

# A New Approach for OSTBC and QOSTBC

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## ABSTRACT

A new approach for Quasi-Orthogonal space time block coding (QO-STBC) is proposed, with simple linear decoding via maximum likelihood detection. The conventional QO-STBC can achieve the full communication rate, but at the expense of the decoding complexity and the diversity gain due to the interference terms in the detection matrix. In this paper we propose a QO-STBC scheme that will eliminate the interference from the detection matrix. The proposed code improves the diversity gain compared with the conventional QO-STBC scheme; it also reduces the decoding complexity. A full rate full diversity order Diagonalized Hadamard Space Time code (DHSTBC) is presented

## Keywords

Quasi-Orthogonal space time block coding (QO-STBC), full rate, full diversity order, Diagonalized Hadamard Space Time code (DHSTBC).

## 1. INTRODUCTION

In traditional communication systems there is only one antenna at both the transmitter and the receiver. This antenna system is known as Single Input Single Output (SISO). SISO systems have a major drawback in terms of the capacity, as stated by the well-known Shannon-Nyquist criterion. In order to increase the capacity of SISO systems to meet the high bit rate transmission demanded by modern communications, the bandwidth and the power have to increase significantly. Fortunately, using the Multiple-Input Multiple-Output (MIMO) system could increase the capacity of the wireless system without the need to increase the transmission power or the bandwidth.

Transmit diversity is a well-known technique to mitigate fading effects over a communication link. Alamouti [1] proposed a transmit diversity scheme that offers maximum diversity gain using two antennas at the transmitter. The pioneering work of Alamouti has been the basis to create Orthogonal space time block coding systems (OSTBCs) for more than two transmit antennas. First of all, Tarokh studied the error performance associated with unitary signal matrices [2]. Sometime later, Ganesan streamlined the derivations of many of the results associated with OSTBC and established an important link to the theory of the orthogonal and ‘amicable orthogonal’ designs [3].

It has been proved that a complex orthogonal design of STBCs that provides full diversity and full transmission rate is not possible for more than two transmit antennas [1]. To achieve full-rate transmission while maintaining much of the orthogonality benefits of OSTBC, Quasi Orthogonal Space Time Block Code (QO-STBC) has been proposed [4-6]. A QO-STBC can achieve full rate but interference terms will appear from the neighboring signals during the signal detection

which will increase the detection complexity and decrease the gain performance.

## 2. CONVENTIONAL QO-STBC

In Quasi-orthogonal code structure, the columns of the transmission matrix are divided into groups. The columns within each group are not orthogonal to each other but those from different groups are orthogonal to each other. By using quasi-orthogonal design, pairs of transmitted symbols can be decoded independently; the loss of diversity in QOSTBC is due to some coupling terms between the estimated symbols.

A QO-STBC scheme for four transmit elements was proposed independently by Jafarkhani [4] and Tirkkonen [5]. Both of the proposed schemes had achieved almost the same BER performance. For example the encoding matrix proposed by Tirkkonen [5] is two  $(2 \times 2)$  Alamouti codes  $X_{12}$  and  $X_{34}$  where:

$$\text{and } X_{34} = \begin{bmatrix} x_3 & x_4 \\ -x_4^* & x_3^* \end{bmatrix} \quad (1)$$

$X_{12}$  and  $X_{34}$  are used in a block structure resulting in what is known as Extended Alamouti QOSTBC,  $X_{ABBA}$ , for four transmit antennas:

$$X_{ABBA} = \begin{bmatrix} X_{12} & X_{34} \\ X_{34} & X_{12} \end{bmatrix} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ x_3 & x_4 & x_1 & x_2 \\ -x_4^* & x_3^* & -x_2^* & x_1^* \end{bmatrix} \quad (2)$$

The Equivalent Virtual Channel Matrix (EVCM)  $H_v$  can be written as:

$$H_v = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & -h_1^* \end{bmatrix} \quad (3)$$

$H_v$  can be described as a highly structured, equivalent, virtual  $(4 \times 4)$  MIMO channel matrix that replaces the  $(4 \times 1)$  received channel vector  $Y$ .

