6.2: Friction

Frictional forces are common and important in our daily lives:
- due to friction we can walk on earth
- car under danger can stop due to friction when breaking
- you can't move heavy crate when pushing it due to friction

Frictional force is due to the interaction of two adjacent surfaces at the atomic or molecular level → force is needed to break the welding between surfaces → frictional force due to action reaction

Friction is always parallel to the surface of interaction and opposite the direction of motion.
6.3: Properties of Friction

- **Property 1.** If the body does not move, then the static frictional force \( f_s \) and the component of \( F \) that is parallel to the surface balance each other. They are equal in magnitude, and is \( f_s \) directed opposite that component of \( F \).

- **Property 2.** The magnitude of \( f_s \) has a maximum value \( f_{s,\text{max}} \) that is given by

\[
f_{s,\text{max}} = \mu_s F_N\]

- **Property 3.** If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value \( f_k \) given by

\[
f_k = \mu_k F_N\]

- Friction is proportional to the normal force.
- Friction is independent of area.
- \( \mu_s \) is generally larger than \( \mu_k \).
- The coefficients do not depend on weight or speed and generally range between 0.003 and 1.

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6.3: Properties of Friction: Example

A car driving with a speed \( v_0 \) makes an emergency breaking and made it to stop in a distance of 290 m as shown. If \( \mu \) between tires and street is 0.6, find the initial speed \( v_0 \).

\[
\sum F_i = ma_x
\]
\[
\Rightarrow -f_k = ma
\]
\[
\Rightarrow -\mu_k F_N = -\mu_k mg = ma
\]
\[
\Rightarrow a = -\mu_k g = -(0.6)(9.8) = -5.88 \text{ m/s}^2
\]
\[
v^2 = v_0^2 + 2a\Delta x
\]
\[
0 = v_0^2 + 2(-5.88)(290)
\]
\[
v_0 = \sqrt{-2(5.88)(290)} = 58 \text{ m/s}
\]

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6.3: Properties of Friction: Example - box on rough surface

- 12 kg Box on surface with friction. If \( \mu_s=0.5 \) and \( \mu_k=0.3 \) find
  a) how large should \( F \) be to get box start moving?
  b) once it’s moving, find the acceleration

**Solution:** analyze \( F \) to its components

- a) Start moving \( \Rightarrow \) not moving \((a=0)\), but \( f_k \) is at its maximum

\[
\sum F_i = ma_x
\]

\[
F \cos \theta - f_k = 0 \quad (a = 0)
\]

\[
F \cos 25^\circ - \mu_k F_N = 0 \quad ........(1)
\]

We can find \( n \) from y-direction

\[
\sum F_i = ma_y
\]
\[
n + F \sin 25^\circ - mg = 0 \quad (a_y = 0)
\]
\[
\Rightarrow n = mg - F \sin 25^\circ \quad ........(2)
\]

\[
\Rightarrow F \cos 25^\circ - \mu_k mg - \mu_k F \sin 25^\circ = 0
\]

\[
F(\cos 25^\circ + \mu_k \sin 25^\circ) = \mu_k mg
\]

\[
\Rightarrow F = \frac{\mu_k mg}{\cos 25^\circ + \mu_k \sin 25^\circ}
\]
\[
F = \frac{0.5(12)(9.8)}{\cos 25^\circ + (0.5)\sin 25^\circ} = 53 \text{ N}
\]

- b) When moving \( \Rightarrow \) we have acceleration \( a \) and kinetic friction \( f_k \) as shown

\[
\sum F_i = ma_x
\]

\[
\Rightarrow F \cos \theta - f_k = ma
\]

\[
F \cos 25^\circ - \mu_k n = ma
\]
\[
\Rightarrow a = \frac{F \cos 25^\circ - \mu_k n}{m}
\]
\[
n = mg - F \sin 25^\circ \quad \text{(from y-direction)}
\]
\[
\Rightarrow a = \frac{F \cos 25^\circ - \mu_k (mg - F \sin 25^\circ)}{m}
\]
\[
a = \frac{53 \cos 25^\circ - (0.3)(12)(9.8) - 53 \sin 25^\circ}{12} = 1.6 \text{ m/s}^2
6.3: Properties of Friction: Example - box on rough incline surface

- 12 kg Box on incline surface with friction.
  If \( \mu_s = 0.5 \) and \( \mu_k = 0.3 \) find
  a) At what angle the object start moving
  b) once it's moving, find the acceleration

