Chapter 30: Magnetic field sources

- Using Biot-Savart Law to calculate the magnetic field produced by current element
- calculate the total magnetic field due to various current distributions.
- Magnetic Force Between Two Parallel Conductors
- Ampère’s law

Chapter 29: Magnetic field sources

- Magnetic field is not only produced by Magnet stones, but it can be produced by current when passing through conductors
- this chapter explores the origin of the magnetic field associated with moving charges.
- Oersted’s discovered in 1819 that a compass needle is deflected by a current-carrying conductor

Chapter 29: Biot-Savart Law

For a length element $ds$ along a current carrying wire, Biot and Savart experimental findings, for the magnetic field $dB$ (of magnitude $dB$) at some point $P$, are summarized by:

1) $dB \perp ds$ and $dB \perp \hat{r}$ where $\hat{r}$ is unit vector directed from $d\hat{s}$ to $\hat{P}$
2) $dB \propto \frac{1}{r^2}$ where $r$ is radial distance from $d\hat{s}$
3) $dB \propto I$, $dB \propto ds$, $dB \propto \sin \theta$
4) B-field $(dB)$ is out at $P$ and B-field is in at $P'$

\[ dB = \frac{\mu_0 I ds \times \hat{r}}{4\pi} \]  
\( \mu_0 = 4\pi \times 10^{-7} \text{Tm/A} \) is permeability of free space

Chapter 30.1: Biot-Savart Law (Magnetic Field Surrounding a Thin, Straight Conductor)

Determine the magnitude and direction of the magnetic field at point $P$ due to current $I$ in the conductor.

\[ B = \frac{\mu_0 I}{4\pi} \int_{a}^{b} \frac{\alpha \csc^2 \theta \sin \alpha \theta \sin \alpha \theta}{a^2 \csc^2 \theta} ds \]

If the wire is very long, $L >> a$

- $\theta_0 = 0^\circ$ then $B = \frac{\mu_0 I}{2a}$
Chapter 29: Biot-Savart Law (magnetic field surrounding a thin, straight conductor)

For long wire, the magnetic field lines are circles concentric with the wire and lie in planes perpendicular to the wire. We can use the right hand to determine the B-field direction as shown: positioning the thumb along the direction of the current. The four fingers wrap in the direction of the B-field.

\[ B = \frac{\mu_0 I}{2\pi a} \]

Chapter 29: Biot-Savart Law (field due to a circular arc of wire)

Calculate the B-field at point O for the current-carrying wire segment shown.

\[ B = \frac{\mu_0 I}{4\pi} \int ds \times \hat{r} \]

For the arc (AC) of length s and radius a

\[ B = \frac{\mu_0 I}{4\pi a^2} \int ds \times \hat{r} \]

\[ B = \frac{\mu_0 I}{4\pi a^2} s \]

But \( s = \alpha \theta \)

Hence \( B = \frac{\mu_0 I\theta}{4\pi a} \)

As shown in previous example

Chapter 29: Magnetic Force Between Two Parallel Conductors

Consider two long, straight, parallel wires separated by a distance \( a \) and carrying currents \( I_1 \) and \( I_2 \) in the same direction.

Since we have B-field from each wire, and both have currents \( I_1 \) & \( I_2 \) same in direction \( \Rightarrow \) attraction

\( F_1 = F_2 = F \) where \( F \) is force between two parallel wires

\( \vec{F}_1 = \vec{F}_2 \) Opposite in direction

\( I_1 \) & \( I_2 \) same in direction \( \Rightarrow \) attraction

\( I_1 \) & \( I_2 \) opposite in direction \( \Rightarrow \) repulsion
Chapter 29: Magnetic Force Between Two Parallel Conductors

The Force per unit length

\[ \frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a} \]

If \( a = 1\) m, \( I_1 = I_2\), and \( F_B/l = 2 \times 10^{-7}\) N/m

\( I\) in both wires is defined to be 1 ampere

Chapter 29: Ampère’s Law

With \( I = 0 \) \( \Rightarrow \) no B-field (a)

