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ORIGINAL PAPER

A linear programming model to optimize the decision-making to managing cogeneration system

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Abstract A mathematical linear programming (LP) model was developed to optimize the decision-making for managing a cogeneration facility as a potential clean-development mechanism project in a hospital in Palestine. The model was developed to optimize the cost of energy and the cost of installation of a small cogeneration plant under constraints on electricity-and-heat supply and demand balances. In the model, the sources of electricity are either from cogeneration or public utilities and it was calculated the least cost to supply electricity and heat to the hospital. The hospital is using heat for their operation and that made the application for the cogeneration to be attractive and feasible. In this study, we will develop the LP model and will show the results and the time schedule for the cogeneration. This developed LP model can be used and run to any cogeneration application with little modification.

Keywords CHP · Palestine · Hospital · Linear programming · Cogeneration

Introduction

Cogeneration is a high-efficiency energy system that produces both electricity (and mechanical power) and valuable heat from a single fuel source. Cogeneration is sometimes known as ‘‘combined heat and power’’, or CHP. It offers major economic and environmental benefits because it turns otherwise wasted heat into a useful energy source. Many industries are using CHP for generating power and meeting the process heat (Tsay and Lin 2000). This greater efficiency means carbon dioxide emissions are cut up by to two thirds when compared with conventional coal-fired power stations. Cogeneration offers an effective method of utilizing the energy from a single primary source, such as coal, oil, natural gas, or biomass fuels, to produce heat and power (Hepbasli and Ozalp 2002).

Businesses, government agencies and facilities most likely to benefit from cogeneration are those that use large quantities of hot water, heat, steam or chilling such as hospitals, large maintenance facilities, aquatic centers and laundries. Others to benefit from cogeneration include hotels, office buildings, food and beverage processors, chemicals and plastics producers, pulp, paper and fiberboard manufacturers, metals processors, textile producers, shopping centers and universities.

Cogeneration recovers the waste heat that is always produced in electricity generation and puts it to good use rather than letting it escape into the atmosphere. In conventional power generation, about two thirds of the energy input is wasted in this way. CHP can recover the majority of this waste heat, creating a far better use of resources and cost savings, and resulting in energy savings of between 20 and 40%. For example, a hospital cogeneration plant could produce some of the power and all the hot water needed for its laundry and hot water system from the waste heat it generates. Similarly, office buildings can produce power for

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electricity and air conditioning from the waste heat generated by its air conditioning engines.

Cogeneration contributes to a more sustainable energy future by minimizing the environmental damage resulting from commercial activities (June Wu and Rosen 1999). The most obvious and well-known benefit is the carbon dioxide savings. On balance, cogeneration results in savings of up to 50% of CO_2 emissions compared with conventional sources of heat and power. Reduced emissions of sulphur dioxide and particulates are further benefits. To achieve these benefits a direct practical help is required (Brown 1996). A well-designed and operated cogeneration plant will always improve energy efficiency and significantly reduce carbon emissions. With a typical supply efficiency of between 70 and 95%, cogeneration is the best all-round solution for the electricity and heat production sector.

Cogeneration provides the most cost-effective option for producing electricity when the savings from heat utilization are taken into account. In countries where a more liberalized electricity market values cogenerated electricity, cogeneration can usually be developed more freely than in markets where regulated tariffs are set.

Optimal models of small cogeneration plant were developed and used to design investment and operation strategies for small CHP plants (Lund and Andersen 2005). These models are used to optimize performance in a market with fluctuating electricity prices. The feasibility of applying CHP to individual homes in UK had showed a saving of 16–39% in annual energy and costs (Newborough 2004). A case study of the installation of a combined CHP facility as a potential clean-development mechanism project in an industrial area in China was studied using a developed mathematical programming model. The model was developed to optimize the installation capacity of the CHP under constraints on electricity-and-heat supply and demand balances. Energy cost and emissions of CO_2 and SO_x were also calculated with the model (Kosugi et al. 2005). A linear programming model was designed to determine the optimal unit sizing of CHP systems in consideration of uncertain energy demands as continuous random variables. The decision variables and the objective function were considered as piecewise linear functions of energy demands by applying a sensitivity analysis in linear programming and an enumeration method in a mixed-integer programming.

