), where

\[ S = \frac{\nu_f - \nu_i}{\nu_f} \]

— The true strain in rolling is defined by

\[ \varepsilon = \ln \frac{t_0}{t_f} \]

— \( \varepsilon \) can be used to determine \( \nu_f \) from

\[ \nu_f = \frac{k \cdot \varepsilon^n}{1 + n} \]

— Friction occurs with a certain coefficient of friction \( \mu \) on either sides of the neutral point. However, both the friction forces act in opposite directions and are not equal.

— The vertical component \( P_v \) of the radial rolling force \( P \) is known as the rolling load. \( P_v \) is
also called the separating force.

- The rolling pressure $P$ equals to:

$$ P = \frac{P_v}{W_0 \cdot L_p} $$

where:

$$ L_p = \text{projected length of } L $$

$$ L_p = \left[ R(t_o - t_f) - \frac{(t_o - t_f)^2}{4} \right]^{1/2} $$

or

$$ L_p \approx \sqrt{R(t_o - t_f)} $$

- A good approximation of $P$ is

$$ P = \overline{Y_f} $$

Therefore, we can write:

$$ P_v = \overline{Y_f} \cdot W_0 \cdot \sqrt{R(t_o - t_f)} \quad (P_v = \text{rolling load}) $$

- The torque in rolling can be estimated by assuming that $P_v$ acts at $L_p/2$. That is

$$ T = P_v \cdot \frac{L_p}{2} $$

or

$$ T = 0.5 \overline{Y_f} \cdot W_0 \cdot R(t_o - t_f) $$

- The power required to drive the rolls is given by

$$ \text{Power} = \left( 2\pi N P_v \frac{L_p}{2} \right)^2 \times N = \text{rev/s} $$

or

$$ \text{Power} = 2\pi N \overline{Y_f} \cdot W_0 \cdot R(t_o - t_f) $$

(12)
The limiting condition for unaided entry of a slab into the rolls is

\[ f \cos \theta \geq \frac{P_n}{P_n} \sin \theta \]

or

\[ \frac{f}{P_n} \geq \tan \theta, \text{ but } f = \mu P_n \]

Therefore, \( \mu \geq \tan \theta \)

* From geometry \( \tan \theta = \frac{L_p}{R - \frac{d}{2}} \approx \sqrt{\frac{d}{R}} \)

but \( \mu \geq \tan \theta = \sqrt{\frac{d}{R}} \)

Therefore, \( d_{\max} \leq \mu^2 R \) \( (d_{\max} \text{ : Max. draft.}) \)

Note that the above equations indicate that force and power to roll a strip of a given width and work material can be reduced by any of the following:

1. Use hot rolling, rather than cold rolling, to reduce \( k \) and \( n \).
2. Reduce draft \( d \).
3. Use a smaller roll radius \( R \), and
4. Use a lower rolling speed \( N \).
Note also that the peak pressure is located at the neutral point. This is shown in the figure below:

![Figure 21.4](image)

**Figure 21.4**: Typical variation in pressure along the contact length in flat rolling. The peak pressure is located at the neutral point. The area beneath the curve, representing the integration in Eq. (21.9), is the roll force $F$.

Rolling Mills: Various rolling mill configurations are available, the basic rolling mill consists of two opposing rolls and is referred to as a two-high rolling mill. Various configurations of rolling mills are shown in the figure below:

![Various configurations of rolling mills](image)

**Figure 21.6**: Various configurations of rolling mills: (a) two high, (b) three high, (c) four high, (d) cluster mill, and (e) tandem rolling mill.
Prob. 21.3: Given:
* \( R = 350 \text{ mm} \)
* \( m = 0.15 \)
* draft is to be equal on each pass.

Find:
(a) minimum \# of passes required,
(b) the draft for each pass.