With \( I \neq 0 \) \( \Rightarrow \) B-field form (b)

\( \Rightarrow \) We can integrate around the closed loop \( \oint B\cdot d\vec{s} \)

\( d\vec{s} \) is displacement element along the loop path

We call the loop where we need to find the B-field by Amperian loop (current must pass through the loop in one direction)

\[ \oint B\cdot d\vec{s} = B\oint ds = \frac{\mu_0 I}{2\pi} (2\pi) = \mu_0 I \]

\[ \oint B\cdot d\vec{s} = \mu_0 I \]

A line integral of \( B\cdot d\vec{s} \) around a closed path equals \( \mu_0 I \), where \( I \) is the current passing through amperian loop surface (surface bounded by the closed path).

Chapter 29: Magnetic Force Between Two Parallel Conductors

Ex: Two infinitely long, parallel wires are lying on the ground 1 cm apart as shown. A third wire, of length 10 m and mass 400 g (0.4 kg), carries a current of \( I_1 = 100\) A and is levitated above the first two wires as shown. What current \( I_2 \) must the infinitely long wires carry so that the three wires form an equilateral triangle?

Since \( I_1 \) and \( I_2 \) are opposite in direction \( \Rightarrow \) repulsive force from Right (\( F_{B,R} \)) and Left (\( F_{B,L} \)) wires with the levitated one

X-components of forces on levitated wire cancels. But y-components add

\[ F_y = 2\left( \frac{\mu_0 I_1 I_2}{2\pi a} \right) \sin 30^\circ \uparrow \]

Since it is levitated

\( F_y \uparrow = F_y \downarrow \)

\[\mu_0 I_1 I_2 (\sin 30^\circ) = mg \Rightarrow I_2 = \frac{mg}{(\sin 30^\circ)\mu_0 I_1} \]

[Equation]

Chapter 29: Ampère’s Law (Long Wire Carrying Current)

Ex: Calculate the B-field a distance \( r \) from the center of the wire carrying current \( I_0 \) in the regions \( r \gg R \) and \( r < R \).

For \( r \gg R \), all current in the wire \( (I) \) pass through whole surface of amperian loop

\[ \oint B\cdot ds = B\oint ds = B(2\pi r) = \mu_0 I_0 \]

\[ B = \frac{\mu_0 I_0}{2\pi r} \]

For \( r < R \)

\[ B_{\text{inner}} = B(2\pi r) = \mu_0 I \]

But, \( \frac{I}{I_0} = \frac{\pi r^2}{a^2} \Rightarrow I = \frac{r^2}{R^2} I_0 \)

\[ \oint B\cdot ds = B(2\pi r) = \mu_0 I \left( \frac{r^2}{R^2} I_0 \right) \]

\[ B = \left( \frac{\mu_0 I}{2\pi R^2} \right) r \]
Chapter 29: Ampère’s Law (The Toroid)

Ex: For a toroid having \( N \) closely spaced turns of wire, calculate the \( B \)-field in the region occupied by the torus, a distance \( r \) from the center.

We construct amperian loop inside the toroid (dashes circle).

By symmetry, \( B \) is constant over the dashed circle and tangent to it.

\[
\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r) = \mu_0 NI
\]

\[\Rightarrow B = \frac{\mu_0 NI}{2\pi r}\]

Outside the toroid: \( B = 0 \)

Chapter 29: The B-Field of a Solenoid

A solenoid is a long wire wound in the form of a helix.

Almost uniform \( B \)-field in the interior.

Net \( B \)-field is the vector sum of the fields resulting from all the turns.

