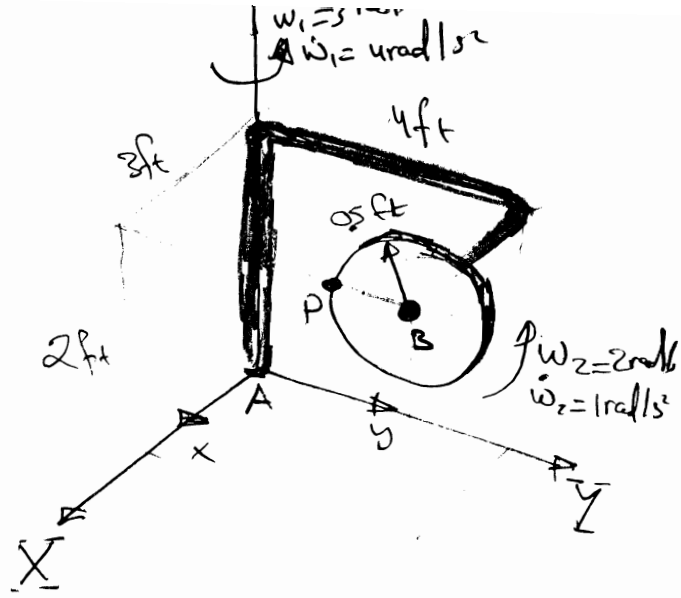
Problem 20-44



$\vec{V}_P$; $\vec{a}_P$

$\vec{\omega} = \vec{\omega}_1 + \vec{\omega}_2$

$\vec{\omega} = (3k + 2i)$ rad/s .

$\dot{\vec{\omega}} = \dot{\vec{\omega}}_1 + \dot{\vec{\omega}}_2$

- to calculate $\dot{\vec{\omega}}_2$ we attach (x,y,z) frame to the rod that rotates with it.

$\Rightarrow \omega_2$ has a constant direction in $(x,y,z) \equiv i$ & $\vec{\Omega} = \vec{\omega}$ of the rod $= \vec{\omega}_1$

$\Rightarrow \dot{\vec{\omega}}_2 = (\dot{\vec{\omega}}_2)_{xyz} + \vec{\omega}_1 \times (\vec{\omega}_2)_{xyz}$

$= 1i + (3k \times 2i) = 1i + 6j$ rad/s²

- to calculate $\dot{\vec{\omega}}_1$ we attach to the base a new frame x', y', z', in this frame ω_1 has a constant direction ; $\vec{\Omega} = \vec{\omega}_{base} = 0$.

$\Rightarrow \dot{\vec{\omega}}_1 = (\dot{\vec{\omega}}_1)_{xyz} + \vec{\Omega} \times \vec{\omega}_1 = 4k$ rad/s²

$\Rightarrow \dot{\vec{\omega}} = 4k + i + 6j = (i + 6j + 4k)$ rad/s².

- point B is located at the end of the rod (moves with it)

i.e, rod AB is one body

$\Rightarrow \vec{V}_B = \vec{V}_A + \vec{\omega}_{AB} \times \vec{r}_{B/A}$; $\vec{\omega}_{AB} = \vec{\omega}_1 = 3k$ rad/s

$\dot{\vec{\omega}}_{AB} = \dot{\vec{\omega}}_1 = 4k$ rad/s² as calculated before

$\vec{V}_B = 0 + 3k \times [3i + 4j + 2k]$

$\vec{V}_B = (-12i + 9j)$ m/s

$\vec{a}_B = \vec{a}_A + \dot{\vec{\omega}}_{AB} \times \vec{r}_{B/A} + \vec{\omega}_{AB} \times (\vec{\omega}_{AB} \times \vec{r}_{B/A}) = 0 + 4k \times (3i + 4j + 2k) + 3k \times [-12i + 9j]$

$\vec{a}_B = -16i + 12j - 27i - 36j = (-43i - 24j)$ m/s²

$\vec{V}_P = \vec{V}_B + \vec{\omega} \times \vec{r}_{P/B} = -12i + 9j + (3k + 2i) \times (-0.5j) = (-10.5i + 9j - k)$ m/s

$\vec{a}_P = \vec{a}_B + \vec{\alpha} \times \vec{r}_{P/B} + \vec{\omega} \times (\vec{\omega} \times \vec{r}_{P/B})$

$= -43i - 24j + (i + 6j + 4k) \times (-0.5j) + (3k + 2i) \times [1.5i - k]$

$= (-4i - 17.5j - 0.5k)$ m/s²