Solution:
* From \( d_{\text{max}} = m^2 R \), we get
  \[ d_{\text{max}} = (0.15)^2 (350) = 7.875 \text{ mm} \]
* but total draft is equal to
  \[ d_{\text{total}} = t_0 - t_f = 50 - 25 = 25 \text{ mm} \]
* minimum \# of passes required is given by
  \[ \text{min. \# of passes} = \frac{d_{\text{total}}}{d_{\text{max}}} = \frac{25}{7.875} = 3.17 \text{ pass} \]
* but \# of passes has to be integer, so the min. \# of passes is 4.

* Now, with \# of passes equals 4, we calculate the corresponding draft as
  \[ d = \frac{d_{\text{total}}}{\text{min. \# of pass}} = \frac{25}{4} \]
  That is
  \[ d = 6.25 \text{ mm} \].
Prob. 21.7: given:
\[
\begin{align*}
\omega_o &= 10 \text{ in} \\
\omega &= 1 \text{ in} \\
\omega_x &= 0.8 \text{ in} \\
R &= 20 \text{ in} \\
V_n &= 50 \text{ ft/min} \\
K &= 35000 \text{ lb/in}^2 \\
N &= 0.2
\end{align*}
\]

Find:
\[
\begin{align*}
P_v &= ? \quad T = ? \quad \text{Power} = ?
\end{align*}
\]

Solution:
* \( V_n = 2\pi NR = (50)(12) = 600 \text{ in/min} \)
  
  from which \( N = 4.78 \text{ rev/min} \)

* \( \varepsilon \) in rolling can be found from
  \[
  \varepsilon = \ln\frac{\omega_o}{\omega_x} = \ln\frac{1}{0.8} = 0.223
  \]

* \( \bar{\gamma}_f \) can be found from
  \[
  \bar{\gamma}_f = \frac{K\varepsilon^n}{n+1} = \frac{35000(0.223)^{0.2}}{1.2}
  \]
  \[
  \therefore \bar{\gamma}_f = 2160.75 \text{ lb/in}^2
  \]

* \( L_p = \sqrt{R(t_o-t_x)} = \sqrt{20(0.2)} = 2 \text{ in} \)

* \( P_v = \bar{\gamma}_f W L_p = (2160.75)(10)(2) = 432150 \text{ lb} \)

* \( T = 0.5 P_v L_p = (0.5)(432150)(2) = 432150 \text{ lb in} \)

* \( \text{Power} = 2\pi N P_v L_p = 2\pi(4.78)(432150)(2) = 25944903.12 \text{ lb in/min} \)

but, we know that \( 1 \text{ hp} = 396000 \text{ lb in/min} \), therefore

\[
\text{Power} = \frac{25944903.12}{396000} = 65.5 \text{ hp}
\]

That is,
\[
\text{Power} = 65.5 \text{ hp}
\]
1.9: Forging:

- Forging is a deformation process in which the work is compressed between two dies. Either impact or gradual pressure is used in forging.

- Forging can be performed hot, warm or cold. In fact, there are three types of Forging:
  (a) Open-die forging
  (b) Impression-die forging
  (c) Flashless forging

The following figure presents these three types:

![Figure 21.10](image-url)  
**FIGURE 21.10** Three types of forging operation illustrated by cross-sectional sketches: (a) open-die forging, (b) impression-die forging, and (c) flashless forging.
19.1: Analysis of Open-die Forging (upsetting):

- If open-die forging is carried out under ideal conditions of no friction between work and die surface, then homogeneous deformation occurs, and the radial flow of the material is uniform throughout its height as pictured in the following Figure.

![Diagram showing homogeneous deformation](image)

**Figure 21.11** Homogeneous deformation of a cylindrical workpiece under ideal conditions in an open-die forging operation: (1) start of process with workpiece at its original height and diameter, (2) partial compression, and (3) final size.

- In this case, case of ideal conditions, the true strain experienced by the work can be found from

\[ \varepsilon = \ln \frac{h_0}{h} \]

Where \( h \) is the instantaneous height of the work part. At the end of the compression stroke, \( h \) reaches its maximum value \( h_f \).
In cold upsetting, the force required to continue the compression at any given height $h$ is given by:

$$F = Y_A$$

where $A$ is the corresponding cross-sectional area. Note that $F$ reaches a maximum value at the end of the forging stroke.

