Ch09: Center Of Mass And Linear Momentum

- The center of mass (com)
- Motion of a system of particles (Newton’s 2nd law)
- Linear momentum
- Impulse
- Conservation of linear momentum
- Inelastic Collision in one dimension
- Elastic collision in 1 and 2 dimensions

9.1: What is physics?

- Analyzing complicated motion of any sort requires simplification via an understanding of physics.
- The complicated motion of a system of objects, such as a car or a ballerina or collection of particles, can be simplified if we determine a special point of the system—the center of mass (com) that system.
- We can apply equations of motion to the center of mass to represent the motion of that system.
- An example for complicated motion is a baseball bat when flipped into air points of bat moves along different path shapes except the center of mass moves a long a simple parabolic path (projectile motion).

9.2: The Center Of Mass

- If we consider a collection of particles or an extended object with a total mass, M.
- We can consider the system as a single particle with the total mass, M, concentrated at a single point called the center of mass (com).
- If a net force external is applied to the system, then you can apply Newton’s second law to the center of mass.
- The center of mass of a system of particles is the point that moves as though:
  - all of the system’s mass were concentrated there
  - all external forces were applied there.

9.2: The Center Of Mass: system of particles

- The position of center of mass $x_{com}$ is the average position of the system’s masses on x-axis.
- For the two particle shown, the $x_{com}$ is
  \[ x_{com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} \]
- For n particles along x-axis
  \[ x_{com} = \frac{\sum_{i=1}^{n} m_i x_i}{M} \]

Total mass of the system = M
9.2: The Center Of Mass: system of particles
- For group of particles in space (more than one dimension), we need to find position vector for the center of mass $\mathbf{r}_{\text{com}}$

\[
\mathbf{r}_{\text{com}} = \frac{\sum r_i m_i}{M} = \frac{1}{M} \sum r_i m_i
\]

\[
\mathbf{r}_{\text{com}} = \frac{\sum m_i \mathbf{r}_i}{M} = x_{\text{CM}} \hat{i} + y_{\text{CM}} \hat{j} + z_{\text{CM}} \hat{k}
\]

\[
x_{\text{com}} = \frac{\sum m_i x_i}{M} \quad y_{\text{com}} = \frac{\sum m_i y_i}{M} \quad z_{\text{com}} = \frac{\sum m_i z_i}{M}
\]

9.2: The Center Of Mass: Solid Bodies
- For an extended solid object (continuous distribution of matter)

\[
\mathbf{r}_{\text{CM}} = \frac{\sum \Delta m_i \mathbf{r}_i}{M}
\]

For $\Delta m_i \rightarrow 0 \rightarrow dm \rightarrow$

\[
\mathbf{r}_{\text{CM}} = \frac{1}{M} \int \mathbf{r} dm
\]

\[
x_{\text{CM}} = \frac{1}{M} \int x dm \quad y_{\text{CM}} = \frac{1}{M} \int y dm \quad z_{\text{CM}} = \frac{1}{M} \int z dm
\]

9.2: The Center Of Mass: Example
- For an extended solid object (continuous distribution of matter)
- For an extended solid object (continuous distribution of matter)

A group of particles in two dimensions is shown in the figure. Find the center of mass if $m_1=1\text{kg}, \quad m_2=1\text{kg}, \quad m_3=2\text{kg}$

we have x and y dimensions $\Rightarrow$

\[
x_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(1)(0) + (1)(2) + (2)(0)}{1 + 1 + 2} = \frac{3}{4} = 0.75 \text{m}
\]

\[
y_{\text{com}} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3} = \frac{(1)(0) + (1)(0) + (2)(2)}{1 + 1 + 2} = \frac{4}{4} = 1 \text{m}
\]

\[
\mathbf{r}_{\text{com}} = (0.75 \hat{i} + 1 \hat{j}) \text{m}
\]

9.2: The Center Of Mass: Example
- A group of particles in two dimensions is shown in the figure.
- Find the center of mass if $m_1=1\text{kg}, \quad m_2=1\text{kg}, \quad m_3=2\text{kg}$

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\]

\[
\mathbf{r}_{\text{com}} = (0.75 \hat{i} + 1 \hat{j}) \text{m}
\]
For a system of particles in motion, we usually interesting in the motion of the center of mass that represents the whole system:

