Chapter 30: Magnetic field sources

- Using Biot-Savart Law to calculate the magnetic field produced by current element
- calculate the total magnetic field due to various current distributions.
- Magnetic Force Between Two Parallel Conductors
- Ampère's law

Chapter 29: Magnetic field sources

- Magnetic field is not only produced by Magnet stones, but it can be produced by current when passing through conductors
- this chapter explores the origin of the magnetic field associated with moving charges.
- Oersted's discovered in 1819 that a compass needle is deflected by a current-carrying conductor → Current in a wire produces magnetic field
- Jean-Baptiste Biot and Félix Savart performed quantitative experiments to calculate the magnetic field ➔ Biot-Savart Law

Chapter 29: Biot-Savart Law

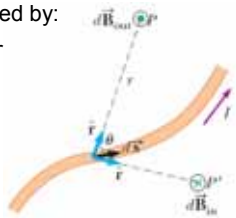
For a length element $d\vec{s}$ along a current carrying wire, Biot and Savart experimental findings, for the magnetic field $d\vec{B}$ (of magnitude dB) at some point P, are summarized by:

1) $d\vec{B} \perp d\vec{s}$ *and* $d\vec{B} \perp \hat{r}$ where $\hat{r}$ is unit vector directed from $d\vec{s}$ to $\vec{P}$

2) $dB \propto \frac{1}{r^2}$ where r is radial distance from $d\vec{s}$

3) $dB \propto I, \quad dB \propto ds, \quad dB \propto \sin\theta$

4) B-field ($d\vec{B}$) is out at P and B-field is in at P'



$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$ **Biot-Savart Law**

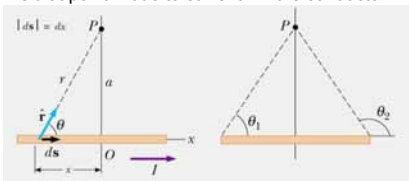
$\mu_0 = 4\pi\times10^{-7}\,T.m/A$ is permeability of free space

➔ the *total* B-field created at some point by a current in a wire is $\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$

Chapter 30.1: Biot-Savart Law (Magnetic Field Surrounding a Thin, Straight Conductor)

Determine the magnitude and direction of the magnetic field at point *P* due to current *I* in the conductor.

$$\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$$



$$|d\vec{s}\times\hat{r}| = |ds\times\hat{r}| = dx(1)\sin\theta$$

$$\tan\theta = \frac{a}{x} \Rightarrow x = -\frac{a}{\tan\theta} = -a\cot\theta$$

$$\frac{dx}{d\theta} = a\csc^2\theta \Rightarrow dx = a\csc^2\theta\, d\theta$$

$$\sin\theta = \frac{a}{r} \Rightarrow r = \frac{a}{\sin\theta} = a\csc\theta$$

$$d\vec{s}\times\hat{r} = a\csc^2\theta\sin\theta d\theta\,\hat{k}$$

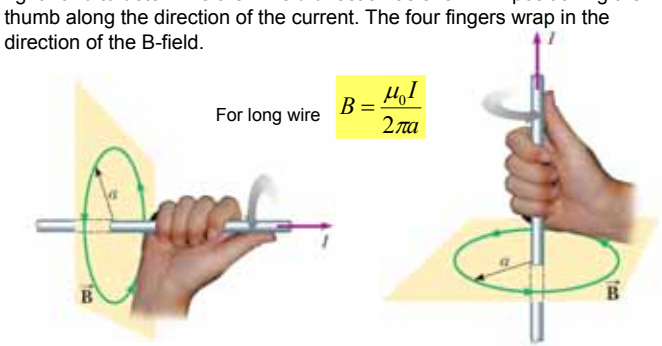
$$B = \frac{\mu_0 I}{4\pi}\int_{\theta_1}^{\theta_2}\frac{a\csc^2\theta\sin\theta d\theta}{a^2\csc^2\theta} \longrightarrow B = \frac{\mu_0 I}{4\pi a}\int_{\theta_1}^{\theta_2}\sin\theta d\theta = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$$

If the wire is very long, L>>a $\theta_1 \approx 0°$, $\theta_2 \approx 180°$ then $B = \frac{\mu_0 I}{2\pi a}$

Chapter 29: Biot-Savart Law (magnetic field surrounding a thin, straight conductor)

➔ as a result, for long wire, the magnetic field lines are circles concentric with the wire and lie in planes perpendicular to the wire. We can use the right hand to determine the B-field direction as shown ➔ positioning the thumb along the direction of the current. The four fingers wrap in the direction of the B-field.