A simple method to decode the QOSTBC is by applying the maximum ratio combining (MRC) technique. MRC can be done by multiplying the received vector  $Y$  with  $H_v^H$  thus:

$$\begin{aligned} X &= H_v^H Y = H_v^H \cdot H_v X_{ABBA} + H_v^H n \\ &= D_4 X_{ABBA} + H_v^H n \end{aligned} \quad (4)$$

$D_4 = H_v^H \cdot H_v$ , is the detection matrix used to decode the received signal,  $H_v^H$  is the Hermitian of  $H_v$  and  $n$  is the noise vector of the AWGN channel. For the OSTBC scheme the detection matrix is always diagonal: this enables the use of simple linear decoding, but in the QOSTBC scheme this cannot be done due to the non-orthogonal detection matrix, as shown in equation (5):

$$D_4 = H_v^H \cdot H_v = \begin{bmatrix} \alpha & 0 & \beta & 0 \\ 0 & \alpha & 0 & \beta \\ \beta & 0 & \alpha & 0 \\ 0 & \beta & 0 & \alpha \end{bmatrix} \quad (5)$$

Where the diagonal elements  $\alpha$  represent the channel gains and  $\beta$  represent the interference from the neighbouring signals, for four transmit antennas

$$\begin{aligned} \alpha &= |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2 \\ \beta &= h_1 h_3^* + h_2 h_4^* + h_1^* h_3 + h_2^* h_4 \end{aligned} \quad (6)$$

The interference terms  $\beta$  in the detection matrix will cause performance degradation; therefore more complex decoding methods were introduced to detect the estimate  $\hat{X}$  as shown in equation (7):

$$\begin{aligned} \hat{X} &= (H_v^H \cdot H_v)^{-1} \cdot H_v^H Y \\ &= (H_v^H \cdot H_v)^{-1} \cdot H_v^H \cdot H_v \cdot X + (H_v^H \cdot H_v)^{-1} \cdot H_v^H \cdot H_v \cdot n \end{aligned} \quad (7)$$

### 3. PROPOSED QO-STBC

In [7] a novel QO-STBC scheme for three and four transmit antennas was proposed by reforming the detection matrix of the conventional QO-STBC by using two given rotation matrices [8]. In this paper a new approach is proposed for the QOSTBC scheme by reforming the detection matrix using a different technique.

Let the detection matrix  $D_4$  be defined in equation (5). The solution of the eigenvalue problem of matrix  $D_4$  can be shown as:

$$D_4 \cdot V - V \cdot D = 0 \quad (8)$$

Where  $D$  is equivalent to  $D = \lambda I$ ,  $\lambda$  being the eigenvalue operator. The solution of equation (8) produces two matrices, these are eigenvalues matrix  $D$  and eigenvectors matrix  $V$  of matrix  $D_4$ , so that to satisfy the  $D_4 \cdot V = V \cdot D$  as follows:

$$D = \begin{bmatrix} \alpha + \beta & 0 & 0 & 0 \\ 0 & \alpha + \beta & 0 & 0 \\ 0 & 0 & \alpha - \beta & 0 \\ 0 & 0 & 0 & \alpha - \beta \end{bmatrix} \quad (9)$$

$$V = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad (10)$$

The eigenvalues matrix ( $D$ ) is the free interference matrix, which can be derived by following these steps:

- Consider the relation between the eigenvalues ( $D$ ) and eigenvectors ( $V$ ) of matrix  $D_4$ :

$$D_4 \cdot V = V \cdot D \quad (11)$$

- Solving equation (11) for  $D$

$$D = V^{-1} \cdot D_4 \cdot V \quad (12)$$

- Substituting equation (5) in (12)

$$D = V^{-1} \cdot H_v^H \cdot H_v \cdot V \quad (13)$$

- The relation between  $V^{-1}$  and  $V^H$  is given by:

$$V^{-1} = \frac{1}{2} V^H \quad (14)$$

- Substitute equation (14) into (13) results:

$$D = \frac{1}{2} V^H \cdot H_v^H \cdot H_v \cdot V \quad (15)$$

Then a new channel matrix can be defined as:

$$H = H_v \cdot V \quad (16)$$

Where  $H$  is given by:

$$H = \begin{bmatrix} h_1 + h_3 & h_2 + h_4 & h_3 - h_1 & h_4 - h_2 \\ h_2^* + h_4^* & -h_1^* - h_3^* & h_4^* - h_2^* & h_1^* - h_3^* \\ h_1 + h_3 & h_2 + h_4 & h_1 - h_3 & h_2 - h_4 \\ h_2^* + h_4^* & -h_1^* - h_3^* & h_2^* - h_4^* & h_3^* - h_1^* \end{bmatrix} \quad (17)$$

$H^H \cdot H$  is diagonal matrix which can achieve simple linear decoding, because of the orthogonal characteristic of the channel matrix  $H$ .

Now, the encoding matrix  $X_{New}$  can be derived corresponding to the channel matrix  $H$ , as in following:

$$X_{New} = \begin{bmatrix} x_1 - x_3 & x_2 - x_4 & x_3 + x_1 & x_4 + x_2 \\ x_4^* - x_2^* & -x_3^* + x_1^* & -x_4^* - x_2^* & x_3^* + x_1^* \\ x_1 + x_3 & x_2 + x_4 & x_3 - x_1 & x_4 - x_2 \\ -x_4^* - x_2^* & x_3^* + x_1^* & x_4^* - x_2^* & -x_3^* + x_1^* \end{bmatrix} \quad (18)$$

Similarly, the detection matrix for three elements scheme can be derived using identically the above method to eliminate the interference terms. The resultant channel and Quasi-Orthogonal coding matrices result in a free interference detection matrix be given as in following:

$$H_3 = \begin{bmatrix} h_1 + h_3 & h_2 & h_3 - h_1 & -h_2 \\ h_2^* & -h_1^* - h_3^* & -h_2^* & h_1^* - h_3^* \\ h_1 + h_3 & h_2 & h_1 - h_3 & h_2 \\ h_2^* & -h_1^* - h_3^* & h_2^* & h_3^* - h_1^* \end{bmatrix} \quad (19)$$

$$X_{3New} = \begin{bmatrix} x_1 - x_3 & x_2 - x_4 & x_3 + x_1 \\ x_4^* - x_2^* & -x_3^* + x_1^* & -x_4^* - x_2^* \\ x_1 + x_3 & x_2 + x_4 & x_3 - x_1 \\ -x_4^* - x_2^* & x_3^* + x_1^* & x_4^* - x_2^* \end{bmatrix} \quad (20)$$

#### 4. PROPOSED SPACE TIME BLOCK CODE FROM DIAGONALIZED HADAMARD MATRIX (DHSTBC)

In this section a full rate full diversity order Diagonalized Hadamard Space Time code (DHSTBC) is presented. The codes generated using this method is orthogonal space time codes,  $XX^H = D$ , where  $D$  is a diagonal matrix.

The generated codes are able to provide full rate and full diversity when the number of the receive antennas are at least equal to the number of transmit antennas, the code matrixes for DHSTBC are limited to the Hadamard matrixes size ( $N = 2^n$ ), where  $n \geq 1$ ).

Let  $S = [s_1, s_2, \dots, s_N]$ , where  $s_1, s_2, \dots, s_N$  are the transmitted symbols. Assume four transmit antennas, then the code matrix can be written in form of a cyclic matrix as follows,

$$S_4 = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2 & s_1 & s_4 & s_3 \\ s_3 & s_4 & s_1 & s_2 \\ s_4 & s_3 & s_2 & s_1 \end{bmatrix} \quad (21)$$