- the velocity of the center of mass ($\vec{v}_{\text{com}}$) of the system is found from

$$\vec{v}_{\text{com}} = \frac{d\vec{r}_{\text{com}}}{dt} = \frac{1}{M} \sum m_i \frac{d\vec{r}_i}{dt} = \frac{1}{M} \sum m_i \vec{v}_i$$

$$\vec{v}_{\text{com}} = \frac{1}{M} \sum m_i \vec{v}_i$$

- Hence, the total mass ($M$) multiplied by the velocity of center of mass ($\vec{v}_{\text{com}}$) represents the all system; it is equivalent to multiply each particle velocity with it's mass and sum them.

\[ M\vec{v}_{\text{com}} = \sum m_i \vec{v}_i = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \ldots \ldots + m_n \vec{v}_n \]

Newton's Second Law for a System of Particles

- Taking the derivative of the $\vec{v}_{\text{com}}$, you can get the acceleration of the center of mass ($\vec{a}_{\text{com}}$)

$$\vec{a}_{\text{com}} = \frac{d\vec{v}_{\text{com}}}{dt} = \frac{1}{M} \sum m_i \frac{d\vec{v}_i}{dt} = \frac{1}{M} \sum m_i \vec{a}_i \Rightarrow \vec{a}_{\text{com}} = \frac{\vec{F}_\text{net}}{M}$$

- Hence, the total mass ($M$) multiplied by the velocity of center of mass ($\vec{v}_{\text{com}}$) represents the all system; it is equivalent to multiply each particle velocity with it's mass and sum them.

\[ M\vec{a}_{\text{com}} = \sum m_i \vec{a}_i = m_1 \vec{a}_1 + m_2 \vec{a}_2 + m_3 \vec{a}_3 + \ldots \ldots + m_n \vec{a}_n \]

- The com of the system will move as if all the mass were there and the net force acted there.
9.4: Linear Momentum

- The linear momentum of a particle with mass \( m \) and velocity \( \vec{v} \) is:

\[
\vec{p} = m\vec{v}
\]

SI unit is \( \text{kg m/s} \equiv N.s \)

- \( \vec{p} \) is a vector quantity and has the direction of the velocity \( \vec{v} \)
- For a particle, any change in momentum due to velocity change is related to some external net force \( \vec{F}_{\text{net}} \)
- From Newton's 2nd law:

\[
\sum \vec{F} = m\vec{a} = m\frac{d\vec{v}}{dt} \quad \Rightarrow \quad \sum \vec{F} = \frac{d(m\vec{v})}{dt} = \frac{d\vec{p}}{dt}
\]

9.5: Linear Momentum For a system of particles

- For a system of particles in motion each has its own velocity and mass \( \rightarrow \) the velocity of the center of mass (\( \vec{v}_{\text{com}} \)) is:

\[
\vec{v}_{\text{com}} = \frac{1}{M} \sum_i m_i \vec{v}_i
\]

- The linear momentum for the center of mass that represents the system particles can be obtained from above equation:

\[
\vec{P}_{\text{com}} = M\vec{v}_{\text{com}}
\]

Momentum for system of particles

where \( \vec{P}_{\text{com}} = M\vec{v}_{\text{com}} = \sum_i m_i \vec{v}_i = m_1 \vec{v}_1 + m_2 \vec{v}_2 + m_3 \vec{v}_3 + \ldots \ldots + m_n \vec{v}_n \)

\[
\Rightarrow \vec{P}_{\text{com}} = M\vec{v}_{\text{com}} = \sum_j \vec{p}_j = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + \ldots \ldots + \vec{p}_n
\]

Hence, the momentum for the system is equivalent to the sum of all individual momentum of particles

9.6: Collision and impulse: Single Collision

- Momentum of particle can only change by a net external force
- Consider a collision between a ball and a bat in baseball game
  \( \rightarrow \) The ball will experience a force \( \vec{F}(t) \) that varies during collision \( \rightarrow \) ball slows down, stop, and reverse direction \( \rightarrow \) \( \vec{p} \) changes due to that force \( \rightarrow \) From Newton's 2nd law:

\[
\vec{F} = \frac{d\vec{p}}{dt} \quad \Rightarrow \quad d\vec{p} = \vec{F}(t) \, dt
\]

