A Proposed Methodology for Modeling Pedestrian Crossing Time at Signalized Crosswalks Considering Bi-directional Flow *

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1. Introduction

Crosswalk geometry and configuration at signalized intersections directly affect the safety, cycle length and resulting delays for all users. Optimizing crosswalk configurations including width, position and angle is an important concern to improve the overall performance of signalized intersections. Quantifying the effect of bi-directional flow and crosswalk width on pedestrian walking speed and crossing time at signalized crosswalks is a prerequisite for improving the geometric design and configuration of signalized crosswalks. Pedestrian crossing time is basically a function of crosswalk length and walking speed. However when pedestrian demand increases at both sides of the crosswalk, crossing time increases due to interaction between conflicting pedestrian flows.

A variety of methods have been developed for determining appropriate pedestrian crossing times at signalized intersections. Although many of these methods have useful applications, most of them have shortcomings when considering the effects of bi-directional flow on crossing time. No consideration is given to deceleration or reduction in walking speed that results from the interaction between the conflicting flows. In this study, a new methodology is proposed to model pedestrian crossing time as a function of pedestrian demand, directional split, and crosswalk width, which is based on aerodynamic drag theory.

2. Literature Review

Few studies addressed the issue of bi-directional flow and its impact on crossing time at signalized crosswalks. Most of the existing works in this respect attempted to investigate the impact of bi-directional flow at other pedestrian facilities such as walkways and sidewalks. However the characteristics of the environment as well as the pedestrian arrival pattern at crosswalks is different from other pedestrian facilities.

Most crossing time estimations have been based on assumptions for start-up delay and a particular walking speed. The *Manual on Uniform Traffic Control Devices* (2003)[1], *Pignataro* (1973)[2], and the Signalized Intersection chapter of the *Highway Capacity Manual* (2000)[3] have formulations similar to Equation (1).

$$T = I + \frac{L}{S_p} + (x\frac{N_{ped}}{w}) \qquad (1)$$

Where T is total time required for all the crossing process (s), I is initial start-up lost time, L is crosswalk length (m), S_p is walking speed (m/s), x is average headway (s/ped/m), N_{ped} is number of pedestrians crossing during an interval p from one side of the crosswalk, and w is crosswalk width (m).

Equation (1) shows that the time spent on the crosswalk itself (L/S_p) is independent from the pedestrian demand, bi-directional effect and crosswalk width.

Lam et al. (2003)[4] investigated the effect of bi-directional flow on walking speed and pedestrian flow under various flow conditions at indoor walkways in Hong Kong. They found that the bi-directional flow ratios have significant impacts on both the at-capacity walking speeds and the maximum flow rates of the studied walkways. However they did not investigate the effect of different walkway's dimensions on the walking speed and the capacity of the walkway.

Virkler, et al. (1984)[5] collected data from some relatively low-volume and high-volume signalized crosswalks and recommended an equation for one-directional flow that also considers platoon size. However they did not consider the impact of bi-directional pedestrian flow.

Golani et al. (2007)[6] proposed a model to estimate crossing time considering start-up lost time, average walking speed, and pedestrian headways as a function of the dominant platoon and the opposite platoon separately. The proposed model is based on *HCM*[3] model which was calibrated by using empirical data. The proposed model relates the impact of bi-directional flow to the headway between pedestrians when they finish crossing. So it is very hard to see how the interaction is happening and what the resulting speed drop or deceleration is.

3. Methodology

The total time needed by a platoon of pedestrians to cross a signalized crosswalk can be divided into two main parts, discharge time and crossing time:

$$T_t = T_d + T_c \qquad (2)$$

Where T_t is the total time needed to cross the crosswalk, T_d is the discharge time necessary for a pedestrian platoon to move from the waiting area and step inside the crosswalk, and T_c is the time necessary to cross the crosswalk.

This study concentrates only on modeling the crossing time T_c as a function of crosswalk length, crosswalk width, and pedestrian demand at both sides of the crosswalk by applying the concept of drag force theory.

