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# A Multi-Antenna Design Scheme based on Hadamard Matrices for Wireless Communications

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## Abstract

A quasi-orthogonal space time block coding (QO-STBC) scheme that exploits Hadamard matrix properties is studied and evaluated. At first, an analytical solution is derived as an extension of some earlier proposed QO-STBC scheme based on Hadamard matrices, called diagonalized Hadamard space-time block coding (DHSBTC). It explores the ability of Hadamard matrices that can translate into amplitude gains for a multi-antenna system, such as the QO-STBC system, to eliminate some off-diagonal (interference) terms that limit the system performance towards full diversity. This property is used in diagonalizing the decoding matrix of the QO-STBC system without such interfering elements. Results obtained quite agree with the analytical solution and also reflect the full diversity advantage of the proposed QO-STBC system design scheme. Secondly, the study is extended over an interference-free QO-STBC multi-antenna scheme, which does not include the interfering terms in the decoding matrix. Then, following the Hadamard matrix property advantages, the gain obtained (for example, in 4x1 QO-STBC scheme) in this study showed 4-times louder amplitude (gain) than the interference-free QO-STBC and much louder than earlier DHSTBC for which the new approach is compared with.

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# 1 Introduction

Multiple antenna systems are useful in achieving excellent broadband technology [1]. They offer high communication data rate by multiplexing large input data over multiple antenna spaces. The signals while traversing the multipath channel environment exploit the channel paths as gains for some advantage. Since the fading for each transmission branch in a multi-antenna system between a pair of transmit and receive antennas are usually independent, the probability that the information is detected correctly is increased [2]. Thus, the probability that the transmit signal will be decoded correctly increases with the diversity order. In this case, the received signal is quite louder (in amplitude with respect to the diversity order) at the receiver so that better throughput and reliable network service is obtained.

One of the commonest methods of deploying the multiple antenna system is by the use of space time block codes (STBC) [3]. This has been extended using Hadamard matrices [3]. Two transmit antenna design using STBC attains full transmission rate and full diversity [4]. However, beyond the two transmit antenna, STBC scheme does not achieve full transmission diversity since the decoding matrix is not a diagonal matrix [5,6]. Full diversity is a criterion that demonstrates the advantage of a space time code and must show reasonable gain at low bit error ratio (BER) values. It (full diversity) depends on the number of transmit antennas, the number of receive antennas and the rank of a temporal correlation matrix of the channel [5,7]. For multicarrier systems, the full diversity criterion also depends on the number of subcarriers available in the design [5,8].

Orthogonal STBC can as well be layered in a special way for more than two transmit diversity. This is usually discussed as the quasi-orthogonal space-time bloc codes (QO-STBC) [9-11]. QO-STBC is used to design multiple antenna systems with more than two transmit antenna to achieve full transmission rate [9]. The QO-STBC of this nature does not achieve full diversity since the decoding matrix is not a diagonal matrix. Earlier methods for achieving full transmission diversity have been pursued in [5,6,12-15]. Most recently, [6] proposed the use of the Hadamard matrix in achieving an interference-free QO-STBC and was extended in [16] for interference-free processing. The method followed in that study does not attain full diversity even though the Hadamard matrix that permit orthogonal column codes with decoding matrix which is a diagonal matrix of louder amplitude was applied. In this study, a method has been proposed to modify the QO-STBC based on Hadamard matrix earlier proposed in [6] towards achieving full diversity by layering the codes in a special way. It will be found that the results obtained in this study have quite significant improvement as expected of full diversity advantage synchronous to the mathematical characteristics of the Hadamard matrix. The proposed method demonstrate that full diversity require that the system performance gets better at lower BER values which is absent in [6]. This study has been carried out for a quadrature phase shift keying (QPSK) scheme only, and can be extended to other mapping techniques.

In Section 2, the system model used in the study is discussed while the traditional Hadamard approach is discussed in Section 3 with the proposed modification. The conclusion is presented in Section 4.

# 2 System Model

In this section, the QO-STBC system model is discussed. It follows from the linear system model. A system that permits linear decoding must satisfy the full diversity design criterion of a multi-antenna system, such as the QO-STBC.

## 2.1 Channel Model

Let there be (in general) $L$ maximum receive antennas and $J$ maximum transmit antennas (where $J = 4$). Then, the received signal at $t$ time slot and on $l$-receive antenna ($0 \leq l \leq L$-1) can be expressed in a simplified form as:

$$R_{t,l} = \sum_{k=0}^{K-1} \alpha_{k,l}\, S_{t,k} + Z_{t,l} \tag{1}$$

where $S_{t,k}$ is the input signal, $\alpha_{k,l}$ is the path gain of the $k^{\text{th}}$ path and $Z_{t,l}$ is the additive white Gaussian noise (AWGN) with zero mean and necessarily a matrix (vector) of the form [13,17]:

$$Z_{t,l} = \begin{bmatrix} z_1 \\ -z_2^* \\ z_2 \\ z_1^* \end{bmatrix} \tag{2}$$