The Hadamard matrix of order four is given as,

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (22)$$

The resultant matrix  $X_4$  is a DHSTBC and hence, the overall expression is given by

$$X_4 = H_4 \cdot S_4 \quad (23)$$

$$X_4 = \begin{bmatrix} s_1 + s_2 + s_3 + s_4 & s_1 + s_2 + s_3 + s_4 & s_1 + s_2 + s_3 + s_4 & s_1 + s_2 + s_3 + s_4 \\ s_1 - s_2 + s_3 - s_4 & s_2 - s_1 - s_3 + s_4 & s_1 - s_2 + s_3 - s_4 & s_2 - s_1 - s_3 + s_4 \\ s_1 + s_2 - s_3 - s_4 & s_1 + s_2 - s_3 - s_4 & s_3 - s_2 - s_1 + s_4 & s_3 - s_2 - s_1 + s_4 \\ s_1 - s_2 - s_3 + s_4 & s_2 - s_1 + s_3 - s_4 & s_2 - s_1 + s_3 - s_4 & s_1 - s_2 - s_3 + s_4 \end{bmatrix}$$

The elements of the DHSTBC matrix have linear combination of the transmitted symbols in which each STBC symbol contains the information of every element of  $S_N$ .

#### 4.1 PROPERTIES OF DHSTBC CODE MATRIX

The generated codes using the proposed method are orthogonal and they have the following properties:

**Property 1:** The product of  $X_N$  and its hermitian transpose  $X_N^H$  is diagonal matrix that is known as diversity product, i.e. the diversity product for  $X_4$  is,

$$X_4 X_4^H = \begin{bmatrix} 4(s_1 + s_2 + s_3 + s_4) & 0 & 0 & 0 \\ 0 & 4(s_1 - s_2 + s_3 - s_4) & 0 & 0 \\ 0 & 0 & 4(s_1 + s_2 - s_3 - s_4) & 0 \\ 0 & 0 & 0 & 4(s_1 - s_2 - s_3 + s_4) \end{bmatrix} \quad (24)$$

Thus, the rows of  $X_2$  and  $X_4$  are orthogonal to each other.

**Property 2:** The summation across the columns of the code matrix returns the original transmitted symbols.

$$\sum_{n=1}^N X_N = [s_1, s_2, \dots, s_N] = S \quad (25)$$

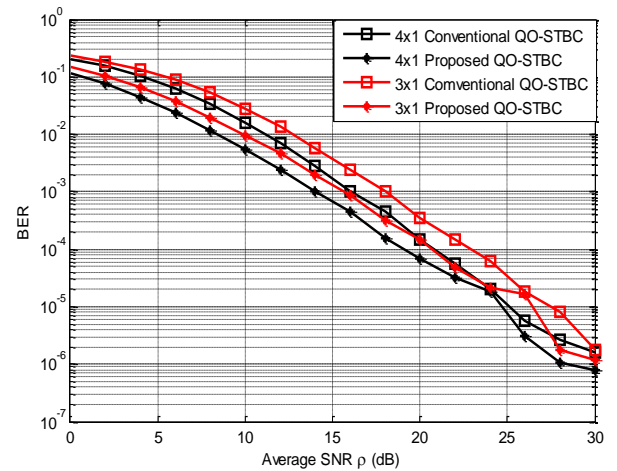
Where  $n=1, 2, \dots, N$  correspond to the rows of  $X_N$ .

#### 5. SIMULATION AND RESULTS

The performance of the proposed schemes in equation (18, 20 and 23) was evaluated over Rayleigh fading channel using a matlab simulation model. The signals were modulated using QPSK, and the total transmit power was divided equally among the number of transmit antennas. The fading was assumed to be constant over four consecutive symbol periods and the channel was known at the receiver.

It should be noted the same data bits and the same mechanism process were used to evaluate the performance of the proposed and conventional schemes.

