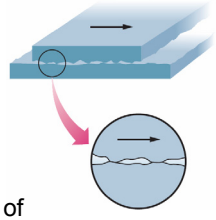
Ch06: Force and Motion - II

- Static Friction
- Kinetic Friction
- Friction properties
- Uniform circular motion
- Centripetal acceleration
- Newton's law in Uniform circular motion

6.2: Friction

- Frictional forces are common and important in our daily lives:
  - due to friction we can walk on earth
  - car under danger can stop due to friction when breaking
  - you can't move heavy crate when pushing it due to friction

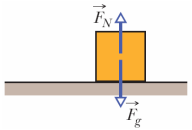
- Frictional force is due to the interaction of two adjacent surfaces at the atomic or molecular level→ force is needed to break the welding between surfaces → frictional force due to action reaction



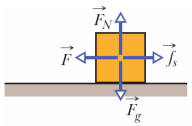
- Friction is always parallel to the surface of interaction and opposite the direction of motion.

6.2: Friction

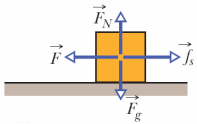
- Frictional Force: motion of a crate with applied forces



With no applied force *F* , There is no attempt at sliding →**no friction**



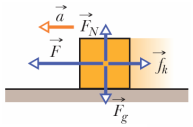
Force *F attempts* sliding but is balanced by the frictional force → No motion. → **Static frictional force f_s**



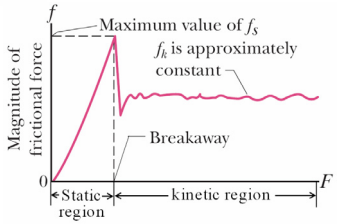
Force *F is now* stronger but is still balanced by the frictional force. → No motion. → **Larger static frictional force f_s**

6.2: Friction

- Frictional Force: motion of a crate with applied forces



Finally, the applied force F has overcome the static frictional force → Block slides and accelerates → **Kinematics friction f_k**



6.3: Properties of Friction

- **Property 1.** If the body does not move, then the static frictional force f_s and the component of **F** that is parallel to the surface balance each other. They are equal in magnitude, and is f_s directed opposite that component of **F**.

- **Property 2.** The magnitude of f_s has a maximum value $f_{s,max}$ *that is given by*

$$f_{s,\max} = \mu_s F_N$$

μ_s is coefficient of static friction

- **Property 3.** If the body begins to slide along the surface, the magnitude of the frictional force rapidly decreases to a value f_k *given by*

$$f_k = \mu_k F_N$$

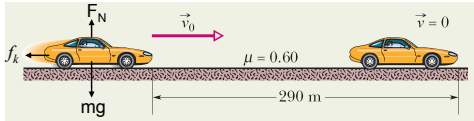
μ_k is coefficient of kinetic friction

- Friction is proportional to the normal force.
- Friction is independent of area.
- μ_s is generally larger than μ_k .

- The coefficients do not depend on weight or speed and generally range between 0.003 and 1.

6.3: Properties of Friction: Example

A car driving with a speed v_0. an emergency breaking made It to stop in a distance of 290 m as shown. If μ_k between tiers and street is 0.6, find it's initial speed v_0



$$\sum F_x = ma_x$$
$$\Rightarrow -f_k = ma$$
$$\Rightarrow -\mu_k F_N = -\mu_k mg = ma$$
$$\Rightarrow a = -\mu_k g =$$
$$= -(0.6)(9.8) = -5.88 m/s^2$$

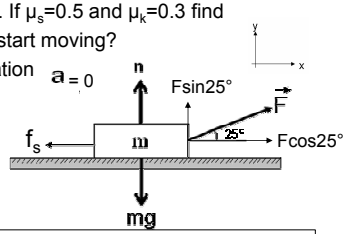
$$v^2 = v_0^2 + 2a\Delta x$$
$$0 = v_0^2 + 2a\Delta x$$
$$\Rightarrow v_0 = \sqrt{-2a\Delta x}$$
$$\Rightarrow v_0 = \sqrt{-2(-5.88)(290)} = 58 m/s$$

6.3: Properties of Friction: Example - box on rough surface

- 12 kg Box on surface with friction. If μ_s=0.5 and μ_k=0.3 find
  a) how large should F be to get box start moving?
  b) once it's moving, find the acceleration

Solution: analyze F to its components

a) Start moving ➔ not moving (a=0), but, f_s is at its maximum



$$\sum F_x = ma_x$$
$$F\cos\theta - f_s = 0 \quad (a = 0)$$
$$F\cos 25° - \mu_s n = 0 \quad .....(1)$$

