An-Najah National University
Faculty of Graduate Studies
Faculty of Science-Department of Physics
Quantum Mechanics I
Spring 2014
1^{st} Exam, Mar. 5^{th} 2014

Name: _____

Number: _____

1. Vectors $|a>$ and $|b>$ belong to a certain abstract vector space such that:

$$|a><a| + |b><b| = 1$$

   (a) (2 points) What is the dimension of the space
   (b) (4 points) Find $\mathrm{Tr}(e^{|a><a|})$
   (c) (2 points) Find $\left[e^{|a><a|}, e^{|a><b|}\right]$

2. Consider a physical system whose three-dimensional state space is spanned by the ortho-normal basis formed by the three kets $|u_1>, |u_2>, |u_3>$. In this basis, the Hamilton operator H and the observable A are written as:

$$H = \hbar\omega_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix} \qquad A = a \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

   where ω_0 and a are positive real constants. The physical state at time t= 0 is the state:

$$|\psi(t=0)> = \frac{1}{\sqrt{2}}|u_1> + \frac{1}{2}|u_2> + \frac{1}{2}|u_3>$$

   (a) (3 points) If operator A is measured at t=0, what values we might get and with what probability.
   (b) (2 points) What is $|\psi(t)>$
   (c) (2 points) What is $<A(t)>$

3. (5 points) Consider a spin-1/2 particle with magnetic moment $\mu = \gamma\sigma$ in a uniform magnetic field that points in the z-direction. If at time t=0 the x-component of the spin as measured and were found to be $\frac{+\hbar}{2}$. At time t, y-component of the spin as measured and were found to be $\frac{+\hbar}{2}$, what is t?

4. A quantum mechanical system is known to posses only two energy eigenstaes denoted $|1>$ and $|2>$. The system also includes three other observables **P**, **Q**, and **R**. The eignestates $|1>$ and $|2>$ are normalized but not necessary to be eigenstates for the observables **P**, **Q**, and **R**. A series of experiment were performed and the following results were obtained:

   1. Experiment 1: $<1|P|1> = \frac{1}{2}$, $<1|P^2|1> = \frac{1}{4}$
   2. Experiment 2: $<1|Q|1> = \frac{1}{2}$, $<1|Q^2|1> = \frac{1}{6}$
   3. Experiment 3: $<1|R|1> = \frac{1}{2}$, $<1|R^2|1> = \frac{5}{4}$, $<1|R^3|1> = \frac{7}{4}$

   (a) (2 points) One of the experiment yields an unphysical results, which one?
   (b) (3 points) Find as many as possible eigenvalues of P and Q as possible, from the given information

Question	Points	Score
1	8	
2	7	
3	5	
4	5	
Total:	25	

Good Luck