In hot upsetting,

$$F = YA$$

where $Y = \text{metal's yield strength} = Y_f$

In actual upsetting operation, because of friction, we have the barreling effect shown below:

*This barreling increases as diameter-to-height ratio of the work part increases, this is because the contact area becomes greater.*
A shape factor can be used to account for the D/h ratio and friction, the actual upsetting force becomes:

\[
F = K_f V_f A
\]

where \( K_f \) is the forging shape factor defined as

\[
K_f = 1 + 0.4 \frac{MD}{h}
\]

where \( D \) is workpart diameter or other dimension representing contact length with die surface.

1.9.2: Impression-die Forging:

Impression-die forging, closed-die forging, is performed with dies that contain the inverse of the desired shape of the part. The process is illustrated in a three steps sequence in the figure below:

FIGURE 21.15 Sequence in impression-die forging: (1) just prior to initial contact with raw workpiece, (2) partial compression, and (3) final die closure, causing flash to form in gap between die plates.
Because of flash formation and the more complex part shapes made with these dies, forces in this process are usually greater and more difficult to be analyzed. However, in impression-die forging the force formula is

\[ F_{\text{max}} = k_f Y_f A \]

Where:

- \( F_{\text{max}} \) = maximum force in the operation, and it is reached at the end of the forging stroke. \( F_{\text{max}} \) determine the required capacity of the press or hammer.
- \( k_f \) = forging shape factor and given in the table below.
- \( A \) = projected area of the part including flash at the end of forging stroke.
- \( Y_f = K E_{av}^n \) (if cold forging); \( Y_f = Y \) (if hot forging)

in which \( E_{av} = \ln \frac{h_0}{h_{fav}} \), where \( h_{fav} \) is given by

\[ h_{fav} = \frac{\text{volume of work part}}{\text{projected area (A)}} = \frac{V}{A} \]

<table>
<thead>
<tr>
<th>TABLE 21.1 Typical ( K_f ) Values for Various Part Shapes in Impression-die and Closed-die Forging.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part Shape</td>
</tr>
<tr>
<td>Impression-die forging</td>
</tr>
<tr>
<td>Simple shapes with flash</td>
</tr>
<tr>
<td>Complex shapes with flash</td>
</tr>
<tr>
<td>Very complex shapes with flash</td>
</tr>
<tr>
<td>Flashless forging</td>
</tr>
<tr>
<td>Coining (top and bottom surfaces)</td>
</tr>
<tr>
<td>Complex shapes</td>
</tr>
</tbody>
</table>

(26)
1.9.3: Flashless Forging:

- In flashless forging, the raw workpiece is completely contained within the die cavity during compression, and no flash is formed. The process is sometimes called closed-die forging. The sequence of the process is illustrated in the figure below:

![Diagram showing the process of flashless forging](#)

**FIGURE 21.18** Flashless forging: (1) just before initial contact with workpiece, (2) partial compression, and (3) final punch and die closure. Symbols \( v \) and \( F \) indicate motion \( (v = \text{velocity}) \) and applied force, respectively.

- Forces in flashless forging reach values comparable to those in impression die forging. Estimates of these forces can be computed using the same methods as for impression die forging.

- Coining: is a special application of flashless forging in which fine details in the die are impressed into the top and bottom surfaces of the workpart.
Note that upsetting (heading) is widely used to form heads on nails, bolts, and similar products. In heading, long wire or bar stock is fed into the machines, the end of the stock is upset forged, and then the piece is cut to length to make the desired hardware item. The process can be performed as open-die forging or closed-die forging. The following figure illustrates the upset forging operation to form a head on a bolt. Note also that the length that can be upset in one blow is three times the diameter of the starting stock.