The path gains are contributed by the frequency responses of the channel, $H$. If the channel matrix is $H$ and the codeword is $\boldsymbol{G}$, the received signal can be expressed as:

$$R_{t,l} = H\boldsymbol{G} + Z \tag{3}$$

In Equation 3, $H$ is the frequency domain content of the channel impulse response defined as [18,19]:

$$h_{t,l} = \sum_{k=0}^{K-1} a_k\, \delta(t - \tau_k(t))\, e^{j\theta_k} \tag{4}$$

The impulse response suggests $k^{th}$-path delay ($\tau_k$) as a function of time discussed, $h_{t,l} \in C^{K\times 1}$ is the channel impulse response of the $k^{\text{th}}$ multipath with $\theta_k$ phase with $a$ -amplitude. If the transmission channel is flat, then $a$ will be uniform for all $K$ paths. For more than one transmit antenna such as $i \in N_T$, each received signal by $l^{th}$-antenna (equivalently $L$=1 in this study) is contributed by;

$$h_{l,i} = \left[h_{t,l}(1), h_{t,l}(2), \cdots, h_{t,l}(N_T)\right]^T, \qquad \begin{array}{c} \forall i = 1, \cdots, 4, \quad i \in N_T \\ L = 1 \end{array} \tag{5}$$

where $(.)^T$ is a transposition. If $H$ is the frequency response of the channel, the transmit signal is $\boldsymbol{G}$, and there are $N_T$-transmit antennas, then the received signal on each branch will be [20];

$$R_{t,l} = \sqrt{\frac{\rho}{N_T}}\,\boldsymbol{G}\,H_{l,i} + \boldsymbol{Z}, \qquad \begin{array}{c} \forall i = 1, \cdots, 4 \\ L = 1 \end{array} \tag{6}$$

For simplicity, $H_{l,i}$ will be replaced by $H_v$ to discuss the equivalent channel matrix, where $\rho$ is the signal to noise ratio (SNR) respective to each transmission branch signal of the codeword $\boldsymbol{G}$ represented as [5];

$$\boldsymbol{G} = \begin{bmatrix} c_1(0) & c_2(0) & \cdots & c_{N_T}(0) \\ c_1(1) & c_2(1) & \cdots & c_{N_T}(1) \\ \vdots & \vdots & \ddots & \vdots \\ c_1(N-1) & c_2(N-1) & \cdots & c_{N_T}(N-1) \end{bmatrix} \tag{7}$$

where $N_T$ is the maximum number of transmit antenna and $N$ is the length of the input symbols. Notice that $\boldsymbol{G} \in C^{N \times N_T}$ matrix such that $C^{N \times N_T}$ is a complex vector of $N_T$-by-$N$ dimension; this is the conventional QO-STBC system. In this study (for 4 transmit antennas and 1 receiver antenna), Equation 7 can be expressed as:

$$\boldsymbol{G} = \begin{bmatrix} g_1 & g_2 & g_3 & g_4 \\ -g_2^* & g_1^* & -g_4^* & g_3^* \\ g_3 & g_4 & g_1 & g_2 \\ -g_4^* & g_3^* & -g_2^* & g_1^* \end{bmatrix} \tag{8}$$

Equation 8 can be decomposed into $G_1$ and $G_2$ to permit maximum-likelihood decoding as [13]:

$$\boldsymbol{G} = G_1(g_1,0,g_3,0) + G_2(0,g_2,0,g_4) \tag{9}$$

This is because, $G_1^H G_2 + G_2^H G_1 = 0$, where $(\cdot)^H$ is the Hermitian operator. Let the equivalent channel matrix be defined as:

$$H_v = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2^* & -h_1^* & h_4^* & -h_3^* \\ h_3 & h_4 & h_1 & h_2 \\ h_4^* & -h_3^* & h_2^* & -h_1^* \end{bmatrix} \tag{10}$$

Now, the decoding method can proceed as:

$$\hat{G} = H_v^H \cdot R = H_v^H H_v \cdot \boldsymbol{G} + H_v^H \cdot Z \tag{11a}$$

Or,

$$\hat{G} = D_4 \cdot \boldsymbol{G} + H_v^H \cdot Z \tag{11b}$$

where $D_4$ is the quasi-orthogonal detection matrix for four transmit antennas and is defined as:

$$D_4 = H_v^H H_v = \begin{bmatrix} \lambda & 0 & \beta & 0 \\ 0 & \lambda & 0 & \beta \\ \beta & 0 & \lambda & 0 \\ 0 & \beta & 0 & \lambda \end{bmatrix} \tag{12}$$