4. Modeling

The force on an object that resists its motion through a fluid is called drag. When the fluid is a gas like air (**Figure 1**), it is called aerodynamic drag (or air resistance). While if the fluid is a liquid like water it is

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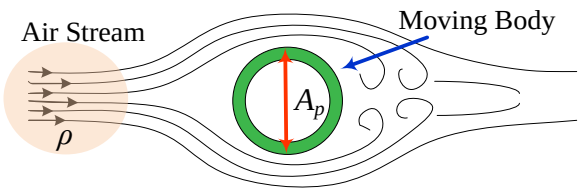


Figure 1: Drag force

called hydrodynamic drag. Drag is a complicated phenomenon and explaining it from a theory based entirely on fundamental principles is exceptionally difficult. *Pugh* (1971)[7] described the relation of drag D, the relative velocity of the air or the fluid, and a moving body in terms of a dimensionless group, the drag coefficient C_d. The drag coefficient is the ratio of drag D to the dynamic pressure q of a moving air stream and is defined by Equation (3):

$$D = C_d q A_p \tag{3}$$

Where D is drag force (kg·m/s^2), C_d is drag coefficient (dimensionless), q is dynamic pressure (force per unit area), and A_p is the projected area (m^2).

The dynamic pressure q which is equivalent to the kinetic energy per unit volume of a moving solid body (*Pugh* (1974)[8]) is defined by Equation (4):

$$q = \frac{1}{2}\rho u^2 \tag{4}$$

Where ρ is density of the air in kilogram per cubic meter, and u is speed of the object relative to the fluid (m/s). By substituting Equation (4) in Equation (3), the final drag force equation is:

$$D = \frac{1}{2} C_d \rho u^2 A_p \tag{5}$$

4.1 'Drag Force' Caused by the Opposite Pedestrian Flow

To use the drag force concept to model the interactions between pedestrian flows, the following assumptions are made:

I. Opposite pedestrian demand is considered as a homogenous flow (**Figure 2**) with a density equal to the number of pedestrian waiting in the beginning of the green interval divided by an area equal to the width of the crosswalk multiplied by 1 meter.

$$\rho = \frac{P_2}{w*1} \qquad (ped/m^2) \tag{6}$$

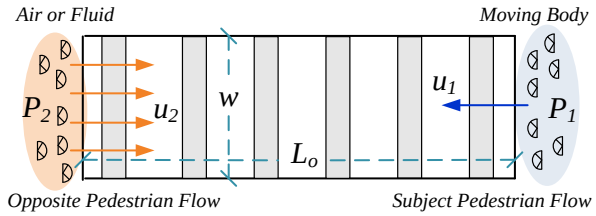
II. The subject pedestrian flow is considered as one body moving against the opposite pedestrian flow. The interactions occur along the projected area of all pedestrians in the subject flow which is defined as the sum of widths of all pedestrians in the subject flow:

$$A_p \approx \beta n \tag{7}$$

Where A_p is the projected area of the subject pedestrian flow (m), β is the average width of one pedestrian, and n is a dimensionless number equal to the number of pedestrian in the subject pedestrian flow P_1, shown in **Figure 2**.

III. The initial speed of the subject and the opposite pedestrian flow when they start crossing is assumed to be equal to the free-flow speed u_o, therefore the relative speed u becomes:

$$u = (u_1 - (-u_2)) = u_o + u_o = 2u_o \tag{8}$$



w: crosswalk width (m), ***L$_o$***: crosswalk length, ***P$_2$***: opposite pedestrian demand, ***P$_1$***: subject pedestrian demand, ***u$_1$***: speed of the subject pedestrian flow, and ***u$_2$***: speed of the opposite pedestrian flow.

