

An Exact Envelope Correlation Formula for Two-Antenna Systems Using Input Scattering Parameters and Including Power Losses

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Abstract – The calculation of the envelope correlation for a two antenna element system in terms of the system's scattering parameters is modified to include power losses. This new expression should reduce the complexity in predicting the spatial envelope correlation, and simplify antenna design where a low envelope correlation is required. This represents a major simplification with respect to the conventional use of the radiation field patterns of the antennas. The accuracy of the technique is illustrated by two examples. **Copyright** © 2010 Praise Worthy Prize S.r.l. - All rights reserved.

Keywords: Antenna Diversity, Radiation Power, Envelope Correlation, Scattering Parameters

Nomenclature

ρ_e Spatial envelope correlation.
 $d\Omega$ Differential spherical area.
 $F_\theta(\theta, \phi)$ Radiation field in \hat{e}_θ direction.
 $F_\phi(\theta, \phi)$ Radiation field in \hat{e}_ϕ direction.
MIMO Multiple Input Multiple Output.
 P_{total} Total power.
 P_{rad} Radiated power.
 P_{loss} Power loss .
 \dagger Hermitian transpose.
 a Incident wave .
 b Reflected wave .
 $J_s(\theta, l)$ Surface current density distribution .
 J_{Si} The total currents on structures on the i^{th} antenna structures.
 a_i The incident wave on the i^{th} antenna structures.
 J_{S1}^1, J_{S1}^2 The normalised currents on structures 1 and 2 due to the incident wave a_1 .
 J_{S2}^1 and J_{S2}^2 The normalised currents on structures 1 and 2 due to the incident wave a_2 .
 P_{li} The power loss on the i^{th} antenna structures.
 L The losses matrix.
 R Is 2x2 radiation matrix.
 D_i The maximum directivity of the i^{th} antenna.
 D_i A lambded resistive element.
 λ The wavelength.
 σ Surface conductivity.

I. Introduction

Mobile communication systems where there is only one antenna at both the transmitter and the receiver are known as Single Input Single Output (SISO) systems. SISO system capacity is limited by the Shannon Nyquist criterion [1]. In order to increase the capacity of SISO systems to meet the high bit rate transmissions demanded by modern mobile communications, the bandwidth and/or the power have to increase significantly. Fortunately, using MIMO systems (Multiple Input, Multiple Output) has the potential to increase the capacity of the wireless system without the need to increase the transmission power or the bandwidth [2]. On the other hand, mutual coupling between the antennas degrades the diversity performance of an antenna system; therefore designers try to minimize the mutual coupling of the antenna system whilst maintaining the matching requirements.

MIMO systems are required to deliver maximum capacity with minimum bit error rate (BER). They can exploit diversity, spatial multiplexing or beam forming and steering techniques, including null steering for interference rejection. Diversity gain requires independent or complementary fading at each antenna element and optimum spacing will depend on the angle of arrival spectrum in the multipath channel. Other techniques will benefit from an array radiation pattern that has high gain, a narrow beamwidth and low side or grating lobes in order to be able to resolve paths that are closely spaced in angle.

But it is challenging to implement multiple antennas in a very small volume such as mobile handsets, PDA's and laptops; therefore the spatial correlation properties of different antenna elements in the array should be considered since this will affect the MIMO channel capacity.

In mobile communication the antenna spacing is usually small, thus the impact of the mutual coupling will be not negligible. Mutual coupling increases the spatial correlation between the array elements. Also, it deforms the radiation pattern of each array element, which affects the diversity gain.

A generalized analysis of signal correlation between any two array elements including non-identical elements and arbitrary load termination of passive antenna ports was presented in [3]. The method is related to the power balance concept and is based on the antenna impedance matrix. In [5] theoretical and simulation studies have been conducted to explain the experimentally observed effect that the correlation between signals of closely spaced antennas is smaller than that predicted using the well known theoretical methods. A simple expression to compute the correlation coefficients from the far field pattern including the propagation environment characteristics and the terminating impedance was introduced in [5].

Three different methods are used to calculate the antenna correlation. The first method is based on the far field pattern [6], the second is based on the scattering parameters at the antennas terminals [7] and the third method is based on Clarke's formula [8]. Calculating correlation using the radiation field pattern of the antenna system is a time consuming method, whether it is done by simulation or using experimental data.

