Ch. 26: Current and Resistance

- Putting electrons in motion
- Electron movement through conductors
- Resistivity and Resistance – Ohm’s Law
- Electrical Power

Ch. 26-1: What is physics

- When charges are in motion (not in electrostatic equilibrium as in 5 chapters before) \( \Rightarrow \) we will have what is called current
- current is used in almost all scientific applications that is connected to technology
- current can be established in materials and cannot be in other materials

Ch. 26-2: Electrical Current: definition

- If there is to be an electric current through a given surface, there must be a net flow of charge through that surface; there will be no current for electrons in motion without net flow through a given surface.
- Hence to have net flow of charges and so have current we need to connect the material with a battery

(a) Loop of cooper wire \( \Rightarrow \) no E-field inside wire \( \Rightarrow \) no net motion for charges \( \Rightarrow \) no current

(b) Loop of cooper wire with battery \( \Rightarrow \) E-field inside wire because of battery potential \( \Rightarrow \) net flow of charges \( \Rightarrow \) current i

\[ q_{I_{av}} = \frac{\Delta q}{\Delta t} \]

(sI unit is Ampere \( A = C/s \))

\[ \Delta q = I_{avg} \Delta t \]

(b) For very small period of time \( \Delta t \) \( \Rightarrow \) instantaneous current

\[ i = \frac{dq}{dt} \]

\( \Rightarrow \) Charge passed in time interval \( t \) can be found by

\[ q = \int dq = \int_{t_i}^{t_f} i dt, \]

- current will be the same in a wire even if it has different cross sectional area because charges pass section aa’ will pass through bb’ and cc’
Ch. 26: Electrical Current: direction

- Current direction: It is defined to be the same direction as the direction of positive charge flow.
- In a metal conductor, the current is due to the motion of the electrons (negatively charged). The direction of the current in a metal conductor is thus opposite the direction of the electrons.
- It is common to refer to a moving charge as a mobile charge carrier.

Ch. 26: Electrical Current

Ex: The amount of charge that passes through the filament of a certain light bulb in 2 s is 1.67 C. Find:
(a) the current in the light bulb, (b) the number of electrons that pass through the filament in 1 second.

(a) \[ I = \frac{\Delta q}{\Delta t} = \frac{1.67}{2} = 0.835 \text{ A} \]

(b) In 1 s, \[ \Delta q = I \Delta t = (0.835)(1) = 0.835 \text{ C} \]

⇒ \[ N = \frac{\Delta q}{q_{\text{electron}}} = \frac{0.835}{1.6 \times 10^{-19}} = 5.22 \times 10^{18} \text{ electrons} \]

Ch. 26: Electrical Current: Macroscopic model – current and drift speed

Consider a conductor of cross-sectional area \( A \) and a segment length \( \Delta x \).

- The total charge pass with a speed \( v_d \) in a given volume \( \Delta V \) covered in time interval \( \Delta t \) is
  \( q = Ne \) (\( N \) is the number of carriers, and \( e \) is the electron charge magnitude)
- Volume of an element of length \( \Delta x \) is: \( \Delta V = A \Delta x \).
- Let \( n \) be the charge density (number of carriers per unit of volume).
  \( n = N/\Delta V \).
- The total number \( N \) of carriers in \( \Delta V \) is: \( N = n \Delta V = n A \Delta x \).
- The charge in this volume is: \( q = Ne = (n A \Delta x)e \). (\( e \) is charge of each carrier)
- Distance traveled at drift speed \( v_d \) by carrier in time \( \Delta t \) is: \( \Delta x = v_d \Delta t \).
- Hence: \( q = (n A v_d \Delta t)e \).
- The average current through the conductor is \( I_{av} = \frac{q}{\Delta t} = ne v_d A \).
Ch. 26: Electrical Current: drift velocity

$\nu_d$ is called drift speed: drift speed is the average speed that carriers move in the electric field. The path of carriers is usually zig zag (appears as a random path) due to its collision with atoms and molecules in the conductor. However, carriers then tend to drift preferentially in one direction with drift velocity $\nu_d$