Figure 2: Applying the concept of drag force on opposing pedestrian flows at crosswalk

After substituting Equation (6), (7), and (8) in Equation (5), then the drag force equation becomes:

$$D = \frac{1}{2}\beta C_d \frac{P_2}{w}(2u_o)^2 n \tag{9}$$

Assuming that the width of one pedestrian β is 0.6m, the drag force D becomes:

$$D = \frac{1}{2}*0.6*C_d \frac{P_2}{w} 4u_0^2 n = \frac{1}{2} C_{Dadj} \frac{P_2}{w} u_o^{\,2} n \tag{10}$$

Where C_{Dadj} is adjusted drag coefficient (dimensionless), and it is defined according to Equation (11).

$$C_{Dadj} = 4\beta C_d = 4*0.6*C_d \tag{11}$$

4.2 Deceleration of the Subject Pedestrian Flow

The net force on a particle observed from an inertial reference frame is proportional to the time rate of change of its linear momentum *(Momentum is the product of mass and velocity):*

$$F = \frac{dM}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} = ma \tag{12}$$

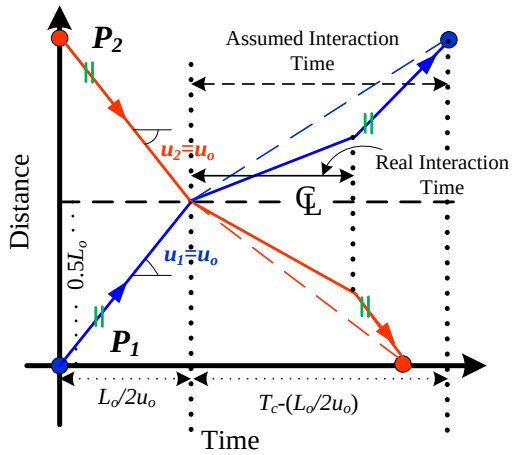
Where m is the mass of the moving body and its equivalent to the subject pedestrian demand P_1, and a is the average deceleration of the subject pedestrian flow.

The final speed of a moving particle in a straight line with constant average deceleration according to the motion equations is:

$$u_f^2 = u_i^2 - 2aL \tag{13}$$

Where u_i is initial speed (*m/s*) which is assumed to be equal to the free-flow speed u_o, u_f is final speed (*m/s*), a is average deceleration of the subject pedestrian demand (*m/s*^{2}), and L is the travelled distance (m).

Figure 3 shows the projection of pedestrian flow trajectory from both sides of a crosswalk. A major assumption of this methodology is that both opposing flows will start walking with the same free-flow speed u_o in a straight line until the middle of the crosswalk where they will meet. In order to avoid the complexity in estimating the real interaction time, the time from the moment when the subject pedestrian flow meets the opposite pedestrian flow at the middle of the crosswalk until the subject pedestrian flow reaches the end of the crosswalk is assumed as the interaction time here. The



L_o: crosswalk length, P_2: opposite pedestrian demand, P_1: subject pedestrian demand, u_1: speed of the subject pedestrian flow, u_2: speed of the opposite pedestrian flow, u_o: free flow speed, and T_c: time needed by subject pedestrian demand to cross the crosswalk.

Figure 3: Time-Space diagram of the conflicting pedestrian flows

resulting deceleration is averaged along the assumed interaction time. Therefore the final speed is:

$$u_f = \frac{L_c - L_o/2}{T_c - L_o/2u_o} \tag{14}$$

Where L_c is average trajectory length of the subject pedestrian demand, T_c is average crossing time of the subject pedestrian demand, L_o is crosswalk length and u_o is free-flow speed. By substituting Equation (14) in Equation (13), the average deceleration becomes:

$$a = \frac{1}{2}\frac{u_o^2 - \left(\frac{L_c - L_o/2}{T_c - L_o/2u_o}\right)^2}{L_c - L_o/2} \tag{15}$$

By substituting the mass and the acceleration in Equation (12), the net force ($ped{\cdot}m/s^2$) becomes:

$$F = ma = \frac{1}{2}P_1\frac{u_o^2 - \left(\frac{L_c - L_o/2}{T_c - L_o/2u_o}\right)^2}{L_c - L_o/2} \tag{16}$$

4.3 Model Development

The drag force caused by an opposite pedestrian flow should be equal to the force that causes the deceleration. By equating Equation (16) and Equation (10), and after solving them for the crossing time T_C, the net equation is:

$$T_c = \frac{(L_c - \frac{L_o}{2})}{\sqrt{u_o^2 - \frac{C_{Dadj}P_2u_o^2(L_c - \frac{L_o}{2})}{w}}} + \frac{L_o}{2u_o} \tag{17}$$

Equation (17) represents how the crossing time varies according to pedestrian demand combinations from both sides of the crosswalk and crosswalk width.