A simple method for the computation of the envelope correlation for two antenna elements using scattering parameters was presented in [7]. This method avoids intensive computations using the radiation field patterns of the antenna system, and may be straightforwardly generalized to the envelope correlation of an N-antenna system [9]. This formulation has been widely adopted in discussing antenna diversity issues [10, 11]. However, the computations in [7] and [9] do not include the power losses in the antenna structures. This accounts for the discrepancy between the envelope correlation results obtained by this method, from that computed directly from the radiation field patterns of the two antenna elements [12]:

$$\rho_e = \frac{\left| \iint_{4\pi} d\Omega F_1(\theta, \phi) \cdot d\Omega F_2(\theta, \phi) \right|^2}{\iint_{4\pi} d\Omega |F_1(\theta, \phi)|^2 \iint_{4\pi} d\Omega |F_2(\theta, \phi)|^2} \quad (1)$$

where $F_i(\theta, \phi) = F_\theta^i(\theta, \phi)\hat{e}_\theta + F_\phi^i(\theta, \phi)\hat{e}_\phi$ is the radiation field of the i^{th} antenna.

In practice it is required to identify envelope correlation between any two sensors in a given array. The correlation is sensitive to the intrinsic power losses in the radiating elements; the methodologies reported in [7] and [9] are based on ideal passive structures, i.e. without losses. Hallbjorner has presented a useful analysis on the effect of antenna efficiency on spatial

correlation estimates [12]. The analysis presented below provides an operational method, with a clear physical basis, for explicitly incorporating the intrinsic (Ohmic) losses into the estimation of the spatial correlation, using the scattering representation for a multi-beam array [13].

The calculation of the envelope correlation for a (NxN) MIMO antenna array in terms of the system's scattering parameters is modified to include power losses [14]. This represents a major simplification with respect to the conventional use of the radiation field patterns of the antennas.

In this paper, a simplified calculation of the envelope correlation in (1) for a lossy two antenna system was intensively evaluated in terms of the scattering parameters and the intrinsic power losses of the antenna structures. The power loss is presented in a matrix formulation, in order to match the presentation in [7,9,14,15]. Two new illustrative examples are presented and discussed to show the contribution of the proposed method. .

I. Format of Manuscript

Consider the electromagnetic geometry suggested by Figure 1, the total power is given by:

$$P_{total} = P_{rad} + P_{loss} \quad (2)$$

where P_{rad} and P_{loss} are the total radiated and loss power respectively; P_{tot} is sometimes called the accepted power, and may be computed in terms of the incident waves by $(a^\dagger a - b^\dagger b)$, where † denotes the Hermitian transpose.

The analysis developed below and the subsequent case studies shown in Figure 2, have been presented for convenience in a wire antenna formulation and the solved using NEC. It should be understood that the underlying concepts are fully general, and can be readily rewritten in terms of general surface and volume currents.

The surface current density distributed on a radiating wire of radius r can be written as;

$$J_s(\theta, l) = \frac{J_s(\theta, l)}{2\pi r} a_l \approx \frac{J_s(l)}{2\pi r} a_l \quad (3)$$

The power loss may be computed in terms of the surface currents on the antenna structures as follows. These currents may be expressed in terms of the incident waves a_1 and a_2 :

$$J_{S1} = \frac{1}{\sqrt{R_S}} \left(a_1 \cdot \frac{J_{S1}^1(l)}{2\pi r} \hat{a}_l + a_2 \cdot \frac{J_{S1}^2(l)}{2\pi r} \hat{a}_l \right) \quad (4)$$

$$J_{S2} = \frac{1}{\sqrt{R_S}} \left(a_1 \cdot \frac{J_{S2}^1(l)}{2\pi r} \hat{a}_l + a_2 \cdot \frac{J_{S2}^2(l)}{2\pi r} \hat{a}_l \right) \quad (5)$$

J_{S1} and J_{S2} are the total currents on structures 1 and 2, respectively, J_{S1}^1 and J_{S1}^2 are the normalised currents on structures 1 and 2 due to the incident wave a_1 , similarly J_{S2}^1 and J_{S2}^2 are due to the incident wave a_2 , R_s is the surface impedance of the antennas. Thus, the power loss on structures 1 and 2 can be expressed by (6) and (7), respectively:

$$P_{l1} = \iint \left(a_1 J_{S1}^1(l) + a_2 J_{S1}^2(l) \right) \cdot \left(a_1 J_{S1}^1(l) + a_2 J_{S1}^2(l) \right)^* d\theta dl \quad (6)$$

$$P_{l2} = \iint \left(a_1 J_{S2}^1(l) + a_2 J_{S2}^2(l) \right) \cdot \left(a_1 J_{S2}^1(l) + a_2 J_{S2}^2(l) \right)^* d\theta dl \quad (7)$$