For long wire $B = \frac{\mu_0 I}{2\pi a}$

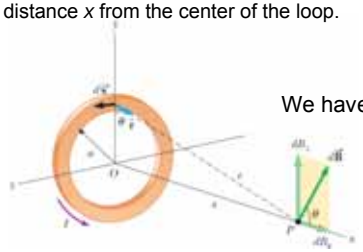


Chapter 29: Biot-Savart Law (Magnetic Field on the axis of a circular current carrying loop)

Calculate the B-field at an axial point *P* a distance *x* from the center of the loop.

$$B = \frac{\mu_0 I}{4\pi}\int \frac{|d\mathbf{s}\times\hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi}\int \frac{ds}{(x^2+a^2)}$$

$$B_\perp cancels (B_y, B_z = 0)$$



We have only B_x ➔ $B = B_x = \frac{\mu_0 I}{4\pi}\oint \frac{ds\cos\theta}{x^2+R^2}$

$$\cos\theta = \frac{a}{r} = \frac{a}{\sqrt{a^2+x^2}}$$

$$B_x = \frac{\mu_0 I a}{4\pi(x^2+a^2)^{3/2}}\oint ds = \frac{\mu_0 I a}{4\pi(x^2+a^2)^{3/2}}2\pi a = \frac{\mu_0 I a^2}{2(x^2+a^2)^{3/2}}$$

$$|d\mathbf{s}\times\hat{\mathbf{r}}| = (ds)(1)\sin 90° = ds$$

$$r^2 = x^2 + a^2$$

Special Cases:

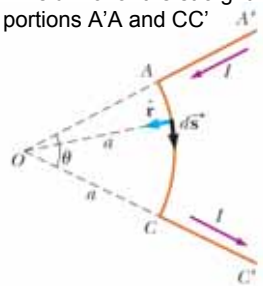
At O (x=0): $B_x \approx \frac{\mu_0 I}{2a}$

At x>>a: $B_x \approx \frac{\mu_0 I a^2}{2x^3} = \frac{\mu_0 \mu}{2\pi x^3}$

Chapter 29: Biot-Savart Law (field due to a circular arc of wire)

Calculate the B-field at point *O* for the current-carrying wire segment shown.

B-field = 0 for the straight portions A'A and CC'



For the arc (AC) of length s and radius a

$$\vec{B} = \frac{\mu_0 i}{4\pi}\int \frac{d\vec{s}\times\hat{r}}{r^2}$$

$$|\vec{B}| = B = \frac{\mu_0 I}{4\pi}\int \frac{ds(1)\sin 90°}{a^2} = \frac{\mu_0 I}{4\pi a^2}\int ds$$

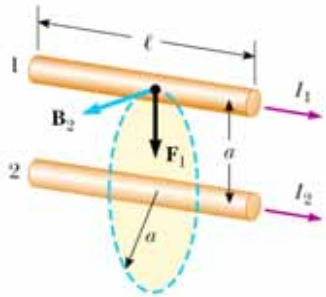
$$\longrightarrow B = \frac{\mu_0 I}{4\pi a^2}s \qquad But\ s = a\theta$$

Hence $B = \frac{\mu_0 I\theta}{4\pi a}$

Full Circle ($\theta = 2\pi$) → $B = \frac{\mu_0 I}{2a}$ As shown in previous example

Chapter 29: Magnetic Force Between Two Parallel Conductors

Consider two long, straight, parallel wires separated by a distance *a* and carrying currents I_1 and I_2 in the same direction



Since we have B-field from each wire, and both have currents ➔ Forces between the wires will appear

$$F_1 = I_1 l B_2 = I_1 l\left(\frac{\mu_0 I_2}{2\pi a}\right) = \frac{\mu_0 I_1 I_2}{2\pi a}l$$

$$F_2 = I_2 l B_1 = I_2 l\left(\frac{\mu_0 I_1}{2\pi a}\right) = \frac{\mu_0 I_1 I_2}{2\pi a}l$$

$\longrightarrow F_1 = F_2 = F_B$ F_B between two parallel wires

$\vec{F}_1 = -\vec{F}_2$ Opposite in direction

I_1& I_2 same in direction ➔ attraction

I_1& I_2 opposite in direction ➔ repulsion

Chapter 29: Magnetic Force Between Two Parallel Conductors

The Force per unit length $\frac{F_B}{l} = \frac{\mu_0 I_1 I_2}{2\pi a}$

If a = 1m, $I_1 = I_2$, and $\mathbf{F_B/l = 2\times10^{-7}}$ N/m

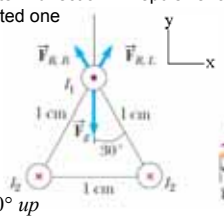
➔ I in both wires is defined to be **1 ampere**