The following figure illustrates a variety of heading applications.
Prob. 21.178 Given:
\[ k = 80,000 \text{ lb/in}^2 \]
\[ n = 0.24 \]
\[ M = 0.1 \]
\[ D_o = \frac{3}{16} \text{ in} \]
\[ D_f = \frac{3}{8} \text{ in} \]
\[ h_f = \frac{1}{16} \text{ in} \]

Find: \( h_o = ? \), \( F_{max} = ? \)

Solution:
(a) Initial volume of head \( V_o \) must equal final volume \( V_f \), that is
\[ V_o = V_f \]
\[ \frac{\pi D_o^2 h_o}{4} = \frac{\pi D_f^2 h_f}{4} \Rightarrow h_o = \frac{D_f^2 h_f}{D_o^2} \]
\[ \therefore h_o = \frac{(\frac{3}{8})^2 (\frac{1}{16})}{(\frac{3}{16})^2} = \frac{1}{4} \text{ in} \]

(b) \( F_{max} = k_f \gamma_f A_f \) (cold work open-die forging)

Where:
\[ k_f = 1 + 0.4 M D_f \]
\[ h_f = 1 + (0.4)(0.1)(\frac{3}{8}) \]
\[ \therefore k_f = 1.24 \]
\[ \varepsilon = \ln \frac{h_o}{h_f} = \ln \left( \frac{\frac{1}{4}}{\frac{1}{16}} \right) \]
\[ \therefore \varepsilon = 1.386 \]

(29)
but \( y_f = k \xi^n = 80000 \times (1.386)^{0.24} \)
\[ y_f = 86523.7 \text{ lb/in}^2 \]

and
\[ A_f = \frac{\pi D_f^2}{4} = \frac{3.14 \times \left(\frac{3}{8}\right)^2}{4} \]
\[ A_f = 0.11 \text{ in}^2 \]

Therefore,
\[ F_{\text{max.}} = (1.24) \times (86523.7) \times (0.11) \]
\[ F_{\text{max.}} = 11843.7 \text{ lb.} \]
1.10: Extrusion:

Extrusion is a compression forming process in which the work metal is forced to flow through a die opening to produce a desired cross-sectional shape. Extrusion can be direct or indirect. Extrusion can be carried out cold, warm or hot. Hollow sections are also possible in direct and in indirect extrusion. The following four figures illustrates the types of direct and indirect extrusion.

**FIGURE 21.31** Direct extrusion.

**FIGURE 21.32** (a) Direct extrusion to produce a hollow or semihollow cross section; (b) hollow and (c) semihollow cross sections.
1.10.1: Analysis of Extrusion:

Consider the following figure as a reference in discussing some of the parameters in extrusion.

The extrusion ratio ($\eta_x$) is defined as:

$$\eta_x = \frac{A_0}{A_f}$$

Assuming ideal deformation occurs with no friction, then
\[ \varepsilon = \ln \frac{r_x}{r_f} = \ln \frac{A_0}{A_f} \]

- Under ideal deformation, the ram pressure is given by

\[ P = \bar{Y}_f \ln r_x \]

*This is because*

\[ \frac{\text{Work}}{\text{Volume}} = \bar{Y}_f \int \varepsilon \, d\varepsilon \]

or

\[ \frac{P \cdot A \cdot L}{V} = \bar{Y}_f \varepsilon \Rightarrow P = \bar{Y}_f \varepsilon \]

- In fact, extrusion is not a frictionless process. Hence, various methods have been suggested to calculate the actual true strain and associated ram pressure. The following is the empirical equation proposed by Johnson for estimating extrusion strain:

\[ \varepsilon_x = a + b \ln r_x \]

where \( \varepsilon_x \) is extrusion strain,

- \( a \) and \( b \) are constants \( a = 0.8 \) and \( b = 1.2 \) to 1.5.
- \( a \) and \( b \) increase by increasing \( \alpha \).
  Where \( \alpha \) is die entry angle.

- The ram pressure to perform indirect extrusion is given by

\[ P = \bar{Y}_f \varepsilon_x \]

*Note that \( \bar{Y}_f \) is calculated based on ideal strain.

(33)
In direct extrusion, because of friction the ram pressure is greater than that for indirect extrusion. This can be seen in the following figure:

![Figure 21.37: Typical plots of ram pressure versus ram stroke (and remaining billet length) for direct and indirect extrusion. The higher values in direct extrusion result from friction at the container wall. The shape of the initial pressure buildup at the beginning of the plot depends on die angle (higher die angles cause steeper pressure buildups). The pressure increase at the end of the stroke is related to formation of the butt.]