Equation 12 is the typical decoding matrix of a QO-STBC system which is not a diagonal matrix .$\lambda$ is the diagonal of the $(4\times 4)I_4$ matrix which is the sum of the channel power (or the path gains) and represented as $\lambda = \sum_{i=1}^{N_T} \|h_i\|^2, \quad \forall i = 1, \cdots, 4$ . Also, $\beta$ represents the interfering terms that degrade the full diversity performance expected of the 4-transmit antenna elements and is computed as: $\beta = h_1 h_3^* + h_2 h_4^* + h_1^* h_3 + h_2^* h_4$ . Thus, $\beta$ will degrade the BER performance of the system so long as the discussed decoding approach is followed. This can be improved by using a more complex decoding approach such that a better estimate of the transmit symbol can be obtained [6, 17]. Let the effective estimate of the transmit symbols be:

$$\begin{aligned} \breve{G} &= (H_v^H \cdot H_v)^{-1} H_v^H \cdot R \\ &= (H_v^H \cdot H_v)^{-1} D_4 \boldsymbol{G} + (H_v^H \cdot H_v)^{-1} H_v^H \cdot Z \\ &= (H_v^H \cdot H_v)^{-1} H_v^H H_v \boldsymbol{G} + (H_v^H \cdot H_v)^{-1} H_v^H \cdot Z \end{aligned} \tag{13a}$$

So that,

$$\breve{G} = \boldsymbol{G} + (H_v^H \cdot H_v)^{-1} H_v^H \cdot Z \tag{13b}$$

$\breve{G}$ is the effective received symbol after the channel is duly compensated. The result of this method is shown in Fig. 1.

## 2.2 QO-STBC System with Linear Decoding

Equation 12 represents the estimate of the equivalent channel detection matrix and the interference terms also. In orthogonal STBC, the detection matrix, $D$, is always a diagonal matrix, and so enables a simple linear decoding [17]. This is not possible in QO-STBC since the matrix is not orthogonal, instead quasi-orthogonal. The detection matrix in this case is not a diagonal matrix, and so would not permit linear decoding and full diversity cannot be attained. An interference-free QO-STBC decoding matrix (diagonal matrix) can be expressed as [6,12,17]:

$$D = \begin{bmatrix} \lambda+\beta & 0 & 0 & 0 \\ 0 & \lambda+\beta & 0 & 0 \\ 0 & 0 & \lambda-\beta & 0 \\ 0 & 0 & 0 & \lambda-\beta \end{bmatrix} \tag{14}$$

In the next section, we follow the Hadamard matrix approach to eliminate the interference term as in [6].We have attempted similar approach in [16].

# 3 Hadamard Matrix Based QO-STBC System

In this section, the conventional Hadamard QO-STBC system is discussed with the proposed modified Hadamard QO-STBC system. In signal processing, the Hadamard matrix can be thought of as being formed from the traditional orthogonal STBC codes. For instance, let the orthogonal codes of the channel matrix for a two transmitter system be defined as [21];

$$H_2 = \begin{bmatrix} h_1 & h_2 \\ h_2 & -h_1 \end{bmatrix} \tag{15}$$

From Equation 15, the eigenvectors of the matrix can be given as [22];

$$V_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \tag{16}$$

The Hadamard matrices are described as matrices of 1's and -1's entries whose columns are orthogonal. It has the property that [23, 24];

$$H_n H_n^H = H_n^H H_n = n.\boldsymbol{I}_n \tag{17}$$

where $\boldsymbol{I}_n$ is an identity matrix for an $n \times n$ order. In terms of the channel matrix, the diagonal terms of Equation 15 are the channel gains. In [21], a real $n \times n$ matrix $G$ with entries $g_1, -g_1, g_2, -g_2, \ldots, g_N, -g_N$, satisfy the following property[21];

$$G^H G = \left(\sum_{i=1}^{N} g_i^2\right)\boldsymbol{I}_n$$
$$= \lambda \boldsymbol{I}_n \tag{18a}$$

Whereas a complex $n \times n$ matrix $G$ has the form,

$$G^H G = b\left(\sum_{i=1}^{N} g_i^2\right)\boldsymbol{I}_n$$
$$= b(\lambda \boldsymbol{I}_n) \tag{18b}$$

where $b$ is a constant, $\lambda = \sum_{i=1}^{N} g_i^2$, $G^H$ is the Hermitian of the matrix $G$ and $\boldsymbol{I}_n$ is the $n \times n$ identity matrix.

In that case, Equation 17 can be discussed that the gain is amplified *n*-times. Since the Hadamard matrix is defined for $rem(n,4)=0$, then in this study where $n=4$, the gain is amplified by 4. We then characterize a 4 transmit element system (with one receiver) following Equation 16as;

$$V_4 = \begin{bmatrix} V_2 & V_2 \\ V_2 & -V_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \tag{19}$$

Now, Equation 19 can be extended to the decoding matrix of QO-STBC systems to linearize the decoding matrix. However, recall that the codes that construct QO-STBC are not orthogonal, instead quasi-orthogonal. Only the codes that construct the orthogonal STBC are orthogonal. But the columns of the QO-STBC system demonstrate the orthogonal characteristics. Constructing the decoding matrix according to a Hadamard matrix explicitly eliminates the interfering terms so that exact full diversity will be achieved. The Hadamard matrix is applied in the design of a QO-STBC that permit linear decoding using the following eigenvectors for 4×4 matrix:

$$V_{Had} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \tag{20}$$

Equation 20 is the Hadamard matrix expected to volunteer the diagonalization property for which the QO-STBC design will attain full diversity. It is expected to eliminate the off-diagonal elements of the decoding matrix of QO-STBC system.