\( \rightarrow \) we can have the change in momentum by integrating both sides over time during collision:

\[
\int_{t_i}^{t_f} d\vec{p} = \int_{t_i}^{t_f} \vec{F}(t) \, dt
\]

A measure for magnitude and duration of a collision force (impulse \( \vec{f} \))
9.6: Collision and impulse: Single Collision

- The impulse on the particle (ball in this case):
  \[ J = \Delta \vec{p} = \int_{t_i}^{t_f} F(t) \, dt \]

Impulse-momentum theorem: Impulse is the net change in momentum due to forces.

where \( \Delta \vec{p} = \vec{p}_f - \vec{p}_i \)

In components (x-components for instant):

\[ J_x = \Delta p_x = p_{fx} - p_{ix} = \int_{t_i}^{t_f} F_x \, dt \]

- Since it is difficult to know \( F(t) \) during collisions, we use the average force magnitude and \( \Delta t \):

\[ J = F_{avg} \Delta t \]

Due to Newton's 3rd law, the ball makes an impulse on the bat of same magnitude but opposite in direction.

A car made an accident with a wall. Its velocity was 15 m/s to left before the accident. After the accident, it is recoil from the wall to the right with velocity 2.6 m/s. If the accident time is 0.15 s, find the impulse caused by the collision and the average force exerted on the car.

\[ m = 1500 \text{ kg}, \quad v_i = -15.0 \text{ m/s}, \quad v_f = 2.6 \text{ m/s}, \quad \Delta t = 0.150 \text{ s} \]

\[ J = ? \quad F_{avg} = ? \]

9.6: Collision and impulse: Example

Note: for x-axis

If \( v \) to the right \( \Rightarrow v \) is +ve

If \( v \) to the left \( \Rightarrow v \) is -ve

The impulse

\[ J = \Delta p \]

\[ J = m(v_f - v_i) = m(v_f - (-v_i)) \]

\[ J = 1500 \times (2.6 - (-15)) \]

\[ = 2.64 \times 10^4 \text{ i N} \cdot \text{s} \]

\[ F_{avg} = \frac{\Delta p}{\Delta t} = \frac{2.64 \times 10^4}{0.15} \]

\[ = 1.76 \times 10^5 \text{ i N} \]

9.6: Collision and impulse: Example: continued from previous slide

Figure below shows the path taken by a race car driver as he collides with the racetrack wall. If driver mass \( m \) is 80 kg find

a) The impulse on the driver due to collision

b) Average force magnitude on driver if the collision lasts for 14 ms

\[ \text{(a)} \quad J = (m(v_f - v_i)) \text{ kg} \cdot \text{m/s} \]

\[ = (80)(-50 \cos 30^\circ - 70 \cos 30^\circ) \]

\[ = -910 \text{ kg} \cdot \text{m/s} \]

\[ \text{(b)} \quad F_{avg} = \frac{J}{\Delta t} \]

\[ = \frac{3616}{0.014} \]

\[ = 255,400 \text{ N} \]

\[ \theta = \tan^{-1} \frac{J_y}{J_x} \]

\[ = 25.5^\circ \]
Consider two particles interact together. We can apply Newton’s Third law:

\[ \vec{F}_{12} = -\vec{F}_{21} \Rightarrow \vec{F}_{21} + \vec{F}_{12} = 0 \]

Apply Newton’s 2nd law:

\[ m_1 \ddot{v}_1 + m_2 \ddot{v}_2 = 0 \]

\[ \frac{d}{dt}(m_1 \dot{v}_1 + m_2 \dot{v}_2) = 0 \]

\[ \dot{p}_1 + \dot{p}_2 = \sum \dot{p} = \text{constant} \]

**9.7: Conservation of Linear Momentum: Example**

Two skaters recoil each other from rest. After recoil Find a) the total momentum b) the momentum for heavy skater1 and c) the velocity for light skater2

- \( m_1 = 100 \text{ kg}, \quad v_1 = 5 \text{ m/s}, \quad m_2 = 50 \text{ kg} \)
  - a) \( \dot{p}_{tot} = ? \)
  - b) \( \dot{p}_1 = ? \)
  - c) \( v_2 = ? \)