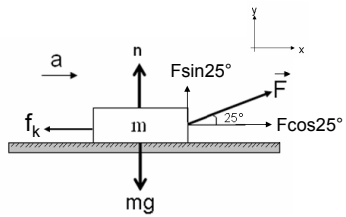
We can find n from y-direction ⟶

$$\sum F_y = ma_y$$
$$n + F\sin 25° - mg = 0 \quad (a_y = 0)$$
$$\Rightarrow n = mg - F\sin 25° .....(2)$$

$$\Rightarrow F\cos 25° - \mu_s mg - \mu_s F\sin 25° = 0$$
$$F(\cos 25° + \mu_s \sin 25°) = \mu_s mg$$
$$\Rightarrow F = \frac{\mu_s mg}{\cos 25° + \mu_s \sin 25°}$$
$$F = \frac{0.5(12)(9.8)}{\cos 25 + (0.5)\sin 25} = 53N$$

6.3: Properties of Friction: Example - box on rough surface

b) When moving ➔ we have acceleration a and kinetic friction f_k as shown



$$\sum F_x = ma_x$$
$$F\cos\theta - f_k = ma$$
$$F\cos 25° - \mu_k n = ma$$
$$\Rightarrow a = \frac{F\cos 25° - \mu_k n}{m},$$
$$n = mg - F\sin 25° \quad \text{(from y - direction)}$$

$$\Rightarrow a = \frac{F\cos 25 - \mu_k (mg - F\sin 25)}{m}$$
$$a = \frac{53\cos 25 - (0.3)((12)(9.8) - 53\sin 25)}{12} = 1.6 m/s^2$$

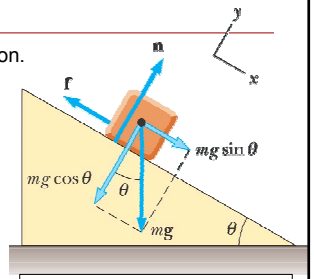
6.3: Properties of Friction: Example - box on rough incline surface

- 12 kg Box on incline surface with friction.

If μ_s=0.5 and μ_k=0.3 find

a) At what angle the object start moving

b) once it's moving, find the acceleration



Solution: a) Object not movng → f_s

$$\sum F_x = ma_x$$
$$mg\sin\theta - f_s = 0 \quad (a = 0)$$
$$\Rightarrow mg\sin\theta - \mu_s n = 0 .....(1)$$

$$\sum F_y = ma_y$$
$$n - mg\cos\theta = 0 \quad (a_y = 0)$$
$$\Rightarrow n = mg\cos\theta .....(2) \quad sub\ for\ F_N\ in\ (1)$$

$$\Rightarrow mg\sin\theta - \mu_s mg\cos\theta = 0$$
$$\Rightarrow \sin\theta = \mu_s \cos\theta$$
$$\Rightarrow \tan\theta = \mu_s$$
$$\Rightarrow \theta = \tan^{-1}\mu_s$$
$$= \tan^{-1} 0.5 = 26.6°$$

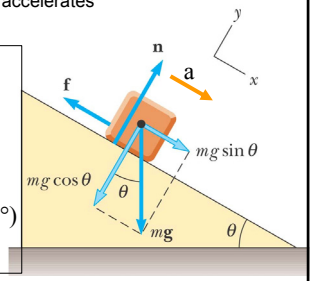
6.3: Properties of Friction: Example - box on rough incline surface

- 12 kg Box on incline surface with friction. If μ_s=0.5 and μ_k=0.3 find

b) once it's moving, find the acceleration

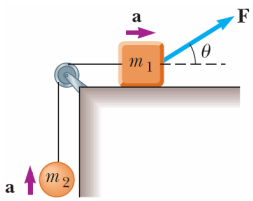
Static friction becomes kinetic → fk<fs → object accelerates at same angle in part (a)

$$\sum F_x = ma_x$$
$$mg\sin\theta - f_k = ma$$
$$\Rightarrow mg\sin\theta - \mu_k mg\cos\theta = ma$$
$$\Rightarrow a = g\sin\theta - \mu_k g\cos\theta$$
$$\Rightarrow a = (9.8)(\sin 26.6°) - (0.3)(9.8)(\cos 26.6°)$$
$$a = 1.7 m/s^2$$



6.3: Properties of Friction: Example – two object connected by a cord

Two objects of masses m_1 and m_2 are connected by a lightweight cord over a lightweight, frictionless pulley as shown. force of magnitude *F* at an angle θ with the horizontal is applied to the block m_1 which is on a rough surface.The coefficient of kinetic friction between the block and surface is μ_k. Determine the magnitude of the acceleration of the two objects.