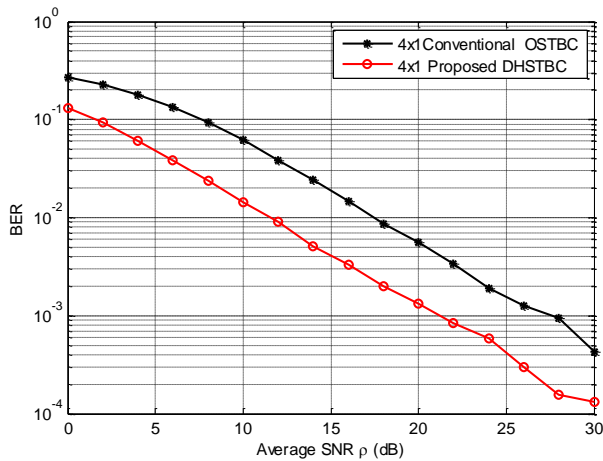
Figure 1 shows the BER performance of the proposed QO-STBC compared to the conventional QO-STBC for four and three transmitter antennas. As it is shown from the Figure 1 that the proposed scheme achieves better performance than the conventional scheme that is about 2dB of additional power gain.



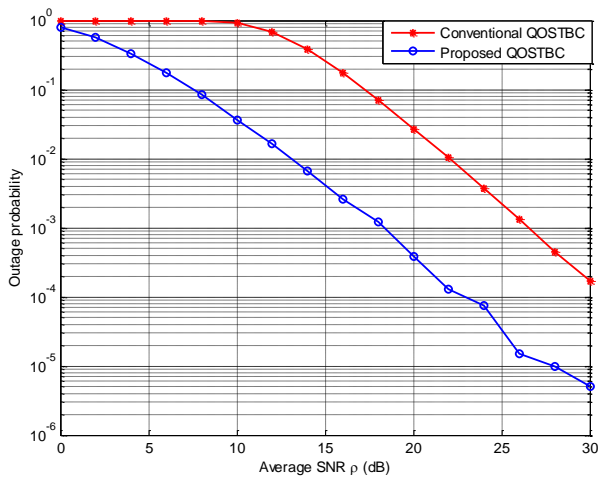
**Figure 1: BER performance of the proposed QO-STBC compared to the conventional QO-STBC.**

Figure 2 shows the performances of the proposed DHSTBC and the conventional OSTBC for four transmit antennas. It is clearly indicated that the proposed scheme achieves better performance than the conventional scheme.

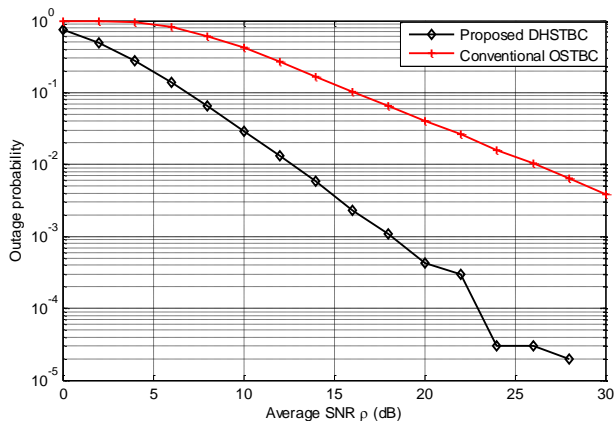
Figure 3 shows the Outage Probability of the proposed QO-STBC compared to the conventional QO-STBC for four transmitter antennas, where it can be observed at high SNR the slope of the probability curves is equal to the diversity gain.



**Figure 2: BER performance of the proposed DHSTBC compared to the conventional OSTBC.**



**Figure 3: Outage Probability proposed QO-STBC compared to the conventional QO-STBC for four transmit antennas.**



**Figure 4: Outage Probability proposed DHSTBC compared to the conventional OSTBC.**

Finally, Figure 4 shows the Outage Probability of the proposed DHSTBC compared to the conventional OSTBC, that indicates the promising results of the proposed method.

## 6. CONCLUSIONS

A new approach for QO-STBC for three and four transmitter antennas schemes was proposed. It was observed that by reforming the detection matrix of the original QO-STBC to eliminate the interference terms, it was able to derive the orthogonal channel matrix that results in simple decoding scheme as in OSTBC. The proposed method has shown better performance than the conventional scheme. In addition, a DiagonalizedHadamard space time code for four transmit antennas was proposed for which the results were shown to be outperformed the conventional one due to the orthogonality of the detection matrix.

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