Solution: a) Object not moving \( \Rightarrow f_s \)

\[
\sum F_i = ma_x
\]

\[
mg \sin \theta - f_s = 0 \quad (a = 0)
\]

\[
mg \sin \theta - \mu_s mg \cos \theta = 0
\]

\[
\Rightarrow \theta = \arctan \left( \frac{\mu_s}{1} \right)
\]

\[
\Rightarrow \text{angle} = 14.05^\circ
\]

b) once it's moving, find the acceleration

Static friction becomes kinetic \( \Rightarrow f_k < f_s \) object accelerates at same angle in part (a)

\[
\sum F_i = ma_x
\]

\[
mg \sin \theta - \mu_k mg \cos \theta = ma
\]

\[
\Rightarrow a = g \sin \theta - \mu_k g \cos \theta
\]

\[
\Rightarrow a = (9.8)(\sin 21.5^\circ) - (0.3)(9.8)(\cos 21.5^\circ)
\]

\[
a = 1.7 \text{ m/s}^2
\]

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6.5: Uniform Circular Motion

- Since object is rotating in circular path \( \Rightarrow \) Centripetal acceleration \( a_c \) directed towards the center of the circle of motion (perpendicular to the velocity \( v \)).

\[
a = \frac{v^2}{R} \quad \text{(centripetal acceleration)}
\]

- Since we have acceleration towards the center \( \Rightarrow \) we have \( F_{net} \) produce \( a_c \)

\[ F_{net} (\Sigma F_i) \text{ is directed towards the center} \]

The string is applying the force to keep the ball rotating in circular path.
When the String Breaks

When the string breaks (meaning no radial force is applied anymore), the ball continues to move at constant velocity, in a straight line along the tangent to circle at the break point. $v$

6.5: Uniform Circular Motion: Newton's law

Newton’s second law in the radial direction (إنجاز نصف المداري) is

$$\sum F_r = ma_r = ma_c = m\frac{v^2}{R}$$

If $F_r$ is towards the center ➔ $+ve$ force
If $F_r$ outwards the center ➔ $-ve$ force

The string is applying the force to keep the ball rotating in a circular path.

6.5: Uniform Circular Motion: Example

A 0.15 kg Object on a string moves in horizontal circle with a speed of 2 rev./s. If the radius of the circle is 0.6m, find the tension in the string

$$\sum F_r = ma_c = m\frac{v^2}{R}$$

$$\Rightarrow T = m\frac{v^2}{R}$$

To find $T$ we need $v$

$$v = 2\text{rev./s} = 2(2\pi r) = 4\pi(0.6) = 7.54 m/s$$

$$\Rightarrow T = m\frac{v^2}{r} = 0.15\frac{(7.54)^2}{0.6} = 14.2 N$$

6.5: Uniform Circular Motion: Example – moving on a vertical circle

At the bottom of the loop

T towards the center (up)
mg out of center (down)

$$\sum F_r = ma_c = m\frac{v^2}{R}$$

$$T - mg = m\frac{v^2}{R}$$

$$\Rightarrow T = mg + m\frac{v^2}{R}$$
Ex: car rounding a curve without skidding?

What is the maximum speed of the car

\[ m = 1500 \text{ kg} \]
\[ r_{\text{curve}} = 120 \text{ m} \]
\[ \mu_s = 0.5 \]

Car rounding curve \( a_c \rightarrow \) force to the center

\( \Rightarrow \) It is static friction force \( f_s \)

Ex: car rounding a curve without skidding?

\[ F_{\text{net}} = ma_c = m \frac{v^2}{R} \]

Maximum speed is when the car is about to skid (at \( f_{s,\text{max}} \))

\[ f_s = m \frac{v^2}{R} \Rightarrow \mu_s n = \mu_s mg = m \frac{v^2}{R} \]
\[ \Rightarrow v^2 = \mu_s gR \Rightarrow v_{\text{max}} = \sqrt{\mu_s gR} \]
\[ \Rightarrow v_{\text{max}} = \sqrt{(0.5)(120)(9.8)} = 24.2 \text{ m/s} \]
Banked curve

- We can reduce the possibility of skidding by designing a banked curve.
- Assume a car driving at speed \( v \) on banked curve with no friction. Find the optimum banking angle without skidding out of the track.

\[
\sum F_i = \sum F_r = ma = m \frac{v^2}{R}
\]

\[
n \sin \theta = m \frac{v^2}{R}
\]

\[
y \text{- direction, } n \cos \theta = mg \]

\[
\Rightarrow n = \frac{mg}{\cos \theta} \Rightarrow \frac{mg}{\cos \theta} \sin \theta = m \frac{v^2}{R} \Rightarrow \tan \theta = \frac{v^2}{gR}
\]

Review

- Static frictional force equals applied force (object is not moving).
- Maximum static frictional force (object about to move).
- Kinetic frictional force (object is moving).
- Newton's law in uniform circular motion

\[
\sum F_r = ma = m \frac{v^2}{R}
\]

Banked curve

If \( v=33 \text{ m/s} \) and curve radius \( r=125 \text{ m} \), what is the banking angle so that the car will not slide.

\[
\tan \theta = \frac{v^2}{gR} \Rightarrow \theta = \tan^{-1} \left( \frac{33^2}{(9.8)(125)} \right) = 42^\circ
\]