This optimization problem was formulated and solved based on a hierarchical optimization algorithm (Gamou et al. 2002). A robust optimal design method was developed to conduct the unit sizing of a cogeneration plant, so that they are robust economically under uncertain energy demands. The values of design variables or equipment capacities, as well as those of operation variables or utility contract demands and energy flow rates, are determined to minimize the maximum normalized regret or, the maximum regret rate in the annual total cost and satisfy all the possible energy demands. This optimization problem was formulated as a multi-level nonlinear programming problem, and its solution was obtained by repeatedly evaluating the upper and lower bounds for the optimal value of the maximum regret rate by means of the fractional, the bi-level and the linear programming (Ito and Yokoyama 2002).

In many Swedish municipalities, there are district heating networks, which are quite commonly supplied by combined heat and power plants (CHP). A mixed integer linear-programming model was constructed to minimize the total cost of supplying electricity and heat to one of the municipalities. The idea was that heat storage can be used to maximise the amount of electricity produced in the CHP plants during peak-price periods. It can also be used to minimise the use of plants with higher operational costs (Rolfsman 2004). In order to provide space heating and domestic hot water, investments could be made on the supply side in power plants. The electricity from the CHP plants is supplied to the municipality but can also be sold to the electricity market, and electricity can, of course, also be bought from the market. The energy system in the Swedish case study was analysed with a mixed-integer linear programming model. The model has 3 h time steps in order to reflect diurnal variations, and an entire year is analysed (Rolfsman 2004).

An optimal design of cogeneration systems by using Hamiltonian algorithm (HA) was studied by many researchers. The HA is efficient to optimize to design and control the complex systems. This method enables the CHP to be optimized in terms of the payback period without the system operation rules following up the electric-power and heat demand (Yangi et al. 1999). A simple linear programming model was developed to determine the optimal strategies that minimize the overall cost of energy for the combined cooling, heating and power production system. The system consists of a gas turbine, an absorption chillers and a heat recovery boiler. The combined cooling, heating and power production system allows a facility to generate its own power and use rejected heat from the turbine to run an absorption chillers or a heat recovery boiler to handle the possible cooling or heating loads. The operational variables in the model are the turbine load fraction and the turbine exhaust heat fraction, which are divided up evenly between the heating and cooling loads (Kong et al. 2004).

Methodology

The aim of the research work is to develop an integrated methodology that can be used to optimize the decision-making for managing cogeneration system in a facility. Linear programming (LP) as a tool of the optimization techniques has been used in a wide range of applications. In this research LP model will be developed aiming to optimize the energy consumption in a facility, it is under the energy management concept which endeavour to conserve the energy and minimize its costs.

The model

The model could be validated to any facility that is qualified of the cogeneration system. By agreeing on the demand of electricity and heat the model produces the optimum solution, which gives the optimum way of using the cogeneration system and other sources of electricity or heat if required.

The decision variables are the amount of energy that is needed or will be exhausted in the system, and KWh is its unit.

Linear programming formulation

The first step in formulating the mathematical programming model is to identify the variables included in the model. These are the decision variables that capture the amount of the KWh required form each source, and the parameters that represent the prices/costs. Therefore, the decision variables are:

Cogeneration systems (Xijk):

i = cogeneration system (1, 2 and 3), where 1 is the gas turbine type, 2 the combined gas/steam turbine type and 3 is the diesel engine type;
j = H or P, where H is the heat and P is the power;
k = S or W, where S is the summer and W is the winter, e.g. X1HS is the heat from the gas turbine in summer season.