## 3.1 The Diagonalized Hadamard QSTBC

The diagonalized Hadamard matrix was used to propose the diagonalized Hadamard space time block coding (DHSTBC) in [6]. The codes are formed such as [5,6];

$$S = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2 & s_1 & s_4 & s_3 \\ s_3 & s_4 & s_1 & s_2 \\ s_4 & s_3 & s_2 & s_1 \end{bmatrix} \tag{21}$$

Equation 21 is for 4 transmit antennas with entries $\begin{bmatrix} s_1, & s_2, & \cdots & s_{N_T} \end{bmatrix} \in C^{N_T \times (N/N_T)}$ where $C^{N_T \times (N/N_T)}$ is a complex vector matrix of $N_T \times (N/N_T)$ dimension and $N$ is the length of the input symbols.

Recall the Hadamard matrix of order four defined in Equation 20; by combining Equation 20 and 21:

$$S_{Had} = V_{Had} \times S$$
$$= \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ s_2 & s_1 & s_4 & s_3 \\ s_3 & s_4 & s_1 & s_2 \\ s_4 & s_2 & s_2 & s_1 \end{bmatrix} \tag{22a}$$

which is equivalent to;

$$S_{Had} = \begin{bmatrix} s_1+s_2+s_3+s_4 & s_1+s_2+s_3+s_4 & s_1+s_2+s_3+s_4 & s_1+s_2+s_3+s_4 \\ s_1-s_2+s_3-s_4 & s_2-s_1-s_3+s_4 & s_1-s_2+s_3-s_4 & s_2-s_1-s_3+s_4 \\ s_1+s_2-s_3-s_4 & s_1+s_2-s_3-s_4 & s_3-s_2-s_1+s_4 & s_3-s_2-s_1-s_4 \\ s_1-s_2-s_3+s_4 & s_2-s_1+s_3-s_4 & s_2-s_1+s_3-s_4 & s_1-s_2-s_3+s_4 \end{bmatrix} \tag{22b}$$

Equation 22 is the encoding matrix according to the Hadamard criteria that designs a QO-STBC code. This matrix has the property similar to Equation 15, such that;

$$(S_{Had})^H . S_{Had} = \begin{bmatrix} 4\beta_1 & 0 & 0 & 0 \\ 0 & 4\beta_2 & 0 & 0 \\ 0 & 0 & 4\beta_3 & 0 \\ 0 & 0 & 0 & 4\beta_4 \end{bmatrix} \tag{23a}$$

where,

$$\begin{aligned} \beta_1 &= (s_1 + s_2 + s_3 + s_4)*(s_1^* + s_2^* + s_3^* + s_4^*) \\ \beta_2 &= (s_1^* - s_2^* + s_3^* - s_4^*)*(s_1 - s_2 + s_3 - s_4) \\ \beta_3 &= (s_1^* + s_2^* - s_3^* - s_4^*)*(s_1 + s_2 - s_3 - s_4) \\ \beta_4 &= (s_1^* - s_2^* - s_3^* + s_4^*)*(s_1 - s_2 - s_3 + s_4) \end{aligned} \tag{23b}$$

This technique constructs quasi-orthogonal space time block codes such that the Hadamard property ($S^H S$) of Equation 17 does not completely exploit the gain to achieve full diversity as in the interference-free QO-STBC form (i.e. Equation 14) as shown in Equation 23or in [16]. Also, Equation 23 is expected to be 4-times louder than Equation 14. Consequently, this method of constructing QO-STBC using the Hadamard matrix does not achieve full diversity.

If the equivalent channel matrix of Equation 21 is constructed as;

$$H_{Had} = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2 & h_1 & h_4 & h_3 \\ h_3 & h_4 & h_1 & h_2 \\ h_4 & h_3 & h_2 & h_1 \end{bmatrix} \tag{24}$$

Thus, the equivalent Hadamard matrix to be combined with Equation 24 will be:

$$V = \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix} \tag{25}$$

By combining Equations 24 and 25, the channel matrix can be constructed as;

$$\begin{aligned} H_{vHad} &= V.H_{Had} \\ &= \begin{bmatrix} 1 & 1 & 1 & 1 \\ -1 & 1 & -1 & 1 \\ -1 & -1 & 1 & 1 \\ -1 & 1 & -1 & 1 \end{bmatrix} \times \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ h_2 & h_1 & h_4 & h_3 \\ h_3 & h_4 & h_1 & h_2 \\ h_4 & h_3 & h_2 & h_1 \end{bmatrix} \end{aligned} \tag{26a}$$