$\nu_d$: Drift velocity

Resistance of charge flow within material depends on how many collisions resist the carrier motion

---

Ch. 26: Resistance: Ohm’s Law

For a conductor of cross-sectional area $A$ carrying a current $I$, the current density $J$ is defined as current per unit area

$$ J = \frac{I}{A} = ne\nu_d $$

( SI unit is A/m²)

$J$ direction: same as current

$J$ and $E$ are established in a conductor whenever a $\Delta V$ exist across the conductor. In some materials (Ohmic materials – Most metals), $J$ is proportional to $E$ according to Ohm's law:

$$ J = \sigma E $$

$\sigma$ is called the conductivity

The inverse of $\sigma$ is called resistivity $\rho = 1/\sigma$

* Note that $\sigma$ or $\rho$ is intrinsic property of material depends on material type and temperature

---

Ex: A copper wire of cross-sectional area $3.31 \times 10^{-6}$ m² carries a current of 10 A. Assuming that each copper atom contributes one free electron to the metal, find the drift speed of the electron in this wire.

The density of copper is 8.95 g/cm³, the molecular weight is 63.5 g/mol

$$ I_w = ne\nu_d A \Rightarrow \nu_d = \frac{I}{neA} $$

$$ n = \frac{N}{V} = \text{no. of free electrons in one mole} $$

$$ \Rightarrow n = \frac{N}{V_{\text{atone}}} = \frac{N}{V_{\text{one mole}}} $$

Because each atom gives 1 free electron

$$ \rho = \frac{m}{V} \Rightarrow V_{\text{one mole}} = \frac{m}{\rho} = \frac{63.5 g}{8.95 g/cm^3} = 7.09 cm^3 = 7.09 \times 10^{-6} m^3 $$

$$ \Rightarrow n = \frac{N}{neA} = \frac{6.02 \times 10^{23}}{7.09 \times 10^{-6}} = 8.49 \times 10^{26} \text{ electron / m}^3 $$

Hence, $\nu_d = \frac{I}{neA} = \frac{10}{(8.49 \times 10^{26})(1.6 \times 10^{-19})(3.31 \times 10^{-6})} = 2.22 \times 10^{-4} m/s$
Ch. 26: Resistance: resistivity

 Resistivity is the inverse of conductivity.
 Both are intrinsic properties of the material.
 Not like resistivity, resistance is a function of the shape and size of the device.

\[ \rho = \frac{1}{\sigma} \rightarrow R = \frac{\rho}{A} \frac{l}{A} = \frac{l}{\sigma A} \]

Resistivity for some materials:
see table 27.2 for other materials

<table>
<thead>
<tr>
<th>Material</th>
<th>( \rho (\Omega \cdot m) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper</td>
<td>1.7 \times 10^{-8}</td>
</tr>
<tr>
<td>Gold</td>
<td>2.4 \times 10^{-8}</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.82 \times 10^{-8}</td>
</tr>
<tr>
<td>Silicon</td>
<td>640</td>
</tr>
<tr>
<td>Rubber</td>
<td>~10^{13}</td>
</tr>
</tbody>
</table>

Ch. 26: Resistance: Ohm’s Law

Ohm’s Law \( V = IR \) is a linear function between \( V \) and \( I \)

- Materials that follow Ohm’s Law (linear function) are called Ohmic materials like metals.
- Materials that do not obey Ohm’s law are called non-ohmic materials like Semiconductor diode

Ex: Calculate the resistance for an aluminum cylinder of length of 10 cm and a cross-sectional area of \( 2 \times 10^{-4} \) m². Repeat the calculation for same cylinder made of glass.