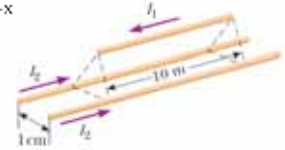
Chapter 29: Magnetic Force Between Two Parallel Conductors

Ex: Two infinitely long, parallel wires are lying on the ground 1 cm apart as shown. A third wire, of length 10 m and mass 400 g (0.4kg), carries a current of I_1 =100 A and is levitated above the first two wires as shown. What current (I_2) must the infinitely long wires carry so that the three wires form an equilateral triangle?

Since I_1 and I_2 are opposite in direction → repulsive force from Right ($F_{B,R}$) and Left ($F_{B,L}$) wires with the levitated one

$$F_{B,L} = F_{B,R} = \frac{\mu_0 I_1 I_2}{2\pi a} l$$





X-components of forces on levitated wire cancels. But y-components add

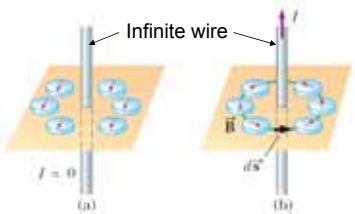
$$\rightarrow F_B = 2\left(\frac{\mu_0 I_1 I_2}{2\pi a} l\right)\sin 30°\ up$$

Since it is levitated

→ F_B up = F_g down

$$\mu_0 \frac{I_1 I_2}{\pi a} l(\sin 30°) = mg \Rightarrow I_2 = \frac{\pi a m g}{(\sin 30°)\mu_0 I_1 l} = \boxed{113\,A}$$

Chapter 29: Ampère's Law



With I=0 ➔ no B-field (a)

With I ≠ 0 ➔ B-filed form Loops about their source (b)

➔ We can Integrate around the closed loop $\oint \vec{B}.d\vec{s}$

$d\vec{s}$ is displacement element along the loop path

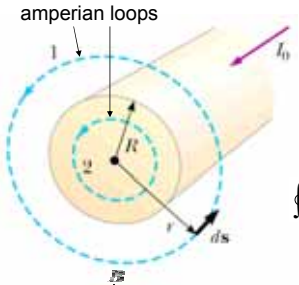
We call the loop where we need to find the B-field by Amperian loop (current must pass through the loop in one direction)

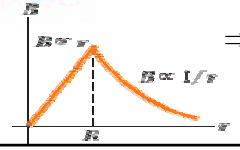
$$\oint \vec{B}.d\vec{s} = B\oint ds = \frac{\mu_0 I}{2\pi r}(2\pi r) = \mu_0 I$$

$$\oint \vec{B}.d\vec{s} = \mu_0 I \rightarrow \text{Ampere's law}$$

A line integral of $\vec{B}.d\vec{s}$ around a closed path equals $\mu_0 I$, where I is the current passing through amperian loop surface (surface bounded by the closed path).

Chapter 29: Ampère's Law (Long Wire Carrying Current) ➔ Ex: Calculate the B-field a distance *r* from the center of the wire carrying current I_0 in the regions r >= R and r < R.





For *r* >= *R* , all curent in the wire (I_0) pass through whole surface of amperial loop

$$\oint \mathbf{B}.d\mathbf{s} = B\oint ds = B(2\pi r) = \mu_0 I_0$$

$$\Rightarrow B = \frac{\mu_0 I_0}{2\pi r}$$

For r < R

$$\oint \mathbf{B}.d\mathbf{s} = B(2\pi r) = \mu_0 I \quad (I \text{ is Current in loop 2})$$

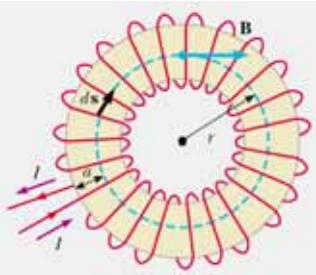
$$But,\ \frac{I}{I_0} = \frac{\pi r^2}{\pi R^2} \Rightarrow I = \frac{r^2}{R^2} I_0$$

$$\Rightarrow \oint \mathrm{B}.ds = B(2\pi r) = \mu_0 I = \mu_0\left(\frac{r^2}{R^2} I_0\right)$$

$$\Rightarrow B = \left(\frac{\mu_0 I_0}{2\pi R^2}\right) r$$

Chapter 29: Ampère's Law (The Toroid)

Ex: For a toroid having N closely spaced turns of wire, calculate the B-field in the region occupied by the torus, a distance r from the center.