5. Estimating the Drag Coefficient (C_{Dadj})

The value of adjusted drag coefficient C_{Dadj} according to aerodynamic drag is dependent on the kinematic viscosity of the fluid, projected area and texture of the moving body. In the pedestrian's case, this value can here be assumed to be dependent on the pedestrian demand at both sides of the crosswalk and their split ratio.

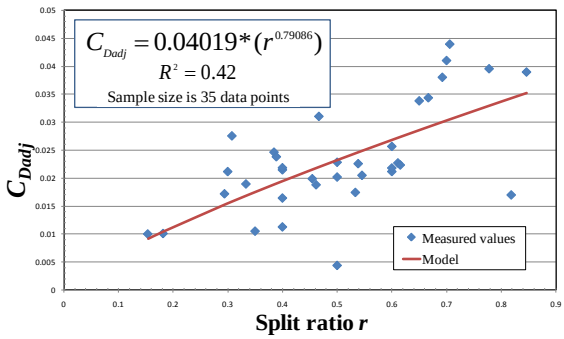


Figure 4: Modeling the drag coefficient as a function of split ratio

In order to define a value for the pedestrian free-flow speed, a 1.5-hour video tape for the crosswalk at the east leg of Nishi-Osu intersection in Nagoya City (6m wide × 25.4m long) was analyzed. Nishi-Osu intersection is characterized by small pedestrian demand with a large crosswalk width. 102 samples of pedestrian's free flow speeds were measured. All the considered pedestrians were leading pedestrians and they did not face any opposite flow or turning vehicles. The average free-flow speed for all the samples is u_o=1.45 m/s. This value is used to estimate C_{Dadj} and crossing time T_c.

To estimate crossing time by using Equation (17), C_{Dadj} was first estimated from empirical data. A 2-hour video tape for the crosswalk at the east leg of Imaike intersection in Nagoya City (9.6m wide × 21.5m long) was analyzed. The pedestrian demand in each cycle at each direction, the average pedestrian trajectory length, and the average pedestrian crossing time in the same cycle were extracted from the video tape. Then by using Equation (17), C_{Dadj} was estimated for 35 samples where the total pedestrian demand was ranging from 5 – 30 pedestrians per cycle (one cycle is 160sec). After analyzing the available data, C_{Dadj} was modeled in terms of the split ratio r which is the ratio of the subject pedestrian demand to the total pedestrian demand. Equation (18) defines the split ratio r.

$$r = \frac{P_1}{P_1 + P_2} \tag{18}$$

Figure 4 shows the relationship between split ratio and C_{Dadj}. As split ratio increases the drag coefficient also increases. Due to the limited sample size and the existence of many external factors that can affect pedestrian behavior, the coefficient of correlation (R^2) is relatively small.

6. Discussion and Validation

After estimating the drag coefficient, Equation (18) can directly be used to calculate the average crossing time for different demand volumes under different crosswalk widths. **Figure 5** shows how the crossing time varies with crosswalk width. When crosswalk width becomes larger and larger for a specific demand, crossing time decreases until it becomes almost constant (free-flow condition). But when crosswalk width