\[ R = \rho \frac{l}{A} \rightarrow R_{Al} = (2.82 \times 10^{-8}) \left( \frac{0.1}{2 \times 10^{-4}} \right) = 1.41 \times 10^{-5} \Omega \]
\[ R_{Glass} = \rho_{Glass} \frac{l}{A} = (3 \times 10^{10}) \left( \frac{0.1}{2 \times 10^{-4}} \right) = 1.5 \times 10^{3} \Omega \]

Ch. 26: Resistance: Resistance of a conductor cylinder

For Nichrome wire Calculate a) the resistance per unit length if it has a radius of 0.321 mm. b) If a potential difference of 10 V is maintained across a 1 m length of the wire, what is the current in the wire? \( \rho = 1.5 \times 10^{-6} \Omega \cdot m \)

a) \[ R = \rho \frac{\ell}{A} \rightarrow R = \frac{\rho}{\ell} \cdot \frac{l}{A} \]
but \[ A = \pi r^2 = \pi (0.321 \times 10^{-3} \text{ m})^2 = 3.24 \times 10^{-7} \text{ m}^2 \]
\[ R = \frac{\rho}{\ell} \cdot \frac{1.5 \times 10^{-6} \text{ m} \cdot \text{m}}{3.24 \times 10^{-7} \text{ m}^2} = 4.6 \text{ \Omega/m} \]

b) \[ I = \frac{V}{R} = \frac{10 \text{ V}}{4.6 \text{ \Omega}} = 2.2 \text{ A} \]
Ch. 26: A Model for Electrical Conduction:
The Drude Model

A regular array of atoms surrounded by a "cloud" of free electrons

Random movement under zero field

Random movement modified by a field → drift velocity

A regular array of atoms surrounded by a "cloud" of free electrons

Atoms

Electrons

The electrons drift under the E-force

\[ F = eE = m_ea \rightarrow a = \frac{eE}{m_e} \]

\[ \Rightarrow J = \sigma E = \frac{n e^2 E}{m_e} \tau \]

but \[ J = \sigma E = \frac{n e^2 E}{m_e} \tau \]

hence \[ v_f = v_i + at = v_i + \frac{eE}{m_e} \tau \]

Take the average over all times between collisions (\( \tau \)) with \( v_i = 0 \)

→ The average velocity

\[ \bar{v}_f = \bar{v}_d = \frac{eE}{m_e} \bar{f} = \frac{eE}{m_e} \tau \]

→ The current density \( J \) value becomes

\[ J = n e \bar{v}_d = \frac{n e^2 E}{m_e} \tau \]

\( \tau \) is related to the average distance \( l \) (mean free path) between collisions

\[ \Rightarrow \tau = \frac{l}{v_d} \]

Ch. 26: Resistivity and temperature

- Resistivity \( \rho \) is an intrinsic property of material. It changes with material type and/or with temperature
- For most conductors, at \( T \uparrow \rho \uparrow \) and hence \( R \uparrow \) as shown in figure
  - Resistivity in metals is linear with temperature over a limited range according to
    \[ \rho = \rho_0 \left[ 1 + \alpha(T - T_0) \right] \]
    \( T \) is in °C or K (Kelvin) where we need to find \( \rho \).
    \( \rho_0 \) is resistivity at room temperature \( T_0 \) (usually taken to be 20 °C or 293 K)
- Since \( R \) is proportional to \( \rho \)
  \[ R = R_0 \left[ 1 + \alpha(T - T_0) \right] \]
  \( \alpha \) is temperature coefficient of resistivity \[ \alpha = \frac{1}{\rho_0} \frac{\Delta \rho}{\Delta T} \] (°C)

Ex: A resistance thermometer is made from platinum and has a resistance of 50 Ω at 20 °C. When immersed in a melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium. For indium \( \alpha = 3.92 \times 10^{-3} \) °C⁻¹

\[ R = R_0 \left[ 1 + \alpha(T - T_0) \right] = R_0 + R_0 \alpha(T - T_0) \]

\[ \Rightarrow \Delta T = \frac{\Delta R}{R_0 \alpha} \Rightarrow T - T_0 = \frac{R - R_0}{R_0 \alpha} = 137° \]

\[ \Rightarrow T = 137° + T_0 = 137 + 20 = 157°C \]
Ch. 26: superconductors

Some times $R$ for some metal class materials goes to zero as temperature goes below a critical temperature $T_c$, these materials are known as superconductors. Above $T_c$, these materials behave like normal metal (Ohmic linear relation).