Equation 26a (when expanded) is equivalent to:

$$H_{vHad} = \begin{bmatrix} h_1+h_2+h_3+h_4 & h_1+h_2+h_3+h_4 & h_1+h_2+h_3+h_4 & h_1+h_2+h_3+h_4 \\ h_2-h_1-h_3+h_4 & h_1-h_2+h_3-h_4 & h_2-h_1-h_3+h_4 & h_1-h_2+h_3-h_4 \\ h_1+h_2-h_3-h_4 & h_1+h_2-h_3-h_4 & h_3-h_2-h_1+h_4 & h_3-h_2-h_1+h_4 \\ h_2-h_1+h_3-h_4 & h_1-h_2-h_3+h_4 & h_1-h_2-h_3+h_4 & h_2-h_1+h_3-h_4 \end{bmatrix} \tag{26b}$$

Consequently,

$$(H_{vHad})^H.H_{vHad} = \begin{bmatrix} 4\beta_1 & 0 & 0 & 0 \\ 0 & 4\beta_2 & 0 & 0 \\ 0 & 0 & 4\beta_3 & 0 \\ 0 & 0 & 0 & 4\beta_4 \end{bmatrix} \tag{27}$$

Using numerical simulation, a QPSK system for 4×1 QO-STBC is used to demonstrate the performance of the traditional QO-STBC, the interference-free QO-STBC [6,12,17] labelled (Dama-QOSTBC) and the earlier proposed QO-STBC based on Hadamard matrix (DHSTBC) [6] in Fig. 1. Notice that Fig. 1 compares the performance of the code words in Equation 22 with both the interference-free and traditional QO-STBC. All results have been presented using MATLAB.

In Fig. 1, it can be observed clearly that the earlier DHTSBC (depicted as old DHSTBC) performed similarly with interference-free QO-STBC (DamaQO-STBC) scheme at 0dB. Also, the DHSTBC performed up to 2dB better than the traditional QO-STBC at low bit energy to noise power ratio (EBN0) values. Meanwhile, beyond the low EBN0 values the inefficiency of the DHSTBC scheme becomes pronounced such that maximum diversity is not experienced at lower BER. It is a requirement that the full diversity must be as large as possible [5,20]. This (full diversity) dependent on the upper bound (or the *rank*) of a pairwise error probability statistic of two different codewords belong to the transmit and the corresponding received codewords [20] which the DHSTC scheme has not exploited.

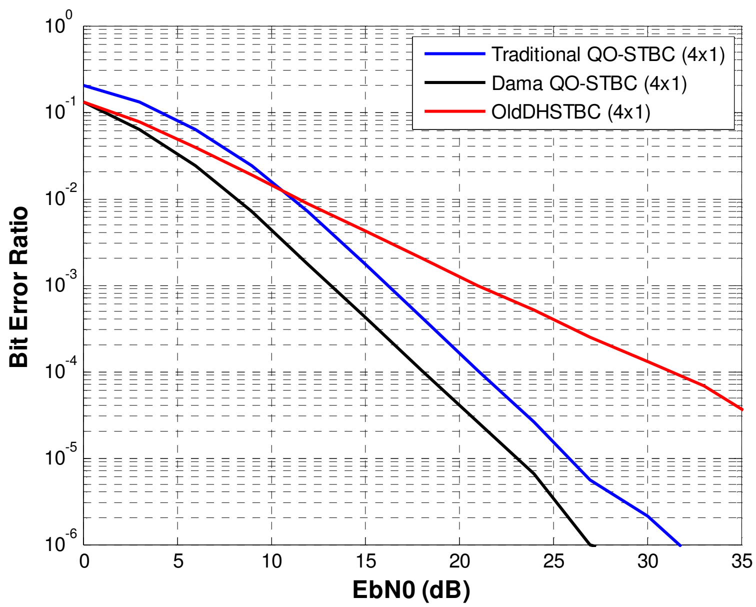


**Fig. 1. Comparison of traditional QO-STBC, Interference-free QO-STBC and Dama-DHSTBC for 4 x 1 MISO System**

## 3.2 Proposed (Improved) QO-STBC

In this section, the QO-STBC system that satisfies the full diversity criterion is constructed. It follows from the Hadamard matrix idea earlier proposed in [6] and is discussed in Section 3.1. Let the channel matrix be constructed as:

$$h_{12} = \begin{bmatrix} h_1 & h_2 \\ h_2 & h_1 \end{bmatrix} \text{ and } h_{34} = \begin{bmatrix} h_3 & h_4 \\ h_4 & h_3 \end{bmatrix} \tag{28a}$$

Then, let the channel matrix of order four according to the Hadamard criterion be:

$$H = \begin{bmatrix} h_{12} & h_{34} \\ -h_{34} & h_{12} \end{bmatrix} \tag{28b}$$

Equation 28 can be thought of, as being orthogonal, although the internal elements can be thought of as being quasi-orthogonal. Also, let the equivalent Hadamard matrix be:

$$V = \begin{bmatrix} v_2 & v_2 \\ -v_2 & v_2 \end{bmatrix} \tag{29a}$$

where,

$$v_2 = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \tag{29b}$$

Combining Equation 29a and Equation 28b such as:

$$H_v = \begin{bmatrix} h_{12} & h_{34} \\ -h_{34} & h_{12} \end{bmatrix} \times \begin{bmatrix} V_2 & V_2 \\ -V_2 & V_2 \end{bmatrix} \tag{30a}$$