$$\sum F_{system} = (\sum m)a$$
$$F\cos\theta - \mu_k n - m_2 g = (m_1 + m_2)a, \quad but\ \ n = m_1 g - F\sin\theta$$

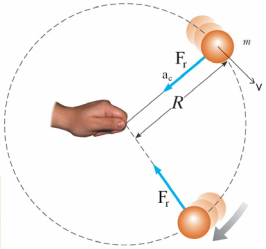
$$a = \frac{F(\cos\theta + \mu_k \sin\theta) - (m_2 + \mu_k m_1)g}{m_1 + m_2}$$

6.5: Uniform Circular Motion

- Since object is rotating in circular path →Centripetal acceleration a_c directed towards the center of the circle of motion (perpendicular to the velocity v).

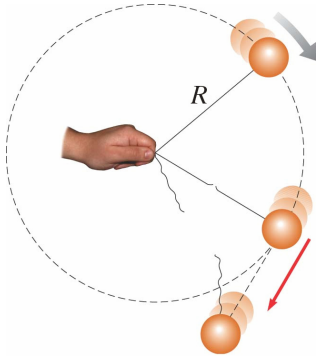
$$a = \frac{v^2}{R} \qquad \text{(centripetal acceleration)}$$

- Since we have acceleration towards the center ➔ we have F_{net} produce a_c
- F_{net} (ΣF_r) is directed towards the center



The string is applying the force to keep the ball rotating in circular path

When the String Breaks



When the string breaks (meaning no radial force is applied anymore), the ball continues to move at constant velocity, in a straight line along the tangent to circle at the break point. v

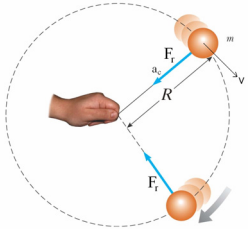
6.5: Uniform Circular Motion: Newton's law

➔ Newton's second law in the radial direction (اتجاه نصف قطري) is

$$\sum \vec{F}_r = m\vec{a}_r = ma_c = m\frac{v^2}{R}$$

If F_r is towards the center ➔ +ve force

If F_r outwards the center ➔ -ve force



The string is applying the force to keep the ball rotating in circular path

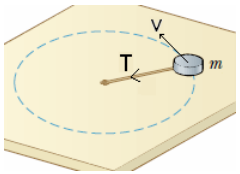
6.5: Uniform Circular Motion: Example

A 0.15 kg Object on a string moves in horizontal circle with a speed of 2 rev./s. If the radius of the circle is 0.6m, find the tension in the string

$$\sum F_r = ma_c = m\frac{v^2}{R}$$

$$\Rightarrow T = m\frac{v^2}{R}$$

To find T we need v



$$v = 2rev./s = 2(2\pi r) = 4\pi(0.6) = 7.54m/s$$

$$\Rightarrow T = m\frac{v^2}{r} = 0.15\frac{(7.54)^2}{0.6} = 14.2N$$

6.5: Uniform Circular Motion: Example – moving on a vertical circle

At the bottom of the loop

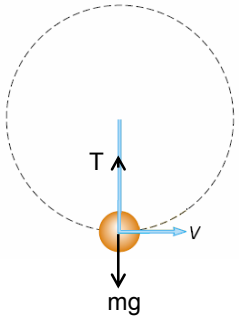
T towards the center (up)

mg out of center (down)

$$\sum F_r = ma_c = m\frac{v^2}{R}$$

$$T - mg = m\frac{v^2}{R}$$

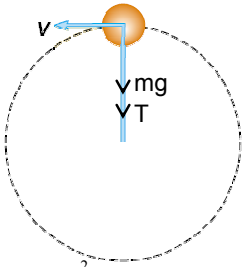
$$\Rightarrow T = mg + m\frac{v^2}{R}$$



6.5: Uniform Circular Motion: Example – moving on a vertical circle

At the top of the loop

Both T towards the center (down)



$$\sum F_r = ma_c = m\frac{v^2}{R}$$

$$T + mg = m\frac{v^2}{R}$$

$$\Rightarrow T = m\frac{v^2}{R} - mg$$

$$m\frac{v^2}{R} > mg \Rightarrow \frac{v^2}{R} > g$$

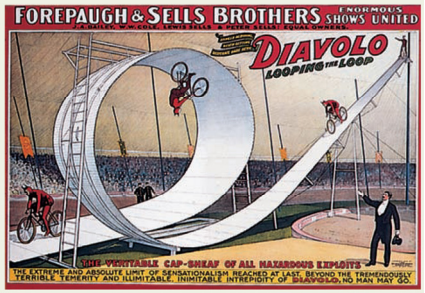
to maintain circular motion
T must be +ve

$\Rightarrow v = \sqrt{gR}$ is minimum speed to maintain circular motion

6.5: Uniform Circular Motion: Example – moving on a vertical circle

Find minimum speed a bicycle can go without falling. R= 2.7 m.