Chapter 29: The B-Field of a Solenoid

Consider long solenoid \( L \gg R \)

Along path 2 and 4, \( \{ \vec{B} \perp d\vec{s} \} \Rightarrow \vec{B} \cdot d\vec{s} = 0 \)

Along path 3, \( \vec{B} = 0 \)

\[
\int \vec{B} \cdot d\vec{s} = \int_{\text{path 1}} \vec{B} \cdot d\vec{s} = B \int ds = Bl
\]

\[
\oint \vec{B} \cdot d\vec{s} = Bl = \mu_0 NI \quad \Rightarrow \quad B = \frac{\mu_0 NI}{l}
\]

\( B = \mu_0 nI \) where \( n = \frac{N}{l} \)

\( n \) is the number of turns per unit length.

Chapter 29: Gauss’s law in magnetism (Magnetic Flux (\( \Phi_B \)))

The definition of \( \Phi_B \) is similar to the electric flux \( \Phi_E \).

If we have element area \( dA \) with magnetic filed \( B \) passing through it, then

\[ \Phi_B = \int \vec{B} \cdot d\vec{A} \quad \text{(Weber=Wb=T.m²)} \]

Where \( d\vec{A} \) is the Surface vector

For a uniform field making an angle \( \theta \) with the surface normal:

\[ \Phi_B = \vec{B} \cdot \vec{A} = BA \cos \theta \]

\[ \Rightarrow \Phi_B = 0 \]

\[ \Phi_B = \Phi_{B,\text{max}} = AB \]
Chapter 29: Gauss’s law in magnetism (Magnetic Flux Through a Rectangular Loop)

\[ \Phi_B = \oint \mathbf{B} \cdot d\mathbf{A} = \int_B dA = \int \frac{\mu_0 I}{2\pi r} dA \]

area element \( dA = b \, dr \).

Because \( r \) is the only variable →

\[ \Phi_B = \frac{\mu_0 I}{2\pi} \ln \left( \frac{a + c}{e} \right) = \frac{\mu_0 I}{2\pi} \ln \left( 1 + \frac{a}{c} \right) \]

Unlike electrical fields, all magnetic field lines always form loops. (always there is a dipole).

Hence, Net flux over any closed surface equal to zero → number of line entering = number of lines leaving

**Summary**

- **Biot–Savart law**
  \[ d\mathbf{B} = \frac{\mu_0 I ds \times \hat{r}}{4\pi r^2} \]

- **Total B-field**
  \[ B = \frac{\mu_0 I}{4\pi} \frac{dx \times \hat{r}}{r^2} \]

- **Force between two parallel wires separated by a distance \( a \)**
  \[ F_a = \frac{\mu_0 I_1 I_2}{2\pi a} \]

- **B-field at distance \( r \) from straight long wire**
  \[ B = \frac{\mu_0 I}{2\pi r} \]

- **B-field due to a circular Arc of Wire radius \( R \)**
  \[ B = \frac{\mu_0 I}{4\pi R} \]

- **B-field at the center of circular loop of radius \( R \) and carrying current \( I \)**
  \[ B = \frac{\mu_0 I}{2R} \]

**Ampère’s law**

\[ \oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \]

the B-field inside a solenoid of \( N \) turns and length \( L \)

\[ B = \frac{\mu_0 N}{\ell} I = \mu_0 Ni \]

The magnitudes of the fields inside a toroid of turns \( N \) and at distance \( r \) from the center magnetic flux \( \Phi_B \) through a surface

\[ \Phi_B = \oint \mathbf{B} \cdot d\mathbf{A} \]

Net magnetic flux \( \Phi_B \) over a closed surface is zero

\[ \oint \mathbf{B} \cdot d\mathbf{A} = 0 \]

**Problems**

1. In Niels Bohr’s 1913 model of the hydrogen atom, an electron circles the proton at a distance of \( 5.29 \times 10^{-11} \) m with a speed of \( 2.19 \times 10^6 \) m/s. Compute the magnitude of the magnetic field that this motion produces at the location of the proton.