This is equivalent to:

$$H_v = \begin{bmatrix} h_1-h_2-h_3+h_4 & h_1+h_2-h_3-h_4 & h_1-h_2+h_3-h_4 & h_1+h_2+h_3+h_4 \\ h_2-h_1+h_3-h_4 & h_1+h_2-h_3-h_4 & h_2-h_1-h_3+h_4 & h_1+h_2+h_3+h_4 \\ h_2-h_1-h_3+h_4 & -h_1-h_2-h_3-h_4 & h_1-h_2-h_3+h_4 & h_1+h_2-h_3-h_4 \\ h_1-h_2+h_3-h_4 & -h_1-h_2-h_3-h_4 & h_2-h_1+h_3-h_4 & h_1+h_2-h_3-h_4 \end{bmatrix} \tag{30b}$$

Recall Equation 18b for complex entries of a matrix, and then by applying the Hadamard property of Equation 17:

$$Q_4 = H_v^H H_v = \begin{bmatrix} 4(\lambda+\beta) & 0 & \alpha & 0 \\ 0 & 4(\lambda+\beta) & 0 & \alpha \\ \alpha & 0 & 4(\lambda-\beta) & 0 \\ 0 & \alpha & 0 & 4(\lambda-\beta) \end{bmatrix} \tag{31}$$

where, $\beta = h_1 h_3^* + h_2 h_4^* + h_1^* h_3 + h_2^* h_4$ , $\lambda = \sum_{n=1}^{N} \|h_n\|^2, \forall n = 1, \cdots, 4$ and $\alpha = 4(h_3 h_1^* - h_1 h_3^* + h_1 h_4^* + h_2 h_3^* - h_3 h_2^* - h_4 h_1^* - h_2 h_4^* + h_4 h_2^*)$ . Meanwhile, notice that in Equation 31, there are some off-diagonal terms (which are necessarily interfering terms,$\alpha$) that may degrade the performance of the QO-STBC scheme. Since the diagonal of the decoding matrix (in Equation 31) already satisfied the interference-free statistic of Equation 14 although 4-times louder (according to the Hadamard criteria of Equation 17), the rest interfering terms can be nulled as follows:

$$Q = Q_4 \otimes V_{null} \tag{32}$$

where $\otimes$ is a kronecker multiplicative operator and $V_{null}$ is defined as:

$$V_{null} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{33}$$

Consequently, Equation 33 will null the interfering terms so that the Hadamard multiplication criterion of Equation 17 is attained and fully diversity exploited. Thus, Equation 32 can be expressed as:

$$Q=(H_v^H H_v)\otimes V_{null}=\begin{bmatrix} 4(\lambda+\beta) & 0 & 0 & 0 \\ 0 & 4(\lambda+\beta) & 0 & 0 \\ 0 & 0 & 4(\lambda-\beta) & 0 \\ 0 & 0 & 0 & 4(\lambda-\beta) \end{bmatrix} \tag{34}$$

where $\lambda=\sum_1^N \|h_n\|^2, \forall n=1,\cdots,4$ and $\beta=h_1h_3^*+h_2h_4^*+h_1^*h_3+h_2^*h_4$

Notice that Equation 34 yields full diversity and is 4-times louder in amplitude compared to the interference-free QO-STBC which has a diagonal matrix. Also the Hadamard property of Equation 17 defined as $(H_v)^H H_v = n\boldsymbol{I}_n$ is well satisfied.

The equivalent encoding matrix for the four transmit element can be formed as:

$$S_{Had}=\begin{bmatrix} s_1+s_2+s_3+s_4 & s_1+s_2+s_3+s_4 & s_3-s_2-s_1+s_4 & s_3-s_2-s_1+s_4 \\ s_2-s_1-s_3+s_4 & s_1-s_2+s_3-s_4 & s_1-s_2-s_3+s_4 & s_2+s_1+s_3-s_4 \\ s_1+s_2-s_3-s_4 & s_1+s_2-s_3-s_4 & s_1+s_2+s_3+s_4 & s_1+s_2+s_3+s_4 \\ s_2-s_1+s_3-s_4 & s_1-s_2-s_3+s_4 & s_2-s_1-s_3+s_4 & s_1-s_2+s_3-s_4 \end{bmatrix} \tag{35}$$

For three transmit elements, the fourth antenna element is nulled in $h_{34}$ such that the term $h_{34}$ in Equation 30, becomes:

$$h_{34}=\begin{bmatrix} h_3 & 0 \\ 0 & h_3 \end{bmatrix} \tag{36}$$

Using Equations 30 and 36, the three transmit element channel matrix can be formed as:

$$H_{3v}=\begin{bmatrix} h_1-h_2-h_3 & h_1+h_2-h_3 & h_1-h_2+h_3 & h_1+h_2+h_3 \\ h_2-h_1+h_3 & h_1+h_2-h_3 & h_2-h_1-h_3 & h_1+h_2+h_3 \\ h_2-h_1-h_3 & -h_1-h_2-h_3 & h_1-h_2-h_3 & h_1+h_2-h_3 \\ h_1-h_2+h_3 & -h_1-h_2-h_3 & h_2-h_1+h_3 & h_1+h_2-h_3 \end{bmatrix} \tag{37}$$

The equivalent encoding symbols can be easily formed by the foregoing discussion. In Fig. 2, the results of the traditional QO-STBC, the interference-free QO-STBC (designated as Dama QO-STBC), the earlier DHSTBC (designated as oldDHSTBC) and the proposed QO-STBC (designated as new DHSTBC) are compared for four-antenna and 3-antenna elements.