At the top, there is a normal force n from track on the bicycle, directed downward, in addition to the weight



$$\sum F_r = ma_c$$

$$n + mg = m\frac{v^2}{R}$$

At minimum speed → bicycle s about to fall → n = 0

$$\Rightarrow mg = m\frac{v^2}{R} \Rightarrow v = \sqrt{gR} = 5.1 m/s$$

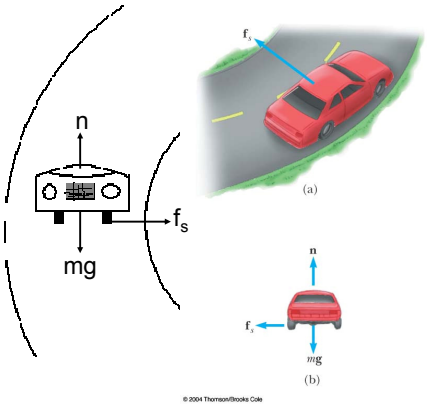
Ex: car rounding a curve without skidding?

What is the maximum speed of the car

m = 1500 kg
r_{curve} = 120 m
μ_s = 0.5

Car rounding curve ➔ a_c ➔ force to the center

➔It is static friction force f_s



Ex: car rounding a curve without skidding?

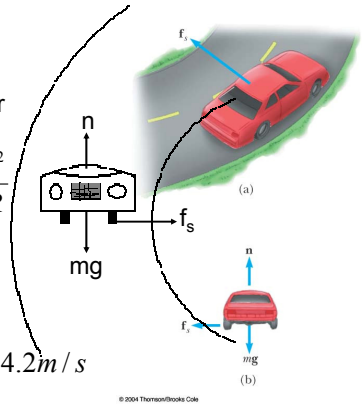
$$F_{net} = ma_c = m\frac{v^2}{R}$$

Maximum speed is when the car is about to skid (at $f_{s,max}$)

$$f_s = m\frac{v^2}{R} \Rightarrow \mu_s n = \mu_s mg = m\frac{v^2}{R}$$

$$\Rightarrow v^2 = \mu_s gR \Rightarrow v_{max} = \sqrt{\mu_s gR}$$

$$\Rightarrow v_{max} = \sqrt{(0.5)(120)(9.8)} = 24.2 m/s$$

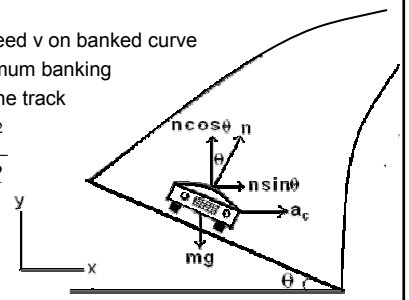


Banked curve

- We can reduce the possibility of skidding by designing a banked curves
- Assume a car driving at speed v on banked curve with **no friction**. Find the optimum banking angle without skidding out of the track

$$\sum F_x = \sum F_r = ma_c = m\frac{v^2}{R}$$

$$n\sin\theta = m\frac{v^2}{R}$$



y - direction, $n\cos\theta = mg$

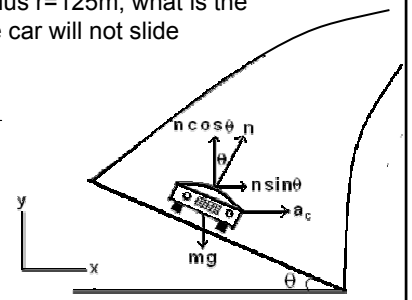
$$\Rightarrow n = \frac{mg}{\cos\theta} \Rightarrow \frac{mg}{\cos\theta}\sin\theta = m\frac{v^2}{R} \Rightarrow \tan\theta = \frac{v^2}{gr}$$

Banked curve

If v=33m/s and curve radius r=125m, what is the banking angle so that the car will not slide

$$\tan\theta = \frac{v^2}{gR} \Rightarrow \theta = \tan^{-1}\frac{v^2}{gR}$$

$$\theta = \tan^{-1}\frac{(33)^2}{(9.8)(125)} = 42°$$



Review

- Static frictional force equals applied force (object is not moving)
- Maximum static frictional force (object about to move) $f_{s,\max} = \mu_s F_N$
- Kinetic frictional force (object is moving) $f_k = \mu_k F_N$
- Newton's law in uniform circular motion $\sum F_r = ma_c = m\frac{v^2}{R}$