---

**The friction force in the direct extrusion is given as**

\[ M_P c \pi D_o L = P_f \frac{\pi D_o^2}{4} \quad \text{--- (x)} \]

where:
- \( P_f \) = additional pressure required to overcome friction
- \( P_c \) = pressure of the billet against the container wall

---

In the worst case, sticking occurs at the container wall, that is \( \tau_i = \tau_f \), or

\[ \frac{M_P c \pi D_o L}{\pi D_o L} = \frac{\bar{V}_f}{2} \quad \Rightarrow \quad P_c = \frac{\bar{V}_f}{2} \]

(34)
Therefore, from equation (*) we get

\[ P_f = \bar{y}_f \frac{2L}{D_0} \]

- Hence, the ram pressure in direct extrusion is

\[ P = \bar{y}_f \left( \varepsilon_x + \frac{2L}{D_o} \right) \]

- Ram force \( F \) is given by

\[ F = PA \]

- Power required is given by

\[ \text{Power} = Fv \]

where \( v \) is ram velocity.

- If the extrudate has a non-circular cross-sectional area, then we have to use a shape factor. This shape factor \( k_x \) can be expressed as

\[ k_x = 0.98 + 0.02 \left( \frac{C_x}{C_c} \right)^{2.25} \]

Where:
- \( C_x \) = perimeter of the extruded cross-section
- \( C_c \) = perimeter of a circle of the same area as the extruded shape, best \( 1 \leq \frac{C_x}{C_c} \leq 6 \).

- With this shape factor, we have

\[ P = k_x \bar{y}_f \varepsilon_x \] (indirect extrusion)

\[ P = k_x \bar{y}_f \left( \varepsilon_x + \frac{2L}{D_o} \right) \] (direct extrusion)

(35)
- Other Extrusion Processes:

(a) Impact Extrusion: performed at higher speeds and shorter strokes than conventional extrusion. The figure below shows several examples of impact extrusion:

(b) Hydrostatic Extrusion: In this process the pilot is surrounded by fluid inside the container, the fluid is then pressurized by the forward motion of the ram as shown in the figure below:

![Impact Extrusion Diagram](image)

![Hydrostatic Extrusion Diagram](image)
Problem 21.27: Given:

\[ L_0 = 3 \text{ in} \]
\[ D_0 = 2 \text{ in} \]
\[ \alpha = 45^\circ \]
\[ D_f = 0.5 \]

\[ a = 0.8 \]
\[ b = 1.3 \]
\[ \bar{Y}_f = Y = 15,000 \text{ lb/m}^2 \]

Find: \( \mathcal{M}_x = ? \)
\[ L = ? \]
\[ P = ? \]
\[ L_f = ? \]

Solution:

\[ \mathcal{M}_x = \frac{A_0}{A_f} = \frac{\pi D_0^2}{\pi D_f^2} = \frac{(2)^2}{(0.5)^2} = 16 \quad \text{Ans.} \]

\[ \therefore \mathcal{M}_x = 16 \]

\[ V_0 = A_0 L_0 = \frac{\pi D_0^2}{4} \]

\[ \therefore V_0 = \frac{\pi (2)^2 (3)}{4} \]

or \[ V_0 = 9.42 \text{ in}^3 \]

but \[ V_s = V_o - V_c \quad (\times) \]

Where \[ V_c = \frac{1}{3} \left[ \frac{\pi D_0^2}{4} \left( \frac{D_0}{2} \right) - \frac{\pi D_f^2}{4} \left( \frac{D_f}{2} \right) \right] \]

or \[ V_c = \frac{1}{3} \left[ \frac{3.14 (4)(2)}{(4)(2)} - \frac{3.14 (0.5)^2 (0.5)}{(4)(2)} \right] \]

\[ \therefore V_c = 1.02 \text{ in}^3 \]