From the results in Fig. 3, it is noticed that even the free-interference QO-STBC in [6] well outperformed the old DHSTBC. However, the proposed QO-STBC outperforms the free-interference QO-STBC circa 4-dB as suggested by the numerical equations.

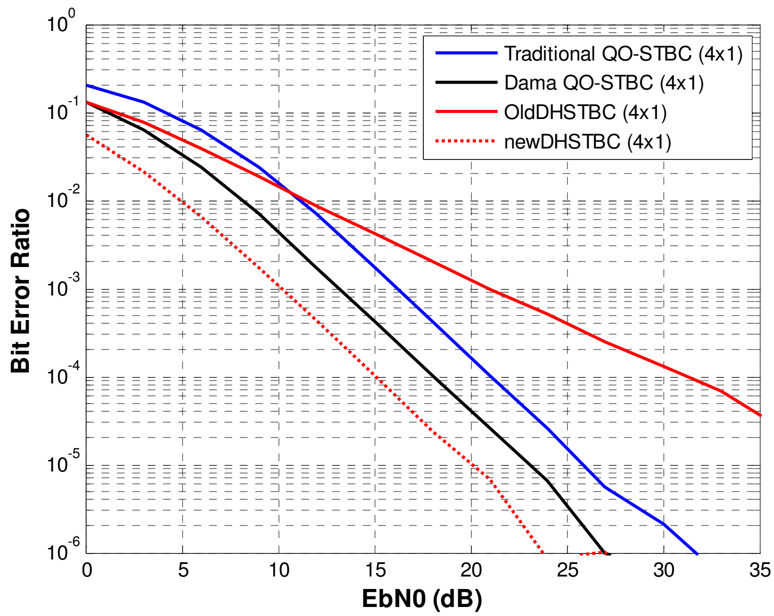


**Fig. 2. Comparison of new QO-STBC with traditional QO-STBC (4x1)**

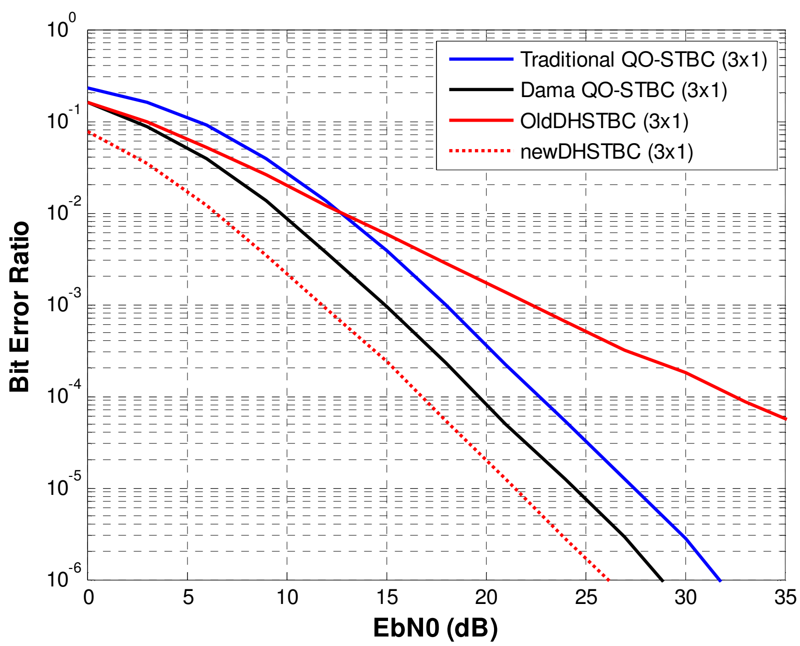


**Fig. 3. Comparison of new QO-STBC with traditional QO-STBC (3x1)**

However, it can be seen that the mathematical description of the new QO-STBC is very evident with about 4-times louder amplitude than the interference-free QO-STBC. This demonstrates the full diversity advantage plus the amplitude gain volunteered by the Hadamard matrix discussed in this work. In comparison to the old DHSTBC, it can be seen that new DHSTBC outperformed the Dama-approach from 0dB EBN0. Thenceforth, the new QO-STBC (which is not limited by non-diversity advantage exploitation) maximally outperformed the old DHSTBC up to 15dB at $10^{-4}$ BER. Notice that this result is consistent in behaviour for 4x1 QO-STBC shown in Fig. 2 and 3x1 QO-STBC also shown in Fig. 3.