Solution: momentum is conserved in x-axis

a) \( \sum \dot{p}_i = \dot{p}_{tot} = 0 \) \; they start from rest

b) \( \dot{p}_1 = m_1 \dot{v}_1 = 100(5) = 500 \text{ kg m/s} \)

c) \( \sum \dot{p}_f = \dot{p}_{tot} \)

\[ \dot{p}_1f + \dot{p}_2f = 0 \]

\[ v_2 = \frac{\dot{p}_1f}{m_2} = \frac{-500}{50} = 10 \text{ m/s} \]

**9.7: Conservation of Linear Momentum: Example**

Two-dimensional explosion: the figure shows a firecracker placed inside a coconut of mass \( M \), initially at rest on a frictionless floor, blows the coconut into three pieces that slide across the floor. Piece \( C \), with mass \( 0.3M \), has final speed \( v_f = 5.0 \text{ m/s} \).

(a) What is the speed of piece \( B \), with mass \( 0.20M \)? (b) What is the speed of piece \( A \)?
9.7: Conservation of Linear Momentum: Example: continued from previous slide

(a) What is the speed of piece B? (b) What is the speed of piece A?

\[ \sum p_f = \sum p_i \Rightarrow \sum \vec{p}_f = \sum \vec{p}_i \text{ and } \sum p_f = \sum p_i \]

Since coconut was at rest before explosion

\[ \sum \vec{p}_f = 0 \Rightarrow m_A v_{x_A} + m_B v_{x_B} + m_C v_{x_C} = 0 \]

- \( 0.5 M v_{x_A} + 0.2 M v_{x_B} \cos 50^\circ + 0.3 M v_{x_C} \cos 80^\circ = 0 \)
- \( -0.5 v_{x_A} + 0.129 v_{x_B} + 0.26 = 0 \)  \( \text{(1)} \)

\[ \sum p_{fi} = 0 \Rightarrow m_A v_{fi_{x_A}} + m_B v_{fi_{x_B}} + m_C v_{fi_{x_C}} = 0 \]

\[ -0.2 M v_{x_B} \sin 50^\circ + 0.3 M v_{x_C} \sin 80^\circ = 0 \]

\[ -0.153 v_{x_B} + 0.148 = 0 \]

\[ \Rightarrow v_{x_B} = 9.67 m/s \]

Sub. in (1) \( \Rightarrow v_{x_B} = 3 m/s \)

9.8: Momentum and Kinetic Energy in Collisions

- Inelastic collision
  - Particles don’t stick together but lose some energy

- Completely inelastic collision
  - Particles stick together and lose some energy (deformation)

- Elastic collision
  - Particles bounce off each other, no loss of energy

\[ \Rightarrow \text{Mechanical Energy is conserved ONLY in elastic collisions} \]

9.9: Inelastic Collision in One dimensions: Inelastic Collision

- Inelastic collision \( \Rightarrow \) Momentum is conserved

\[ \sum \vec{p}_i = \sum \vec{p}_f \]

\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]

- Mechanical energy is not conserved

\[ K_i = \frac{1}{2} \left( m_1 v_{i_1}^2 + m_2 v_{i_2}^2 \right) \quad \text{and} \quad K_f = \frac{1}{2} \left( m_1 v_{f_1}^2 + m_2 v_{f_2}^2 \right) \]

\[ \Rightarrow \text{The Energy loss} = E_{loss} = K_f - K_i \]

9.9: Inelastic Collision in One dimensions: Completely Inelastic Collision

- Completely inelastic collision \( \Rightarrow \) Momentum is conserved

\[ \sum \vec{p}_i = \sum \vec{p}_f \]

\[ m_1 \vec{v}_1 + m_2 \vec{v}_2 = (m_1 + m_2) \vec{v}_f \]

- Mechanical energy is not conserved

\[ K_i = \frac{1}{2} \left( m_1 v_{i_1}^2 + m_2 v_{i_2}^2 \right) \quad \text{and} \quad K_f = \frac{1}{2} \left( m_1 + m_2 \right) v_f^2 \]

\[ \Rightarrow \text{The Energy loss} = E_{loss} = K_f - K_i \]
9.9: Example: A 900 kg car had a velocity of 20 m/s to the right collides with other 1800 kg car that was stopped. If the two car stuck together, find the final velocity after collision.

\[ V_i = 20 \text{ m/s} \quad V_o = 0 \quad V_f = ?? \]