We construct amperian loop inside the toroid (dashes circle)→By symmetry, B is constant over the dashed circle and tangent to it

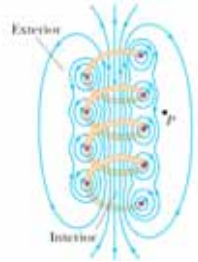
$$\oint \vec{B}.d\vec{s} = B\oint ds = B(2\pi r) = \mu_0 NI$$

$$\Rightarrow B = \frac{\mu_0 NI}{2\pi r}$$

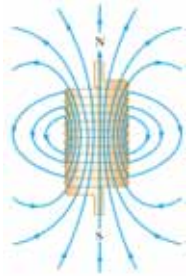
Outside the toroid: $B = 0$

Chapter 29: The B-Field of a Solenoid

A **solenoid** is a long wire wound in the form of a helix



Almost uniform **B**-field in the interior

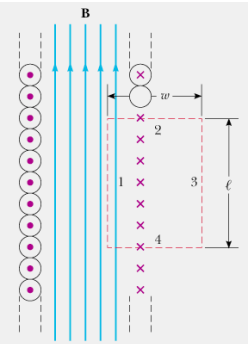


uniform **B**-field in the interior

net **B**-field is the vector sum of the fields resulting from all the turns.

Chapter 29: The B-Field of a Solenoid

Consider long solenoid L >> R



Along path 2 and 4, ($\vec{B} \perp d\vec{s}$) ➔ $\vec{B}.d\vec{s} = 0$

Along path 3, $\vec{B}$=0

$$\oint \vec{B}.d\vec{s} = \int_{path1} \vec{B}.d\vec{s} = B\int_{path1} ds = Bl$$

$$\oint \vec{B}.d\vec{s} = Bl = \mu_0 NI \longrightarrow B = \frac{\mu_0 NI}{l}$$

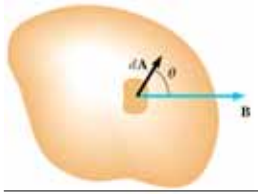
$$B = \mu_0 nI \quad \text{where} \quad n = N/l$$

n is the number of turns per unit length.

Chapter 29: Gauss's law in magnetism (Magnetic Flux (Φ_B))

The defenition of Φ_B is similar to the electric flux Φ_B.

If we have element area *d*A with magnetic filed *B* passing through it, then

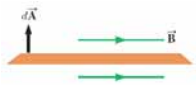


$$\Phi_B = \int \vec{B}.d\vec{A}$$ (Weber=Wb=T.m²)

Where $d\vec{A}$ is the Surface vector

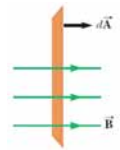
For a uniform field making an angle θ with the surface normal:

$$\Phi_B = \vec{B}.\vec{A} = BA\cos\theta$$



$For\ \vec{B} \perp d\vec{A}$

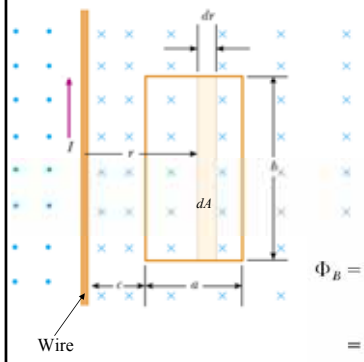
$\Rightarrow \Phi_B = 0$



$For\ \vec{B} // d\vec{A}$

$\Phi_B = \Phi_{B,\max} = AB$

Chapter 29: Gauss's law in magnetism (Magnetic Flux Through a Rectangular Loop)



$$\Phi_B = \int \vec{\mathbf{B}} \cdot d\vec{\mathbf{A}} = \int B\,dA = \int \frac{\mu_0 I}{2\pi r}\,dA$$

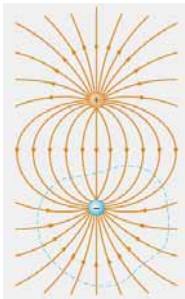
area element $dA = b\,dr$.
Because r is the only variable ➔