Electricity:

X4W is the electricity from the grid in winter;
X4S is the electricity from the grid in summer.
Heat:

X5W is the heat from boilers in winter;
X5S is the heat from boilers in summer.
The parameters:

egw is the electricity cost from the grid in winter;
egs is the electricity cost from the grid in summer;
hcw is the heat cost from the boiler in winter;
hcs is the heat cost from the boiler in summer;
Cij is the Cost from cogeneration system i (i = cogeneration system (1, 2 and 3), where 1 is the gas turbine type, 2 the combined gas/steam turbine type and 3 is the diesel engine type. J is the heat or power).

The objective of the model is to minimize the total annual amount of energy consumed at the enterprise, which means minimizing the annual costs of energy and the annual pollution because of the emissions resulted from generating various types of energy:

$$\min\left(\sum_{i=1}^{3}\mathrm{Cih}(\mathrm{XiHS}+\mathrm{XiHW})+\mathrm{Cip}(\mathrm{XiPS}+\mathrm{XiPW})\right)$$
$$+\,\mathrm{egw}\,\mathrm{X4W}+\mathrm{egs}\,\mathrm{X4S}+\mathrm{hcw}\,\mathrm{X5W}+\mathrm{hcs}\,\mathrm{X5S}.$$

There are several relationships that limit the values that can be taken on by the decision output variables. These constraints are categorized as follows.

Energy demand

Heat/power demand in KWh must be accounted in seasons, winter and summer:

$$\left(\sum_{i=1}^{3}\mathrm{XiWS}\right)+\mathrm{X5W}\geq\text{Heat demand in winter (HDW)}$$

$$\left(\sum_{i=1}^{3}\mathrm{XiHS}\right)+\mathrm{X5S}\geq\text{Heat demand in summer (HDS)}$$

$$\left(\sum_{i=1}^{3}\mathrm{XiPW}\right)+\mathrm{X4W}\geq\text{Power demand in winter (PDW)}$$

$$\left(\sum_{i=1}^{3}\mathrm{XiPS}\right)+\mathrm{X4S}\geq\text{Power demand in summer (PDS)}.$$

Cogeneration performance restrictions

The cogenerations systems have specific performance in producing the heat and power as follows, e.g. gas turbine system output is 60% heat and 40% electricity so an equation like this can represent this ratio in the model ($X_1PS = 0.6X_1HS$). And so for the others:

$$X_1PW - 0.6X_1HW = 0,$$
$$X_2PS - 0.89X_2HS = 0,$$
$$X_2PW - 0.89X_2HW = 0,$$
$$X_3PS - 0.89X_3HS = 0,$$
$$X_3PW - 0.89X_3HW = 0.$$

Conflicts

The model will choose the best system that fulfills the objective function. For example, the heat must be fulfilled from one source/system. The equations below meet these requirements:

$$\mathrm{X4W} - \mathrm{PDW\,y1} = 0,$$
$$\mathrm{X1PW} - \mathrm{PDW\,y2} = 0,$$
$$\mathrm{X2PW} - \mathrm{PDW\,y3} = 0,$$
$$\mathrm{X3PW} - \mathrm{PDW\,y4} = 0,$$
$$\mathrm{y1} + \mathrm{y2} + \mathrm{y3} + \mathrm{y4} = 1,$$
$$\mathrm{X4S} - \mathrm{PDS\,j1} = 0,$$
$$\mathrm{X1PS} - \mathrm{PDS\,j2} = 0,$$
$$\mathrm{X2PS} - \mathrm{PDS\,j3} = 0,$$
$$\mathrm{X3PS} - \mathrm{PDS\,j4} = 0,$$
$$\mathrm{j1} + \mathrm{j2} + \mathrm{j3} + \mathrm{j4} = 1.$$

The above equations should be replaced by the following ones, and then the solution that gives the minimum value is chosen:

$$X_5W - HDW\,y_1 = 0,$$
$$X_1HW - HDW\,y_2 = 0,$$
$$X_2HW - HDW\,y_3 = 0,$$
$$X_3HW - HDW\,y_4 = 0,$$
$$y_1 + y_2 + y_3 + y_4 = 1,$$
$$X_5S - HDS\,j_1 = 0,$$
$$X_1HS - HDS\,j_2 = 0,$$
$$X_2HS - HDS\,j_3 = 0,$$
$$X_3HS - HDS\,j_4 = 0,$$
$$j_1 + j_2 + j_3 + j_4 = 1.$$

Replacing the set of the previous equations aims at checking two cases; the first one when the user prefers to have excess in heat, and the second when he/she needs excess in electricity.