**Before collision**

**After collision**

Solution:

Momentum is conserved

\[ \sum \vec{p}_i = \sum \vec{p}_f \]

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_f \]

\[ \Rightarrow \vec{v}_f = \frac{m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i}}{m_1 + m_2} = \frac{900(20) + 0}{900 + 1800} = 6.67 \text{ m/s} \]

9.10: Elastic Collision in One dimensions

**Before collision**

**After collision**

- Momentum is conserved
  \[ \sum \vec{p}_i = \sum \vec{p}_f \]

\[ m_1 \vec{v}_{1i} + m_2 \vec{v}_{2i} = (m_1 + m_2) \vec{v}_{1f} + (m_1 + m_2) \vec{v}_{2f} \] (1)

- Energy is also conserved
  \[ \sum K_i = \sum K_f \]

\[ \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \] (2)

From equations 1 & 2

\[ \vec{v}_{1f} - \vec{v}_{2f} = -\vec{v}_{1f} - \vec{v}_{2f} \]

9.9: Example: Ballistic pendulum: A bullet \((m_1)\) is fired into a large block of wood \((m_2)\) suspended from some light wires. The bullet embeds in the block, and the entire system swings through a height \(h\). How can we determine the speed of the projectile from a measurement of \(h\)?

**Before collision**

**Immediately after collision**

To find the speed of bullet \(v_{1a}\) we need to find \(v_{2a}\)

- From conservation of energy (only for system after collision from B to C)

\[ m_1 \vec{v}_{1a} + 0 = (m_1 + m_2) \vec{v}_{2a} \]

\[ \Rightarrow \vec{v}_{1a} = \frac{(m_1 + m_2) \vec{v}_{2a}}{m_1} \]

We have inelastic collision

\[ \sum \vec{p}_i = \sum \vec{p}_f \]

\[ m_1 \vec{v}_{1a} + 0 = (m_1 + m_2) \vec{v}_{2a} \]

\[ \Rightarrow \vec{v}_{1a} = \frac{(m_1 + m_2) \vec{v}_{2a}}{m_1} \]

9.10: Elastic Collision in One dimensions: Example

Two metal spheres, suspended by vertical cords, initially just touch. Sphere 1, with mass \(m_1 = 30 \text{ g}\), is pulled to the left to height \(h_1 = 8 \text{ cm}\), and then released from rest. After swinging down, it undergoes an elastic collision with sphere 2, whose mass \(m_2 = 75 \text{ g}\). What is the velocity \(v_1\) of sphere 1 just after the collision?

Sphere 1 speed before collision

\[ E_f = E_i \quad \Rightarrow \quad \frac{1}{2} m_1 v_{1i}^2 = m_1 gh_1 \]

\[ \Rightarrow \quad v_{1i} = \sqrt{2gh_1} = 1.252 \text{ m/s} \]

\[ m_1 \vec{v}_{1o} + m_2 \vec{v}_{2o} = m_1 \vec{v}_{1f} + m_2 \vec{v}_{2f} \]

\[ 0.047 + 0 = 0.03 \vec{v}_{1f} + 0.075 \vec{v}_{2f} \] (1)

\[ \vec{v}_{1f} \cdot \vec{v}_{2f} = - (\vec{v}_{1f} \cdot \vec{v}_{2f}) \]

\[ 1.252 - 0 = -\vec{v}_{1f} + \vec{v}_{2f} \] (2)

Solve the two equations

\[ \begin{align*}
  v_{1f} &= -0.537 \text{ m/s} \\
  v_{2f} &= 0.715 \text{ m/s}
\end{align*} \]
9.10: Example: A block \((m_1=1.6 \text{ kg})\) initially moving to the right at 4 \text{ m/s} collides with a spring attached to a second block \((m_2=2.1 \text{ kg})\) moving to the left with at 2.5 \text{ m/s}. The spring constant is 600 \text{ N/m}. Find the velocities of the two blocks after the collision.