(37)
Now, from equation (x) \( V_s \) equals to

\[
V_s = 9.42 - 1.02
\]

\[
\therefore V_s = 8.4 \text{ in}^3
\]

but \( V_s = \frac{\pi D_o^2}{4}L = \frac{\pi(2)^2}{4}L = 8.4 \text{ in}^3 \)

\[\Rightarrow L = 2.675 \text{ in} \quad \text{Ans.}\]

\[
\varepsilon = \ln \frac{f_x}{f_x}
\]

\[\text{or} \quad \varepsilon = \ln(16)\]

\[\therefore \varepsilon = 2.772\]

Now, \( \varepsilon_x = 0.8 + 1.3 \times (2.772) \)

\[\therefore \varepsilon_x = 4.4\]

\[
P = \frac{\bar{f}}{f_x} \left( \varepsilon_x + \frac{2L}{D_0} \right)
\]

\[P = 15000 \left( 4.4 + \frac{2(2.675)}{2} \right)\]

\[\therefore P = 106125 \text{ lb/in}^2 \quad \text{Ans.}\]

\[
V_s = A_fL_f = \frac{\pi D_f^2}{4}L_f
\]

\[\text{or} \quad V_s = \frac{3.14}{4} \cdot (0.5)^2L_f = 8.4 \text{ in}^3\]

from which,

\[L_f = 42.8 \text{ in} \quad \text{Ans.}\]
1.11: Wire and Bar Drawing:
- Drawing is an operation in which the cross-section of a bar or wire is reduced by pulling it through a die opening.
- Bar drawing is the term used for large-diameter bar stock, while wire drawing applies to small diameter stock.

1.11.1: Analysis of Drawing:
- Consider the figure below which illustrates the drawing process.

![Figure 21.43 Drawing of bar, rod, or wire.](image)

- Area reduction in drawing is defined as
  \[
  \eta = \frac{A_o - A_f}{A_o}
  \]
- The draft in drawing is defined as
  \[
  d = D_o - D_f
  \]
- If no friction occurred in drawing, then the ideal
Strain is given by:
\[ \varepsilon = \ln \frac{A_0}{A_f} = \ln \frac{1}{1 - \eta} \]

Therefore, the stress that results from this ideal strain is given by:
\[ \sigma = \frac{\sqrt{f}}{f} \varepsilon \]

or \[ \sigma = \frac{\sqrt{f}}{f} \ln \frac{A_0}{A_f} \]

However friction is present in drawing. In addition, die angle affects the drawing stress. In fact, Schey has given a formula for the drawing stress which accounts for friction and die angle. This formula is given as:
\[ \sigma_d = \frac{\sqrt{f}}{f} \left( 1 + \frac{M}{\tan \alpha} \right) \phi \ln \frac{A_0}{A_f} \]

where:
\[ \alpha = \text{die angle in degrees} \]
\[ \phi = \text{a factor that accounts for inhomogeneous deformation} \]

and,
\[ \phi = 0.88 + 0.12 \frac{D}{L_c} \]

In which:
\[ D = \frac{D_0 + D_f}{2} \]
\[ L_c = \frac{D_0 - D_f}{2 \sin \alpha} \]
The corresponding draw force is given as

\[ F = A_f \bar{Y} (1 + \frac{M}{\tan \alpha}) \phi \ln \frac{A_o}{A_f} \]

The power required in drawing operation is the draw force multiplied by exit velocity of work.

1.11.2: Maximum Reduction Per Pass:

If the reduction is large enough, draw stress will exceed the yield strength of the exiting metal.

Under the assumption that \( \bar{Y}_f = Y \) that is \( (n = 0) \), and ideal drawing conditions, then

\[ \delta_f = \bar{Y} \ln \frac{A_o}{A_f} = Y \ln \frac{A_o}{A_f} = \delta \]

from which,

\[ \ln \frac{A_o}{A_f} = 1 \]

or \[ \frac{A_o}{A_f} = 2.718 \] (maximum possible theoretical area ratio)

Which means, \( \eta_{\text{max}} = 0.632 \) (The theoretical maximum reduction possible in a single draw)

However, in practice, draw reductions per pass are well
below the theoretical limit. Reductions of 0.5 for single-draft bar drawing and 0.3 for multiple draft wire drawing seem to be the upper limits in industrial practice.