B-field at the center of the circle is (Ex. 30.3):

\[ B = \frac{\mu_0 I}{2\pi R} \]

But, \( I = q/t \),

\[ t = \text{distance/speed} = \frac{2\pi R}{v} \]

\[ I = q/(2\pi R) \]

\[ \Rightarrow B = \frac{\mu_0 I}{2R} = \frac{\mu_0 q/(2\pi R)}{2R} = 12.5 \, \text{T} \]
2. The segment of wire in Figure carries a current of $I = 5.00\, \text{A}$, where the radius of the circular arc is $R = 3.00\, \text{cm}$. Determine the magnitude and direction of the magnetic field at the origin.

For the straight sections $\int ds \times \mathbf{r} = 0$

For quarter circle $\int 1/4 \text{ B-field of full circle}$

$B = \frac{1}{2} \frac{\mu_0 I}{R}$ into the paper

$B = \frac{4\pi \times 10^{-7} \, \text{T} \cdot \text{m/A}}{5.00 \, \text{A}} \times \frac{8}{0.030 \, \text{m}}$

$= 26.2\, \mu\text{T}$ into the paper

Or, B-field due to circular curve is $B = \frac{\mu_0 I}{8R} = 26.2\, \mu\text{T}$ into the paper

3. (a) What is the magnitude of the magnetic field created by $I_1$ at the location of $I_2$? (b) What is the force per unit length exerted by $I_1$ on $I_2$? (c) What is the magnitude of the magnetic field created by $I_2$ at the location of $I_1$? (d) What is the force per length exerted by $I_1$ on $I_2$?

4. A long straight wire lies on a horizontal table and carries a current of $1.20\, \mu\text{A}$. In a vacuum, a proton moves parallel to the wire (opposite the current) with a constant speed of $2.50 \times 10^4 \, \text{m/s}$ at a distance $d$ above the wire. Determine the value of $d$.

What forces affect the proton?
1) $mg$ downward 
2) $F_B$ upward

Balance between the weight of the proton and the magnetic force

$mg = F_B$ \quad $mg = qvB$, but $B = \mu_0 I/2\pi d$

$mg = qvB = q\mu_0 I/2\pi d$

$d = \frac{qvB}{2\pi mg} = \frac{(1.60 \times 10^{-19} \, \text{C}) \cdot (2.30 \times 10^4 \, \text{m/s}) \cdot (4\pi \times 10^{-7} \, \text{T} \cdot \text{m/A}) \cdot (1.20 \times 10^{-6} \, \text{A})}{2\pi (1.67 \times 10^{-27} \, \text{kg}) \cdot (9.80 \, \text{m/s}^2)} = 5.40\, \text{cm}$

5. What current is required in the windings of a long solenoid that has 1,000 turns uniformly distributed over a length of 0.400 m, to produce at the center of the solenoid a magnetic field of magnitude $1.00 \times 10^{-4} \, \text{T}$?

$L = \frac{\mu_0 N}{\mu_0 N} = \frac{1.00 \times 10^{-4} \, \text{T}}{0.400 \, \text{m}} = 31.8 \, \text{mA}$
6. A cube of edge length $\ell = 2.50\text{ cm}$ is positioned as shown in Figure P30.35. A uniform magnetic field given by $\mathbf{B} = (5\hat{i} + 4\hat{j} + 3\hat{k})\text{T}$ exists throughout the region. (a) Calculate the flux through the shaded face. (b) What is the total flux through the six faces?

(a) $\Phi_B = \mathbf{B} \cdot \mathbf{A} = B \cdot A \cdot (2.50 \times 10^{-2} \text{ m})^2$ $\hat{i}$

$\Phi_B = 3.12 \times 10^{-3} \text{ T} \cdot \text{m}^2 = 3.12 \times 10^{-3} \text{ Wb} = 3.12 \text{ mWb}$

(b) $\Phi_{\text{total}} = \oint \mathbf{B} \cdot d\mathbf{A} = 0$ for any closed surface (Gauss's law for magnetism)