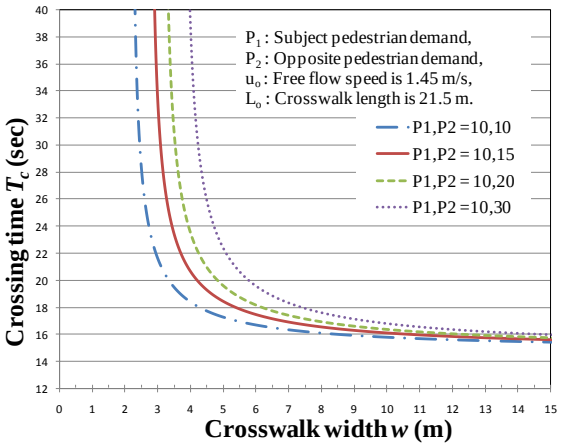


Figure 5: Changes in crossing time with changing demand and crosswalk width

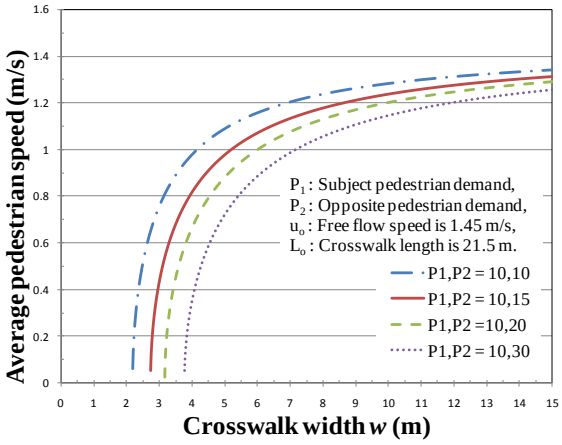


Figure 6: Drop in average walking speed due to bi-directional flow effect

becomes smaller for a specific demand, crossing time increases, as the interactions between the opposing flows increases, until it reaches a point where the opposing flows block each other causing a drastic increase in crossing time.

Figure 6 shows the drop in average walking speed due to the interactions between the opposing flows. As the crosswalk width decreases for a specific pedestrian demand, the interactions increase causing reduction in the average walking speed. The drop in the walking speed continues with reducing crosswalk width until a point where the speed drops drastically. This tendency is reasonable if we assume that pedestrian cannot walk outside the crosswalk, therefore it is expected that as the demand increases for a specific crosswalk width, the average walking speed will drop, until it reaches almost zero where every pedestrian cannot walk any more.

To validate the proposed model, the average crossing time was measured for 38 cycles under different demand ratios and compared with the estimated crossing time from the proposed model. **Figure 7** illustrates the differences between the measured and the estimated crossing times. A paired t-test was performed and the result showed that the estimated values were not significantly different from the measured values at the 95% confidence level.

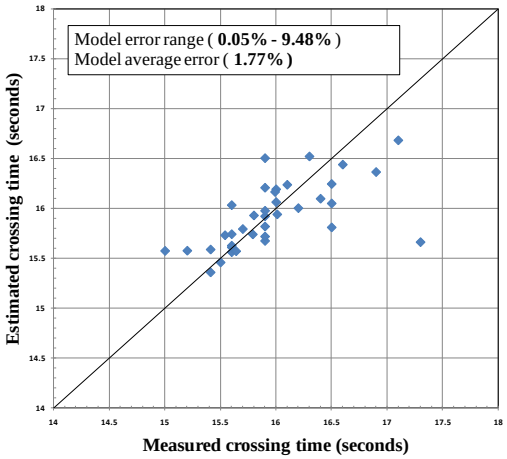


Figure 7: Model validation

7. Conclusions and Future Works

A new methodology in modeling the interactions between opposing pedestrian flows at signalized crosswalks was proposed in this paper. This methodology is based on the aerodynamics drag concept. The final equation of crossing time T_c provides a rational quantification for the effect of crosswalk length, pedestrian demand, demand split ratio, and crosswalk width on crossing time. However, the proposed model needs to be validated and compared with the real pedestrian behavior at crosswalks with higher demands. The nature of the drag coefficient C_{Dadj} is a key factor in estimating crossing time. Collecting and analyzing more data especially for high demand crosswalks are necessary to develop more concrete formula for the drag coefficient with higher correlation. The proposed methodology will be used as a basis to define the required crosswalk width for different pedestrian demand volumes and split ratios.

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