Expanding these expressions gives,

$$P_{l1} = \frac{1}{2\pi} \int |a_1|^2 J_{S1}^1(l) J_{S1}^{1*}(l) + a_1 a_2^* J_{S1}^1(l) J_{S1}^{2*}(l) + \dots \dots a_2 a_1^* J_{S1}^2(l) J_{S1}^{1*}(l) + |a_2|^2 J_{S1}^2(l) J_{S1}^{2*}(l) dl \quad (8)$$

$$P_{l2} = \frac{1}{2\pi} \int |a_1|^2 J_{S2}^1(l) J_{S2}^{1*}(l) + a_1 a_2^* J_{S2}^1(l) J_{S2}^{2*}(l) + \dots \dots a_2 a_1^* J_{S2}^2(l) J_{S2}^{1*}(l) + |a_2|^2 J_{S2}^2(l) J_{S2}^{2*}(l) dl \quad (9)$$

Hence, (8) and (9) can be expressed in the matrix notation as,

$$P_{loss} = (a, La) = (a^\dagger La) \quad (10)$$

where,

$$L = L' + L'' = \begin{pmatrix} L'_{11} & L'_{12} \\ L'_{21} & L'_{22} \end{pmatrix} + \begin{pmatrix} L''_{11} & L''_{12} \\ L''_{21} & L''_{22} \end{pmatrix} \quad (11)$$

The elements L'_{ij} may be read across as,

$$\begin{pmatrix} \frac{1}{2\pi} \int J_{S1}^1(l) J_{S1}^{1*}(l) dl & \frac{1}{2\pi} \int J_{S1}^1(l) J_{S1}^{2*}(l) dl \\ \frac{1}{2\pi} \int J_{S1}^2(l) J_{S1}^{1*}(l) dl & \frac{1}{2\pi} \int J_{S1}^2(l) J_{S1}^{2*}(l) dl \end{pmatrix} \quad (12)$$

and similarly for L''_{ij} . From the properties of the Hermitian inner product, it follows that $L_{ij} = L_{ji}^*$.

Equation (2) can now be expressed in terms of the incident waves a_1 and a_2 .

$$a^\dagger (1 - S^\dagger S) a = (a^\dagger La) + (a^\dagger Ra) \quad (13)$$

where the scattering matrix S of the two-antenna system, includes the power loss. R is the 2×2 matrix defined in [7] as,

$$R = \begin{pmatrix} \frac{D_1}{4\pi} \iint d\Omega |F_1(\theta, \phi)|^2 & \frac{\sqrt{D_1 D_2}}{4\pi} \iint d\Omega F_1(\theta, \phi) \cdot F_2^*(\theta, \phi) \\ \frac{\sqrt{D_2 D_1}}{4\pi} \iint d\Omega F_2(\theta, \phi) \cdot F_1^*(\theta, \phi) & \frac{D_2}{4\pi} \iint d\Omega |F_2(\theta, \phi)|^2 \end{pmatrix} \quad (14)$$

Where D_i is the maximum directivity of the i^{th} antenna. Therefore, from (13) the equivalent elements of (14) can be expressed as follows:

$$\frac{D_1}{4\pi} \iint d\Omega |F_1(\theta, \phi)|^2 = 1 - (1 - |S_{11}|^2 + |S_{21}|^2) - (L'_{11} + L''_{11}) \quad (15)$$

$$\frac{\sqrt{D_1 D_2}}{4\pi} \iint d\Omega F_1(\theta, \phi) \cdot F_2^*(\theta, \phi) = -(S_{11}^* S_{21} + S_{21}^* S_{22}) \dots \dots \dots - (L'_{12} + L''_{12}) \quad (16)$$

Thus, the envelope correlation for the two-antenna system geometry (Fig. 1) can be expressed in terms of the scattering parameters, and the intrinsic power losses, as in (17).