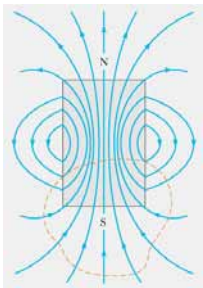
$$\Phi_B = \int \frac{\mu_0 I}{2\pi r}\,b\,dr = \frac{\mu_0 I b}{2\pi}\int \frac{dr}{r}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi}\int_c^{a+c} \frac{dr}{r} = \frac{\mu_0 I b}{2\pi}\ln r\Big|_c^{a+c}$$

$$= \frac{\mu_0 I b}{2\pi}\ln\left(\frac{a+c}{c}\right) = \frac{\mu_0 I b}{2\pi}\ln\left(1+\frac{a}{c}\right)$$

Chapter 29: Gauss's law in magnetism





Unlike electrical fields, all magnetic field lines always form loops. (always there is a dipole).

Hence,

Net flux over any closed surface equal to zero ➔ number of line entering = number of lines leaving

Electric Field Lines enter Gauss surface

Magnetic Field Lines enter and leave Gauss surface

$$\oint \vec{B}.d\vec{A} = 0$$

Summary

Biot–Savart law ➔ Total B-filed

$$d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{I d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\int \frac{d\mathbf{s}\times\hat{\mathbf{r}}}{r^2}$$

force between two parallel wires separated by a distance a is

$$F_B = \frac{\mu_0 I_1 I_2}{2\pi a}l$$

B- field at distance r from straight long wire

$$B = \frac{\mu_0 I}{2\pi r}$$

B-field due to a circular Arc of Wire radius R

$$B = \frac{\mu_0 i\theta}{4\pi R}$$

B-field at the center of circular loop of radius R and carrying current i

$$B = \frac{\mu_0 i}{2R}$$

Ampère's law

$$\oint \vec{B}.d\vec{s} = \mu_0 I$$

the B-field inside a solenoid of N turns and length L

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$

The magnitudes of the fields inside a toroid of turns N and at distance r from the center

$$B = \frac{\mu_0 N I}{2\pi r}$$

magnetic flux Φ_B through a surface

$$\Phi_B = \int \vec{B}.d\vec{A}$$

Net magnetic flux Φ_B over a closed surface is zero

$$\oint \vec{B}.d\vec{A} = 0$$

Problems

1. In Niels Bohr's 1913 model of the hydrogen atom, an electron circles the proton at a distance of 5.29×10^{-11} m with a speed of 2.19×10^{6} m/s. Compute the magnitude of the magnetic field that this motion produces at the location of the proton.

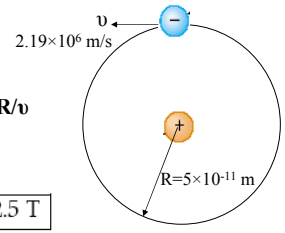
B-field at the center of the circle is (Ex. 30.3):

$$B = \frac{\mu_0 I}{2R}$$

But, I = q/t ,

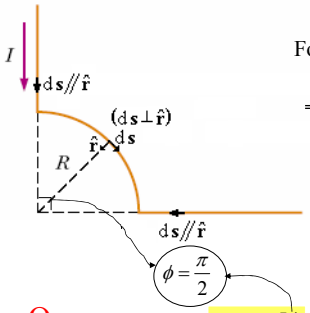
t =distance/speed= 2πR/υ

➔ I = q(υ/2πR)



$$\Longrightarrow B = \frac{\mu_0 I}{2R} = \frac{\mu_0 q(v/2\pi R)}{2R} = \boxed{12.5\ \text{T}}$$

2. The segment of wire in Figure carries a current of $I = 5.00$ A, where the radius of the circular arc is $R = 3.00$ cm. Determine the magnitude and direction of the magnetic field at the origin.



$\boxed{\text{For the straight sections } d\mathbf{s} \times \hat{\mathbf{r}} = 0}$

For quarter circle ⟹ 1/4 B-field of full circle

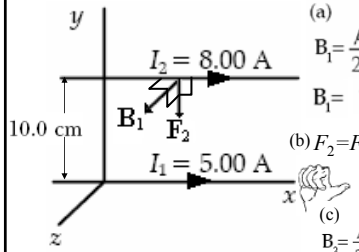
$$\Longrightarrow B = \frac{1}{4}\frac{\mu_0 I}{2R} = \frac{\mu_0 I}{8R} \text{ into the paper}$$

$$B = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A})}{8(0.030\,0\ \text{m})}$$

$$= \boxed{26.2\ \mu\text{T into the paper}}$$

Or, B-field due to circular curve is $B = \frac{\mu_0 I \phi}{4\pi R}$ ⟹ $B = \frac{\mu_0 I}{8R} = \boxed{26.2\ \mu\text{T into the paper}}$

3. (a) What is the magnitude of the magnetic field created by I_1 at the location of I_2? (b) What is the force per unit length exerted by I_1 on I_2? (c) What is the magnitude of the magnetic field created by I_2 at the location of I_1? (d) What is the force per length exerted by I_2 on I_1?