# 3. Conclusion

In this paper, the Hadamard based QO-STBC has been presented. Such matrix which is widely used in engineering is shown to be an important matrix in this study. It was discussed as a matrix of 1's and -1's. An earlier method for using the scheme for designing QO-STBC to achieve full diversity has been modified. This was studied for three-transmit antenna QO-STBC and four transmit antenna QO-STBC systems. Following mathematical proofs, results obtained showed *n*-time louder amplitude gain (*n* is number of transmit antennas considered) when the Hadamard matrix was used in designing QO-STBC than the interference elimination approach. That was a significant improvement compared to the conventional QO-STBC. This scheme achieves full diversity and as well satisfies the Hadamard matrix properties.

# Competing Interests

Authors have declared that no competing interests exist.

# References

[1] Ghosh A, Ratasuk R, Mondal B, Mangalvedhe N, Thomas T. LTE-advanced: next-generation wireless broadband technology [Invited Paper]. IEEEWireless Communications. 2010;17:10-22.

[2] Zhang W, Xiang-Gen X, Ben Letaief K.. Space-time/frequency coding for MIMO-OFDM in next generation broadband wireless systems. IEEE Wireless Communications. 2007;14:32-43.

[3] Anoh K, Ochonogor O, Abd-Alhameed R, Jones S, Mapuka T. Improved Alamouti STBC multi-antenna system using hadamard matrices. Int'l J. of Communications, Network and System Sciences. 2014;2014.

[4] Alamouti SM. A simple transmit diversity technique for wireless communications. IEEE Journal on Selected Areas in Communications. 1998;16:1451-1458.

[5] Fazel F, Jafarkhani H. Quasi-orthogonal space-frequency and space-time-frequency block codes for MIMO OFDM channels. IEEE Transactions on Wireless Communications. 2008;7:184-192.

[6] Dama Y, Abd-Alhameed R, Ghazaany T, Zhu S. A New Approach for OSTBC and QOSTBC. International Journal of Computer Applications. 2013;67:45-48.

[7] Su W, Safar Z, Liu KR. Towards maximum achievable diversity in space, time, and frequency: performance analysis and code design. IEEE Transactions on Wireless Communications. 2005;4:1847-1857.

[8] Lu B, Wang X. Space-time code design in OFDM systems, in IEEEGlobal Telecommunications Conference, 2000. GLOBECOM'00. 2000:1000-1004.

[9] Jafarkhani H. A quasi-orthogonal space-time block code. IEEE Transactions on Communications. 2001;49:1-4,.

[10] Jafarkhani H, Tarokh V. Multiple transmit antenna differential detection from generalized orthogonal designs. IEEE Transactions onInformation Theory. 2001;47:2626-2631.

[11] Tirkkonen O, Boariu A, Hottinen A. Minimal non-orthogonality rate 1 space-time block code for 3+ Tx antennas, in 2000 IEEE Sixth International Symposium on Spread Spectrum Techniques and Applications. 2000:429-432.

[12] Dama YAS, Abd-Alhameed RA, Jones SMR, Migdadi HSO, Excell PS. A new approach to quasi-orthogonal space-time block coding applied to quadruple mimo transmit antennas. Fourth International Conference on Internet Technologies & Applications. 2011:2011.

[13] Sharma N, Papadias CB. Improved quasi-orthogonal codes through constellation rotation. IEEE Transactions onCommunications. 2003;51:332-335.

[14] Wang D, Xia X.-G. Optimal diversity product rotations for quasiorthogonal STBC with MPSK symbols. IEEE Communications Letters. 2005;9:420-422.

[15] Xian L, Liu H. Optimal rotation angles for quasi-orthogonal space-time codes with PSK modulation. IEEE Communications letters. 2005;9:676-678.

[16] Anoh K, Dama Y, Abd-Alhameed R, Jones S. A Simplified Improvement on the Design of QO-STBC Based on Hadamard Matrices. Int'l J. of Communications, Network and System Sciences. 2014;7:37.

[17] Park U, Kim S, Lim K, Li J. A novel QO-STBC scheme with linear decoding for three and four transmit antennas. IEEE Communications Letters. 2008;12:868-870.

[18] Parsons JD, Parsons PJD. The mobile radio propagation channel, vol. 2: Wiley New York; 2000.

[19] Tse D, Viswanath P. Fundamentals of wireless communication: Cambridge University Press; 2005.

[20] Su W, Safar Z, Olfat M, Liu KR. Obtaining full-diversity space-frequency codes from space-time codes via mapping. IEEE Transactions on Signal Processing. 2003;51:2905-2916.

[21] Proakis J, Salehi M. Digital Communications, Fifth ed. Asia: McGraw-Hill; 2008.

[22] Horadam KJ. Hadamard matrices and their applications: Princeton University Press; 2007.

[23] Seberry J, Wysocki BJ, Wysocki TA. On some applications of Hadamard matrices. Metrika. 2005;62:221-239.

[24] Hedayat A, Wallis W. Hadamard matrices and their applications. The Annals of Statistics. 1978;6:1184-1238.

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