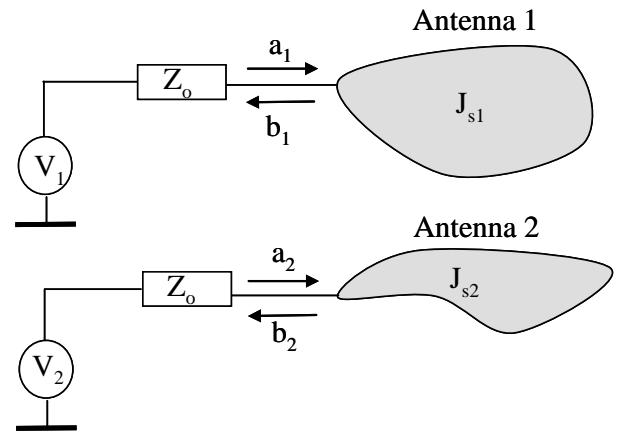


Fig. 1. Basic geometry for two antenna element system.

II. Simulation and Results

In order to verify (17), the envelope correlation has been computed for two parallel half-wavelength wire dipoles in free space, as a function of their separation. The antenna far fields and scattering parameters were obtained using NEC [8]. The wire radius of each dipole was set at 0.002 wavelengths.

Two different sources of loss were considered for the purpose of validation. In the first instance, both dipoles were loaded by two lumped resistive loads, each of 25Ω , as in Fig. 2. Secondly, the dipoles were loaded by surface conductivity along the antenna geometry.

$$\rho_e = \frac{|1 - (S_{11}^* S_{12} + S_{21}^* S_{22}) - (L'_{12} + L''_{12})|^2}{\{1 - (|S_{11}|^2 + |S_{21}|^2) - (L'_{11} + L''_{11})\} \{1 - (|S_{22}|^2 + |S_{12}|^2) - (L'_{22} + L''_{22})\}} \quad (17)$$

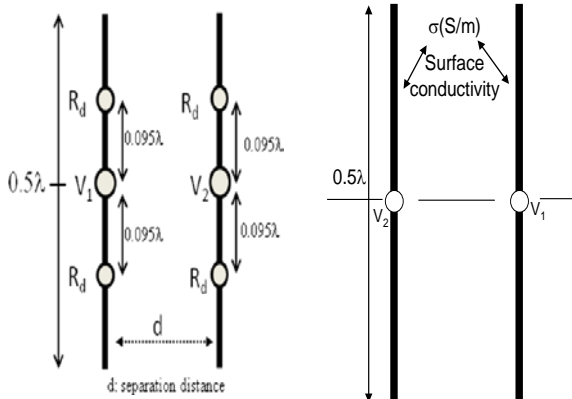


Fig. 2. Examples under test; (left: antennas loaded by lumped resistive loads, right: antennas loaded by surface conductivity)

Variations between the proposed method and the lossless approach were checked by simulation. The spatial envelope correlation calculated using the far field parameters, versus the dipole separation distance, are given in Fig. 3 for both lossless and lossy cases. There is good agreement between the proposed method and the results calculated from (1) for the lossy case. The envelope correlation for dipole separations less than 0.5 wavelengths can take values bigger than the achieved S_{21} values. It is also interesting to note that the nulls of the envelope correlation are shifted compared with those obtained for the lossless approach.

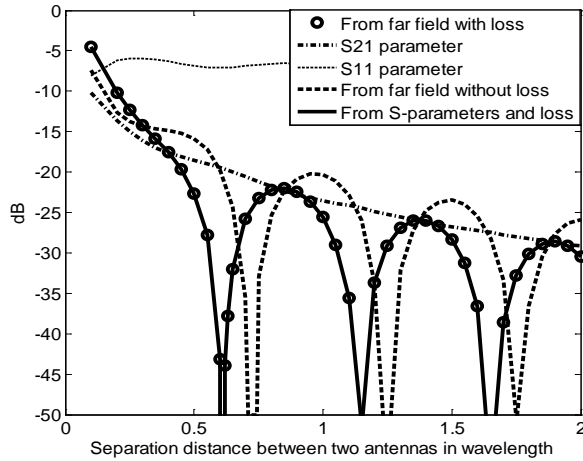


Fig. 3. Computed spatial envelope correlations and S-parameters for two half wavelength dipoles against their separated distance.