Season restrictions

Another relationship; the same system should be used in winter and summer:

$$y_1 - j_1 = 0,$$
$$y_2 - j_2 = 0,$$
$$y_3 - j_3 = 0,$$
$$y_4 - j_4 = 0.$$

Adding non-negativity constraints for each of the decision variables yield the final formulation:

$$\min\left(\sum_{\mathrm{i=1}}^{3}\mathrm{Cih(XiHS + Xi\,HW) + Cip(Xi\,PS + XiPW)}\right) + \mathrm{egw}X_4W + \mathrm{egs}X_4S + \mathrm{hcw}X_5W + \mathrm{hcs}X_5S,$$
$$\text{subject to:}\ \left(\sum_{\mathrm{i=1}}^{3}\mathrm{XiWS}\right) + X_5W \geq HDW,$$
$$\left(\sum_{\mathrm{i=1}}^{3}\mathrm{XiHS}\right) + X_5S \geq HDS,$$
$$\left(\sum_{\mathrm{i=1}}^{3}\mathrm{XiPW}\right) + X_4W \geq PDW,$$
$$\left(\sum_{\mathrm{i=1}}^{3}X_iPS\right) + X_4S \geq PDS,$$
$$X_1PS - 0.6X_1HS = 0,$$
$$X_1PW - 0.6X_1HW = 0,$$
$$X_2PS - 0.89X_2HS = 0,$$
$$X_2PW - 0.89X_2HW = 0,$$
$$X_3PS - 0.89X_3HS = 0,$$
$$X_3PW - 0.89X_3HW = 0.$$
$$X_4W - PDW\,y_1 = 0,$$
$$X_1PW - PDW\,y_2 = 0,$$
$$X_2PW - PDW\,y_3 = 0,$$
$$X_3PW - PDW\,y_4 = 0,$$
$$y_1 + y_2 + y_3 + y_4 = 1,$$
$$X_4S - PDSj_1 = 0,$$
$$X_1PS - PDSj_2 = 0,$$
$$X_2PS - PDSj_3 = 0,$$
$$X_3PS - PDSj_4 = 0,$$
$$j_1 + j_2 + j_3 + j_4 = 1,$$

or

$$
\begin{aligned}
X_5W - HDW\,y_1 &= 0,\\
X_1HW - HDW\,y_2 &= 0,\\
X_2HW - HDW\,y_3 &= 0,\\
X_3HW - HDW\,y_4 &= 0,\\
y_1 + y_2 + y_3 + y_4 &= 1,\\
X_5S - HDS\,j_1 &= 0,\\
X_1HS - HDS\,j_2 &= 0,\\
X_2HS - HDS\,j_3 &= 0,\\
X_3HS - HDS\,j_4 &= 0,\\
j_1 + j_2 + j_3 + j_4 &= 1.\\
y_1 - j_1 &= 0,\\
y_2 - j_2 &= 0,\\
y_3 - j_3 &= 0,\\
y_4 - j_4 &= 0,\\
\text{XiHS} &\geq 0 \quad \forall i,\\
X_iHW &\geq 0 \quad \forall i,\\
X_iPS &\geq 0 \quad \forall i,\\
\text{XiPW} &\geq 0 \quad \forall i,\\
X_4W &\geq 0,\\
X_4S &\geq 0,\\
X_5W &\geq 0,\\
X_5S &\geq 0,
\end{aligned}
$$

where y_1, y_2, y_3, y_4 and j_1, j_2, j_3, j_4 are defined as integers.