Before collision:
- \(v_{i1} = (4.0) \text{ m/s}\)
- \(v_{i2} = (-2.5) \text{ m/s}\)

After collision:
- \(v_{f1}\)
- \(v_{f2}\)

Befor collision
Momentum is conserved
\[ m_1v_{i1} + m_2v_{i2} = m_1v_{f1} + m_2v_{f2} \]

For elastic collision, we have
\[ 1.15 = (1.6) v_{f1} + (2.1) v_{f2} \] (1)

From eqn’s (1) and (2)
\[ v_{f1} = -3.38 \text{ m/s} \]
\[ v_{f2} = 3.12 \text{ m/s} \]

9.11: Collisions In Two Dimensions

- Conservation of momentum still stands:
  \[ \sum p_i = \sum p_f \]
  \[ m_1v_{i1x} + m_2v_{i2x} = m_1v_{f1x} + m_2v_{f2x} \]
  \[ m_1v_{i1y} + m_2v_{i2y} = m_1v_{f1y} + m_2v_{f2y} \]

- Analyze each component separately:
  \[ \sum p_{ix} = \sum p_{fx} \Rightarrow m_1v_{f1x} + m_2v_{f2x} = m_1v_{i1x} + m_2v_{i2x} \]
  \[ \sum p_{iy} = \sum p_{fy} \Rightarrow m_1v_{f1y} + m_2v_{f2y} = m_1v_{i1y} + m_2v_{i2y} \]

- Use conservation of kinetic energy if collision is elastic:
  \[ K_f = K_i \]
  \[ \frac{1}{2} (m_1v_{f1x}^2 + m_2v_{f2x}^2) = \frac{1}{2} (m_1v_{i1x}^2 + m_2v_{i2x}^2) \]

9.11: Example: If before collision, \(m_2\) was at rest, find magnitude and direction of speed for \(m_2\) after collision \((m_1=1 \text{ kg}, m_2=2 \text{ kg})\)

Before collision:
- \(v_{i1} = 10 \text{ m/s}, \quad v_{i2} = 5 \text{ m/s}\)
- \(\theta = 30^\circ\)

2-variables \(\rightarrow 2\-eqn’s\)

Momentum is conserved in the two dimensions \(x\) and \(y\)

X-component
\[ \sum p_{ix} = \sum p_{fx} \Rightarrow m_1v_{f1x} + m_2v_{f2x} = m_1v_{i1x} + m_2v_{i2x} \]
\[ m_1v_{f1x} \cos 30 + m_2v_{f2x} \cos \phi = m_1v_{i1x} + 0 \]
\[ 4.33 + 2v_2 \cos \phi = 10 \Rightarrow v_2 \cos \phi = v_{f2x} = 2.83 \text{ m/s} \]

Y-component
\[ \sum p_{iy} = \sum p_{fy} \Rightarrow m_1v_{f1y} + m_2v_{f2y} = m_1v_{i1y} + m_2v_{i2y} \]
\[ m_1v_{f1y} \sin 30 - m_2v_{f2y} \sin \phi = 0 \]
\[ -v_{f2y} \sin \phi = v_{f2y} = -1.25 \text{ m/s} \]

Hence,
\[ v_{f2y} = \sqrt{v_{f2x}^2 + v_{f2y}^2} = 3.1 \text{ m/s}, \quad and \quad \phi = \tan^{-1} \frac{-1.25}{2.83} = -23.8^\circ \]
Collision for system of particles

Example: Rocket fired vertically, explode at altitude of 1000 m into three fragments, as shown. The 3 fragments are of equal masses. Find the velocity of the third part after collision.

\[ \vec{p}_{\text{com},i} = \vec{p}_{\text{com},f} \Rightarrow M\vec{v}_{\text{com},i} = M\vec{v}_{\text{com},f} \Rightarrow \vec{v}_{\text{com},i} = \vec{v}_{\text{com},f} \]

For isolated system of particles → Momentum is conserved →

\[ \sum \vec{p}_i = \sum \vec{p}_f = \vec{p}_{\text{com}} \]

\[ Mv_{i,x} = Mv_{f,x} \Rightarrow 0 = M\left( \frac{m_1v_{1,x}}{M} + \frac{m_2v_{2,x}}{M} + \frac{m_3v_{3,x}}{M} \right) \Rightarrow 0 = m_1v_{1,x} + m_2v_{2,x} + m_3v_{3,x} = 0 + \left( \frac{M}{3} \right)(240) + \left( \frac{M}{3} \right) \]

\[ \Rightarrow v_{3,x} = -240 \text{ m/s} \]