Figures 4 and 5 illustrate variations in the spatial envelope correlation, versus the lumped resistive loads, and surface conductivities, respectively.

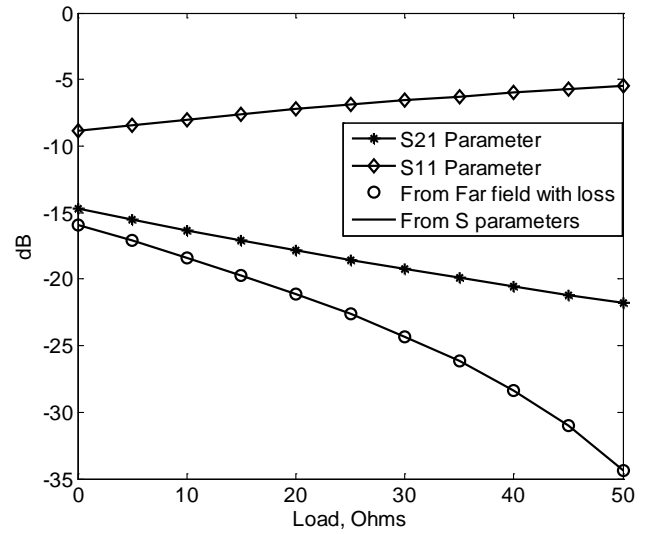


Fig. 4. Envelope correlation and S-parameters for two half wavelength dipoles against the lumped resistive loads shown in Fig. 2.

The dipole separation was kept constant at 0.5 wavelengths. These envelope correlation results prove the concept of the proposed method, i.e. equation (17), against those obtained from the far field patterns. It can be seen in fig. 4 that the envelope correlation values closely approach the S_{21} values as the load is reduced. For the surface conductivity case, they are much closer for small wire conductivity values.

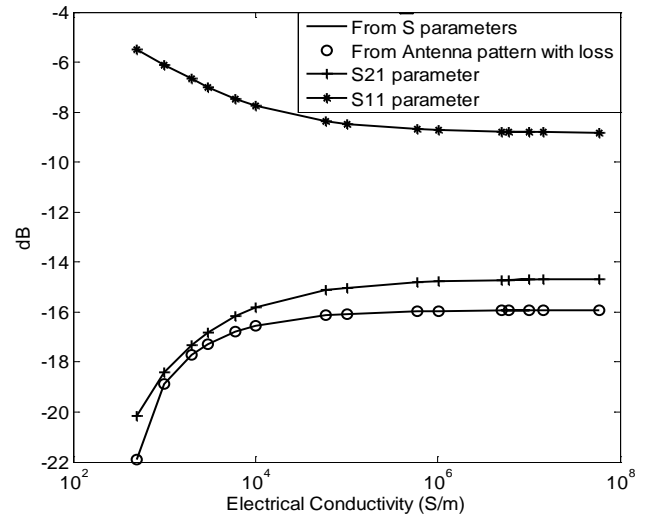


Fig.5. Envelope correlation and S-parameters for two half wavelength dipoles against the surface electric conductivity on both dipoles.

In summary, the analysis described here, based on the conceptual framework assumed by equation (17), provides a direct and accurate forecast of spatial

envelope correlation, as compared with the far field analysis in equation (1).

It should be noted that several empirical approaches exist for the direct measurement of the radiation efficiency of passive antennas. These include radiometry [17, 18], random field analysis [19] and reverberation chamber techniques [20], and when applied to multi-port passive structures they can provide an independent check on the diagonal terms in the L-matrix (i.e. equation (11)). Such practical implementations will be considered for future developments of this work.

III. Conclusions

A method of calculation, for the spatial envelope correlation of a two-antenna system, which includes losses, using the system scattering parameters has been presented. This new expression should reduce the complexity in predicting the spatial envelope correlation, and simplify antenna design where a low envelope correlation is required. Two validation examples were presented, which demonstrate good agreement between the proposed method, and the explicit calculation using far field pattern data. Two examples have been presented to validate the technique. The results have shown close agreement between the proposed method and the full computation using the far field pattern data.

Practical implementations for the direct measurement of the radiation efficiency of passive antennas will be the challenge for the future work.

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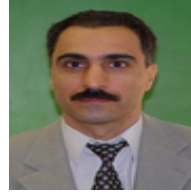
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