11.3: Drawing Equipment and Dies:

Bar drawing is accomplished on a machine called a draw bench. The arrangement of a draw bench is shown in the following figure:

---

Wire drawing is done on continuous drawing machines that consist of multiple draw dies, separated by accumulating drums between the dies, as shown in the figure below. Each drum, called a capstan, is motor driven to provide the proper pull force.
The following figure identifies the features of a typical draw die. Four regions of the die can be distinguished: (1) entry, (2) approach angle, (3) bear surface, and (4) back relief.

Note: \[ 6^\circ \leq \alpha \leq 2^\circ \cdot \]

1.11.4: Tube Drawing

The drawing process can be used to reduce the diameter or wall thickness of seamless tubes and pipes. Tube drawing can be carried out either with or without a mandrel. The following figure illustrates Tube Drawing with mandrel.

---

**Figure 21.46** Draw die for drawing of round rod or wire.

**Figure 21.48** Tube drawing with mandrels: (a) fixed mandrel and (b) floating plug.
Given:
\[ D_0 = 0.125 \text{ in} \]
\[ \eta = 0.2 \]
\[ k = 40000 \text{ lb/in}^2 \]
\[ n = 0.15 \]
\[ \alpha = 12^\circ \]
\[ M = 0.1 \]

\[ \eta = 0.9 \]
\[ h_p = 1.5 \text{ hp for each motor.} \]

Find: \( v_2 = ? \) (wire speed as it exits second die)

Solution:

\[ A_o = \frac{\pi (D_0)^2}{4} \]
or
\[ A_o = \frac{\pi (0.125)^2}{4} = 0.01227 \text{ in}^2 \]

\[ \eta = \frac{A_o - A_1}{A_o} = 0.2 \]
from which
\[ A_1 = 0.009813 \text{ in}^2 \]
but
\[ A_1 = \frac{\pi D_1^2}{4} \]

So,
\[ D_1 = 0.1118 \text{ in} \]

Now,
\[ F_1 = A_1 \bar{Y}_f (1 + \frac{M}{\tan \alpha}) \phi \ln \frac{A_o}{A_1} \]

Where:
\[ \bar{Y}_f = \frac{k \, E^n}{n+1} = \frac{k \left( \ln \frac{A_o}{A_1} \right)^n}{n+1} \]

or
\[ \bar{Y}_f = 40000 \left( \ln \frac{0.01227}{0.009813} \right)^{0.15} \]

\[ \therefore \bar{Y}_f = 27786.13 \text{ lb/in}^2 \]

(44)
\[
\phi = 0.88 + 0.12 \frac{D}{L_c}
\]

or

\[
\phi = 0.88 + 0.12 \left[ \frac{0.125 + 0.1118}{2} \right] \left[ \frac{0.125 - 0.1118}{2 \sin(12^\circ)} \right]
\]

\[
\implies \phi = 1.3276
\]

Now, from equation (44) we get

\[
F_1 = (0.098128)(2778.63) \left(1 + \frac{0.1}{\tan(12^\circ)}\right)(1.3276) \ln \left(\frac{0.01227}{0.0098}\right)
\]

\[
\implies F_1 = 119.6 \text{ lb}
\]

Now, power = \(F_1 \nu_1\)

\[
(0.9)(1.5)(396000) = (119.6) \nu_1
\]

\[
\implies \nu_1 = 4470 \text{ in/min}
\]

We know that

\[
\frac{A_1 - A_2}{A_1} = \gamma = 0.2
\]

from which \(A_2 = 0.00785 \text{ in}^2\)

but \(A_1 \nu_1 = A_2 \nu_2\)

\[
\nu_2 = \frac{A_1 \nu_1}{A_2} = \frac{0.098128}{0.00785} (4470)
\]

\[
\implies \nu_2 = 5587.5 \text{ in/min.}
\]

(45)