Two long, parallel conductors

(a)
$$B_1 = \frac{\mu_0 I}{2\pi r}\hat{\mathbf{k}} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A})}{2\pi(0.100\ \text{m})}\hat{\mathbf{k}}$$

$B_1 = 1.00 \times 10^{-5}$ T out of the page

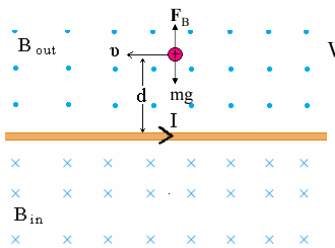
(b) $F_2 = F_B = I_2 l B_1$ ➔ $F_B/l = I_2 B_1 = (8\text{A})(1\times10^{-5}\ \text{T})$
$= 8\times10^{-5}$ N/m downward

(c)
$$B_2 = \frac{\mu_0 I}{2\pi r}(-\hat{\mathbf{k}}) = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(8.00\ \text{A})}{2\pi(0.100\ \text{m})}(-\hat{\mathbf{k}})$$

$B_2 = \boxed{1.60 \times 10^{-5}\ \text{T into the page}}$

(d) $F_1 = F_B = I_1 l B_2$ ➔ $F_B/l = I_1 B_1 = (5\text{A})(1.6\times10^{-5}\ \text{T})$
$= 8\times10^{-5}$ N/m upward

4. A long straight wire lies on a horizontal table and carries a current of 1.20 μA. In a vacuum, a proton moves parallel to the wire (opposite the current) with a constant speed of 2.30×10^4 m/s at a distance d above the wire. Determine the value of d.



What forces affect the proton?
1) mg downward 2) $\mathbf{F}_B$ upward
balance between the weight of the proton and the magnetic force

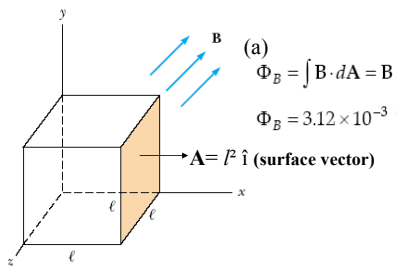
mg = F_B ➔ mg = qυB , but B = $\mu_0 I/2\pi d$
➔ mg = $q\upsilon\mu_0 I/2\pi d$

$$d = \frac{q\upsilon\mu_0 I}{2\pi mg} = \frac{(1.60\times10^{-19}\ \text{C})(2.30\times10^{4}\ \text{m/s})(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(1.20\times10^{-6}\ \text{A})}{2\pi(1.67\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)} = \boxed{5.40\ \text{cm}}$$

5. What current is required in the windings of a long solenoid that has 1 000 turns uniformly distributed over a length of 0.400 m, to produce at the center of the solenoid a magnetic field of magnitude 1.00×10^{-4} T?

$$B = \mu_0 \frac{N}{\ell} I \text{ so } I = \frac{Bl}{\mu_0 N} = \frac{(1.00\times10^{-4}\ \text{T})0.400\ \text{m}}{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})1\,000} = \boxed{31.8\ \text{mA}}$$

6. A cube of edge length $\ell = 2.50$ cm is positioned as shown in Figure P30.35. A uniform magnetic field given by $\mathbf{B} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$T exists throughout the region. (a) Calculate the flux through the shaded face. (b) What is the total flux through the six faces?



(a)

$$\Phi_B = \int \mathbf{B}\cdot d\mathbf{A} = \mathbf{B}\cdot\mathbf{A} = \left(5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}}\right)\text{ T}\cdot\left(2.50\times10^{-2}\text{ m}\right)^2\hat{\mathbf{i}}$$

$$\Phi_B = 3.12\times10^{-3}\text{ T}\cdot\text{m}^2 = 3.12\times10^{-3}\text{ Wb} = \boxed{3.12\text{ mWb}}$$

(b) $(\Phi_B)_{\text{total}} = \oint \mathbf{B}\cdot d\mathbf{A} = \boxed{0}$ for *any* closed surface (Gauss's law for magnetism)