<table>
<thead>
<tr>
<th>Material</th>
<th>$T_c$ (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>HgBa$_2$Ca$_2$Cu$_2$O$_y$</td>
<td>134</td>
</tr>
<tr>
<td>Tl–Ba–Ca–Cu–O</td>
<td>125</td>
</tr>
<tr>
<td>Bi–Sr–Ca–Cu–O</td>
<td>105</td>
</tr>
<tr>
<td>YBa$_2$Cu$_3$O$_y$</td>
<td>92</td>
</tr>
<tr>
<td>Nb$_3$Ge</td>
<td>25.2</td>
</tr>
<tr>
<td>Nb$_3$Sn</td>
<td>18.05</td>
</tr>
<tr>
<td>Nb</td>
<td>9.46</td>
</tr>
<tr>
<td>Pb</td>
<td>7.18</td>
</tr>
<tr>
<td>Hg</td>
<td>4.15</td>
</tr>
<tr>
<td>Sn</td>
<td>3.72</td>
</tr>
<tr>
<td>Al</td>
<td>1.19</td>
</tr>
<tr>
<td>Zn</td>
<td>0.88</td>
</tr>
</tbody>
</table>

$R$ versus $T$ for mercury (Hg) sample ($T_c$=4.15 K)

Ch. 26: Electrical power

The rate at which the system loses potential energy is called average electrical power.

$$P_{ave} = \frac{E}{t} = \frac{U}{t} = \frac{qV}{t} = I_{ave}V$$

For $\Delta t \to 0 \Rightarrow$ instantaneous Electrical power

$$P = \frac{dU}{dt} = \frac{dq}{dt}V = IV$$

(SI for power is unit is $V.A = $Watts(W))

Hence, power Dissipated on a Resistor

$$P = IV = I^2R = \frac{(V)^2}{R}$$

Ch. 26: Electrical power

Ex: In the following circuit, rank the currents in at marked points (a, b, c, d, e, and f)

$I_a = I_b$

$I_c = I_d$

$I_e = I_f$

$I_a = I_c + I_e$

$P_{60W} > P_{30W}$

$I_cV > I_eV$\n
$\Rightarrow I_c > I_e$ \n
$\Rightarrow I_a = I_b > I_c = I_d = I_e = I_f$
Ch. 26: Electrical power

Ex: An electric heater is constructed by applying a potential difference of 120 V to a Nichrome wire that has a total resistance of 8.00 Ω. a) Find the current carried by the wire and the power rating of the heater, b) if the energy cost is 0.7 NIS/KWh, and the heater is used for 90min, what is the cost?

a)

\[
I = \frac{V}{R} = \frac{120 \text{ V}}{8.00 \text{ Ω}} = 15.0 \text{ A}
\]

\[
\mathcal{P} = I^2R = (15.0 \text{ A})^2(8.00 \text{ Ω}) = 1.80 \times 10^3 \text{ W} \quad \mathcal{P} = 1.80 \text{ kW}
\]

b)

Energy transferred

\[
E = Pt = (1.8KW)(90 \text{ min})(1h / 60 \text{ min}) = 2.7 \text{ KWh}
\]

\[
\text{coast} = \text{(energy)(cost)} = (2.7 \text{ KWh})(0.7 \text{ NIS / KWh}) = 1.89 \text{ NIS}
\]

Summary

- Current is the net rate of charge flow.
- Electrons move at the drift velocity.
- Resistance is the ratio of voltage applied to current. The ratio is linear for most metals.
- Resistivity is a material property.
- Energy will be dissipated on a resistor. The rate of dissipation is the power.