In case of selling the excess electricity

Depending on the enterprise situation, they might be able to sell the excess of the electricity. So the model should be modified by adding new decision variables and new constraints.

The two variables (X_6 and X_7), which represent the excess of the electrical energy in winter and summer respectively, should be subtracted from the objective function equation.

And the new constraints are: $X_1PW + X_2PW + X_3PW – X_6 = PDW$ and $X_1PS + X_2PS + X_3PS – X_7 = PDS$.

Choosing the optimum situation

As mentioned before, the model should be run twice; firstly aiming to fulfil the heat demand, and secondly to fulfil the power demand. Then the user can decide referring to his/her criteria about the system that will meet the organizational requirements.

Depending on the strategic objective of the user; other factors might affect the choice of the most excellent situation. The quality of the service, the reliability of the system, the working environment and many other intangible factors should be considered.

In addition, the possibility of selling electricity can also affect the decision-maker in the enterprise in adopting his/her priorities to install and manage the system on the ground.

Why linear programming?

Linear programming (LP) is one of the most widely used techniques in management science. It is primarily concerned with the determination of the best allocation of scarce resources and aims to optimize the decisions either by maximizing the benefit or minimizing the costs.

In cogeneration application, a decision should be made to choose the most adequate system on the site and how this system should be managed; LP model can solve this problem and guide the user to the optimum choice and the best approach to manage the chosen system by minimizing the costs of the energy consumed at the enterprise.

Results

Having formulated the distribution problem as a linear programming problem, the next step is to implement the model on a real case which was a hospital in Nablus (Nablus Speciality Hospital). LINDO software (developed by LINDO Systems, Inc.) was used for solving the model.

The output of the model is shown in Figs. 1 and 2.

And from the results shown in Figs. 1 and 2, it is worthy for the strategic management in the hospital to go for case 1 (fulfilling the heat demand) and take the investment decision. In this case the total saving in the course of implementing cogeneration system is around $51,000. And as a result, a considerable saving in pollution emissions has been gained; Table 1 summarizes the total saving.

The solution of the LP model from LINDO:

LP OPTIMUM FOUND AT STEP 16

OBJECTIVE VALUE = 49044.9922

VARIABLE	VALUE	REDUCED COST
Y4	1.000000	30280.634766
J4	1.000000	-5585.421875
X3HS	530561.81250	0.000000
X3HW	856184.25000	0.000000
X3PS	472200.00000	0.000000
X3PW	762004.00000	0.000000

Fig. 1 In case of fulfilling heat demand

The solution of the LP model from LINDO:
LP OPTIMUM FOUND AT STEP 18
OBJECTIVE VALUE = 65188.9141

VARIABLE	VALUE	REDUCED COST
Y4	1.000000	4578.960449
J4	1.000000	-13525.450195
X3HS	129600.000000	0.000000
X3HW	129470.000000	0.000000
X3PS	115344.000000	0.000000
X3PW	115228.296875	0.000000
X4S	356856.000000	0.000000

Fig. 2 In case of fulfilling power demand

Table 1 Total saving

Gas type	Total saving (tonnes/year)
CO_2	23.5872
NO_x	–0.19327
SO_2	6.882978

Conclusion

The problem of the combined heat cogeneration (CHP) system is large and complex that the existing engineering tools alone cannot provide good solutions. Within a set of resources, the use of management decision techniques such as linear mathematical modeling can help in satisfactorily solving some of these problems in a systematic way. In this paper, we have investigated a method of CHP optimal design from the economical and environmental viewpoints. This method uses a mathematical linear programming model to optimize the decision-making for managing a cogeneration facility as a potential clean-developed mechanism. The model was designed to optimize the cost of energy and the cost of installation of a small cogeneration plant under constraints on electricity-and-heat supply and demand balances. The model was developed to check the sources of electricity, either from cogeneration or public utilities. It was calculated the least cost to supply electricity and heat to a hospital in Palestine. The results showed that about US $51,000 will be the savings for the hospital.

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